Risk, Incentives, and Contracting Relationships

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Abstract

The aim of this paper is to understand the impact of optimal provision of both risk and incentives on the choice of contracting partners. I study a risky setting where heterogeneously risk-averse employers and employees must match to be productive. They face a standard one-sided moral hazard problem: mean output increases in the noncontractible input of the employee. Better insurance comes at the cost of weaker incentives, and this tradeoff differs across partnerships of different risk compositions. I show that this heterogeneous tradeoff determines the equilibrium matching pattern, and focus on environments in which assortative matching is the unique equilibrium. This endogenous matching framework enables a concrete and rigorous analysis of the interaction between formal and informal insurance. In particular, I show that the introduction of formal insurance crowds out informal insurance, and may leave those individuals acting as informal insurers in the status quo strictly worse off.

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1 Introduction

Risk is an unavoidable feature of life, but its pervasiveness and the methods which exist to manage it differ in developed and in developing countries. People in developing countries face high levels of risk: disease is widespread, the climate is punishing, and occupations are hazardous (Dercon (2005), Fafchamps (2008)). Instead of steady salaries, incomes are typically highly variable and depend on factors beyond individual control. For example, a farmer's livelihood is bound to the whims of nature, the vagaries of health, and the caprice of crop prices.

In addition, people in developing countries are more vulnerable to the risks they face. They live close to or at subsistence levels, and thus have no buffer with which to cushion negative shocks. Moreover, the formal insurance and credit institutions, credibly backed and enforced by stable governance and legal systems, which are available to people in developed countries, are notably absent in developing environments (Dercon (2005), Fafchamps (2008)).

Consequently, the poor must depend on creative ways of incorporating risk protection into their interactions with each other (Alderman and Paxson (1992), Morduch (1995), Dercon (2005), Fafchamps (2008)). For example, two farmers working together in a cropping group to grow a risky harvest could smooth each other's consumption by agreeing to a sharing rule of the income from the realized harvest. These farmers could also try to establish sharing rules with outsiders, but committing to a division of pooled income becomes prohibitively difficult if the contracting parties do not jointly observe or cannot easily verify income. As a result, the two farmers have compelling reasons to adapt their working partnership with each other to accommodate risk concerns. This "costly state verification" (Townsend (1979)) often causes subsets of people ostensibly matched for other purposes to incorporate risk management into their existing relationships. (See Townsend and Mueller (1998) for examples of costly state verification and informal insurance in the Indian village of Aurepalle.) Hence, the absence of formal institutions not only induces individuals to rely on their relationships with each other, it also causes these relationships to address *layers* of needs. These relationships and the arrangements within them operate outside of formal insurance and credit channels and can therefore be thought of as "informal institutions". Indeed, Bardhan relates in his 1980 paper on interlocking agrarian factor markets: "Generalizing from his experience with the hill peasants of Orissa, Bailey (1966) notes: 'The watershed between traditional and modern society is exactly this distinction between single-interest and multiplex relationships.'"

In this paper, I study the implications of this multidimensionality of interpersonal relationships arising from the absence of formal institutions by focusing specifically on the influence of missing formal insurance institutions on the equilibrium formation of productive partnerships between risk-averse people and subject to one-sided moral hazard. A classic example is sharecropping: a farmer lives on and crops a plot belonging to a landowner and pays rent as a share of the realized harvest. Sharecropping has thrived in different parts of the world for centuries, and continues to be prevalent in parts of the world today, for example in rice farms in Bangladesh (Akanda et. al. (2008)) and in Madagascar (Bellemare (2009)). If inputs (including effort) are difficult to monitor and are noncontractible, however, the contract that provides first-best incentives for the farmer should be the contract that makes the farmer the residual claimant (the farmer should pay a fixed rent). So why the share contract? Stiglitz (1974) suggested that the share contract emerged because employment relationships also provided protection from risk, in the absence of formal insurance: while a fixed rent contract would induce the right incentives, it would force the agent to bear the full risk of the stochastic harvest. A fixed wage contract would fully insure the agent, but the agent would have no incentive to exert costly effort.

However, while the effect of the tradeoff between incentive and insurance provision on a contract between a given pair of principals and agents has been studied extensively, the effect of this tradeoff on the *formation* of employment relationships has received far less attention. This latter analysis is essential for rigorously understanding the true strength of informal insurance–that is, the level of risk-sharing achieved within the network of equilibrium employment relationships which emerges in the absence of formal insurance and credit institutions, and not merely the risk-sharing achieved with a single, isolated group of individuals.

To perform this analysis rigorously, I build a model of endogenous one-to-one matching between heterogeneously risk-averse principals and agents who face a classic moral hazard problem. Principals each own a unit of physical capital, but have infinite marginal cost of effort, while agents own zero units of physical capital, but have the same finite marginal cost of effort. Physical capital must be combined with human capital in order to produce output. For example, in a sharecropping setting, landowners would be principals, farmers would be agents, and land and farming expertise would need to be combined to produce any sort of harvest.

The distribution of output depends additively on the unobservable and noncontractible effort exerted by the agent, as well as on the level of riskiness of the environment. More specifically, an increase in effort exerted by the agent leads to an increase in mean output but has no impact on the variance, while the level of riskiness of the environment determines the variance of output. A matched principal and agent can commit *ex ante* to a return-contingent sharing rule of their jointly-produced output.

The difficulty of characterizing the equilibrium wage schedule in moral hazard models in which both the principal and the agent are risk-averse (and have differing risk attitudes) is well-known. Instead of using a Holmstrom and Milgrom (1987) type story where the principal observes some coarser aggregate of output than the agent does to justify linear contracts in a CARA-Normal framework, I develop a model of one-sided moral hazard in which principals and agents have CARA utility and returns are distributed Laplace. The Laplace distribution has two key features: first, it resembles the normal distribution (for example, it is symmetric about the mean), but has fatter tails, and second, its likelihood ratio is a piecewise constant with discontinuity at the mean. (Please see Appendix 1 for further details of the Laplace distribution.) This framework is optimally suited to analyzing the formation of equilibrium networks of relationships subject to one-sided moral hazard and risk-sharing, since the equilibrium wage cleanly separates incentive and insurance provision. The equilibrium wage schedule is piecewise linear with discontinuity at the mean, where the linearity at output levels away from the mean captures efficient risk-sharing between the risk-averse principal and agent, and the discontinuity at the mean captures incentive provision. (Please see Appendix 1 for a detailed proof and discussion of this result.)

I characterize sufficient conditions for the unique equilibrium matching between principals and agents to be negative-assortative in risk attitude (the i^{th} least risk-averse principal works with the i^{th} most risk-averse agent, and so on), and for the unique equilibrium matching to be positive-assortative in risk attitude (the i^{th} least risk-averse principal works with the i^{th} least risk-averse agent). Intuitively, the equilibrium risk composition of partnerships is determined by the tradeoff between incentive and insurance provision in the following way: a less risk-averse principal who hires a more risk-averse agent can charge that agent a risk premium for insurance, but the more that the principal insures the agent, the less effort that agent will exert. Moreover, a more risk-averse agent has a higher effective marginal cost of effort in the first place (because a more risk-averse agent is "more concerned" about the possibility of exerting high effort but being unlucky and realizing a low output draw).

I show that the key determinants of equilibrium group composition are: the curvature of the cost of effort function, the riskiness of the environment, and the distribution of risk types in the economy. The curvature of the cost of effort function influences the equilibrium match because it characterizes the tradeoff between insurance and incentive provision across partnerships of different risk attitudes. Consideration of an extreme example provides the intuition. Suppose the cost of effort function were "infinitely convex". Then, regardless of risk attitude, any agent would exert near zero effort. But this means that no agent is cheaper to incentivize than any other agent–that is, all agents are equally expensive to incentivize. Hence, from the principals' perspective, incentive provision is effectively held fixed across agents of different risk attitudes, and the matching becomes driven purely by insurance provision. So, the equilibrium match will be negative-assortative in risk attitudes, since the less risk-averse principals are differentially more willing to provide insurance to the more risk-averse agents, who are differentially more willing to purchase it. As the cost of effort function becomes less extreme, however, the less risk-averse agents become notably cheaper to incentivize than the more risk-averse agents, and positive-assortative matching may arise.

The riskiness of the environment is a particularly interesting determinant of the equilibrium match. It influences the matching in two important ways: first, the safer the environment is, the lower the premium for insurance. Second, the safer the environment, the more informative output is as a signal of agent effort. In other words, insurance provision becomes less differentiated across agents of different risk attitudes when the environment is safer, because even the more risk-averse agents don't need so much insurance, while incentive provision more sharply differentiates across agents of different risk attitudes when the environment is safer, because there is less room for hidden action.

Finally, the distributions of risk types in the group of principals and in the group of agents affect the equilibrium match[1]. This is because effort is supplied only by the agent, and not by the

[1] This is in contrast with the distribution-free result from Wang (2012a) which studied the impact of the trade-off between two informal insurance techniques (income-smoothing and consumption-smoothing) on the equilibrium formation of relationships.

principal–hence, the risk attitude of a principal has a different effect on equilibrium effort than does the risk attitude of an agent. The one-sidedness of moral hazard generates an asymmetry in the model which makes the distribution of risk types important for matching: equilibrium partnerships depend both on "within group heterogeneity", that is, how much risk attitudes vary among principals, and how much risk attitudes vary among agents, as well as "across group heterogeneity", that is, how different the risk attitudes of principals are form the risk attitudes of agents.

Loosely, the features of the environment in which negative-assortative matching emerges as the unique equilibrium are: a cost of effort function which is either close to linearity or is extremely convex, a highly risky environment (output is a very noisy signal of effort, and the premium paid for insurance is high), and a group of principals who are distinctly more risk-averse than the agents. By contrast, the features of the environment where positive-assortative matching emerges as the unique equilibrium are: a "moderately convex" cost of effort function, a safe environment (output is a quite precise signal of effort, and the premium paid for insurance is low), and a group of principals who are distinctly less risk-averse than the agents. To be slightly more specific, the distribution of risk types influences the equilibrium match by affecting the bounds which delineate these "close to linear", "moderatey convex", and "extremely convex" descriptors of the curvature of the cost of effort function.

After establishing the main theoretical results, I work through a numeric example which demonstrates the policymaking importance of understanding the risk composition of the equilibrium network of partnerships: evaluating policy requires understanding how people will re-optimize upon its introduction, and understanding how people will re-optimize in response to the introduction of a policy requires understanding how people optimize in the status quo. That is, in order to predict what effect the introduction of a formal institution would have, it is necessary first to have a complete picture of the operations of the equilibrium informal institutions. For example, I discuss the impact of introducing formal insurance. I show that such a policy leads to a "crowding out" of informal insurance, and in particular may leave the providers of informal insurance worse off. This framework of endogenous matching is exactly suited to analyzing concretely and rigorously the "crowding out" effect, which is often discussed in the literature.

A small body of existing literature examines the relationship between the insurance and incentive provision tradeoff and the endogenous formation of contracting relationships under one-sided moral hazard. Ghatak and Karaivonov (2013) develop a model of endogenous pairwise matching between risk-neutral landlords and tenants who are heterogeneous in ability and must work together to be productive. As in the sharecropping model of Eswaran and Kotwal (1985), landlords specialize in managerial tasks while tenants specialize in labor tasks. Production requires the completion of both kinds of tasks, and inputs are not contractible. A landlord can complete both tasks herself, or she can "sell the farm" to the tenant, who then completes both tasks, or she can offer the tenant a sharecrop contract, and they each complete the task of their specialty. In the first-best, the equilibrium match is positive-assortative in ability, but, if the agent's abilities are substitutes or weak complements in production, Ghatak and Karaivonov show that the equilibrium matching becomes negative-assortative in the second-best. (With sufficient complementarity, the equilibrium matching is positive-assortative in the second-best.)

Franco et. al. (2011) consider a risk-neutral principal who requires two (risk-neutral) agents to operate her machinery, where agents differ in their marginal cost of effort (high or low). The principal must decide what teams of agents to form–low marginal cost with low marginal cost (positive-assortative), or low with high (negative-assortative). Moral hazard is double-sided–the key new force behind the matching decision is the principal's inability to condition compensation on an individual agent's contribution. The main result is that the super/submodularity of the production technology in workers' inputs no longer drives assortative matching in the presence of this double-sided moral hazard. When the production technology is modular, so that in the absence of moral hazard there is no matching prediction, Franco et. al. show that the presence of moral hazard can still lead to positive- or negative-assortative matching, depending on the optimal compensation scheme, which depends on the types and output of the team. For example, if types exerting higher input are rewarded more according to the scheme, then negative matching is optimal, to reduce the likelihood of "accidental payment", that is, paying an individual who exerts low effort for the high effort exerted by his partner. When the technology exhibits complementarities, a scenario is described where increasing complementarity does not lead to positive-assortative matching because of the double-sided moral hazard.

Serfes (2008) studies a setting where risk-neutral principals owning exogenously-assigned projects match with risk-averse agents, where the projects of principals vary in riskiness (riskier projects have higher mean and higher variance), and the risk aversion of agents also varies. A principal and an agent jointly produce output, where output depends additively on unobservable and noncontractible effort exerted by the agent. Serfes shows that the equilibrium match is often not globally assortative in risk attitude, and that the relationship between the riskiness of the environment and the power of the contract can be ambiguous, due to the endogeneity of matching.

Importantly, the approach to studying the tradeoff between incentive and insurance provision in this paper is markedly different from approaches in papers such as the one by Serfes, where principals are assumed to be risk-neutral, but own projects which vary in riskiness. While such a model generates a tractable equilibrium wage (because risk-neutrality ensures linearity), the endogenous assignment problem of heterogeneously risk-averse agents to risk-neutral principals with heterogeneously risky projects is really answering the question, "What risky project is a risk-averse agent assigned to when the output of the project depends on the noncontractible effort of the agent, and a risk-neutral insurer is available who sells insurance?" That is, this approach can be thought of as focusing on the impact of a formal institution on equilibrium activities undertaken in a village, when there is some sort of monitoring problem. By contrast, my paper builds a model of endogenous matching between heterogeneously risk-averse principals and heterogenously risk-averse agents, where only one project is available (for example, wheat is the only crop that can be grown). This approach focuses on the emergence, structure, and performance of informal insurane institutions in the status quo. Hence, the first question that this paper answers is, "How well-insured are heterogenously risk-averse individuals when a lack of formal institutions pushes their interpersonal relationships to address multiple needs, including the need for risk protection?"

Legros and Newman (2007) studied more generally the problem of endogenous matching under nontransferable utility. They present techniques to characterize stable matchings in nontransferable utility settings by generalizing the Shapley and Shubik (1972) and Becker (1974) supermodularity and submodularity conditions for matching under transferable utility. Under nontransferable utility, the indirect utility of each member of the first group given a partnership with each member of the second group can be calculated, fixing the second member's level of expected utility at some level v. Then, this indirect utility expression, which depends on both members' types and v, is analyzed for supermodularity and submodularity in risk types.

A number of papers have attempted to empirically detect the inverse relationship between the riskiness of the environment and the power of the contract predicted by the basic principal-agent model with risk and moral hazard. For example, Allen and Lueck (1992) study sharecropping relationships in the American Midwest in 1986, and observe that the strength of dependence of a sharecropper's rent contract on realized harvest is *positively* correlated with the riskiness of the crop grown. From this, they conclude that risk is not a problem for farmers, since risk considerations do not appear to influence contract design. However, if more risk-averse farmers work for less risk-averse landowners, cultivating safer crops under low-powered contracts, while less risk-averse farmers work for more risk-averse landowners, cultivating riskier crops under high-powered contracts, the same empirical observations would emerge, but risk concerns would be playing a significant role in contract design, through the unaccounted-for channel of contracting partner choice. Alternatively, we expect formal institutions to be stronger in the United States–hence landowner-farmer relationships may not be as multidimensional as they are in developing countries. This would imply that this is not the right dataset to test for the theoretically-predicted relationship between risk and incentives.

The remainder of the paper proceeds as follows. The next two sections present the model and the results. Section 4 works through a hypothetical policy example and shows that the evaluation of policy may change drastically if it accounts for the endogenous network response. Section 5 concludes.

2 The Model

In this section, I introduce a framework designed to analyze the formation of equilibrium contracting relationships under one-sided moral hazard when the contract balances insurance provision with incentive provision.

The framework consists of the following elements:

The population of agents: the economy is populated by two groups of agents, $G1$ and $G2$, where $|G1| = |G2| = Z$, Z a finite, positive integer. Call members of $G1$ "principals" and members of $G2$, "agents". All principals own one unit of physical capital, but have infinite marginal cost of effort; all agents lack physical capital but have finite and identical marginal costs of effort. The cost of effort for all agents is $c(a)$, $c(a) > 0$, $c'(a) > 0$, $c''(a) > 0$.

Principals and agents both have CARA utility, $u(x; r) = -e^{-rx}$, where individuals differ in their degree of risk aversion. Let r_1 represent a principal, and r_2 represent an agent. Principals and agents are identical in all other aspects. There are no assumptions on distributions of risk types.[2]

The risky environment: A principal-agent partnership can only produce positive output if one unit of physical capital is combined with human capital. For example, a landowner owns land, and a farmer has agricultural experience and skill, and a successful harvest requires the landowner and farmer to combine their capital. Output is given by $R|a = \gamma a + \varepsilon$, where the riskiness of the environment is captured by $\varepsilon \sim f_\varepsilon$, an exogenously-given, well-defined, differentiable probability distribution function with support on $(-\infty, \infty)$. The effort of an agent, a, increases the mean but doesn't affect the variance of returns.

Information and commitment: All agents know each other's risk types. In the first-best environment, an agent's effort is both observable and contractible. In the second-best environment, the agent's effort is neither observable nor contractible.

A given matched pair (r_1, r_2) observes the realized output of their partnership, and is able to commit *ex ante* to a return-contingent sharing rule $s(R_{p_{12}})$, where $R_{p_{12}}$ is the realized return of (r_1, r_2)'s joint project p_{12}. More precisely, $s(R_{p_{12}})$ specifies the wage paid to the agent r_2 when the realized return is $R_{p_{12}}$, where $s : \mathbb{R} \to \mathbb{R}$ (there are no limited liability assumptions). In order to be *feasible*, the income the principal r_1 receives must be less than or equal to $R_{12} - s(R_{p_{12}})$. Since all individuals have monotonically increasing utility, r_1's share will be equal to $R_{12} - s(R_{p_{12}})$.

The equilibrium: An equilibrium is:

1. A match function $\mu(r_1) = r_2$, where $\mu(\cdot)$ assigns each r_1 to at most one agent r_2, and distinct people have distinct partners.

   Moreover, the matching pattern described by $\mu(\cdot)$ must be stable. That is, it must satisfy two properties:

   (a) *No blocks*: no unmatched principal and agent should be able to write a feasible wage contract such that both of them are happier with each other than they are with the partners assigned to them by μ.

   (b) *Individual rationality*: each agent must receive a higher expected utility from being in the match $\mu(\cdot)$ than from remaining unmatched.

[2] Of course, in reality, types are multidimensional, and matching decisions are not exclusively based on risk attitudes. It is worth noting that the model can account for this. For example, kinship and friendship ties are important, in large part because of information (they know each other's risk types), and commitment (they trust each other, or can discipline each other). Kinship and friendship ties would enter into this theory in the following way: an individual would first identify a pool of feasible risk-sharing partners. This pool would be determined by kinship and friendship ties, because of good information and commitment. Following this, individuals would choose risk-sharing partners from these pools. This choice would be driven by risk attitudes, as addressed in this benchmark with full information and commitment.

Thus, this theory can be thought of as addressing the stage of matching that occurs after pools of feasible partners have been identified.

2. A set of sharing rules and effort choices by the agents, one sharing rule and one effort choice for each matched pair. The agent chooses an effort level which is optimal for her–she should not be able to choose a different effort level and become better off. Furthermore, no pair should be able to choose a different sharing rule which leaves both partners weakly better off, and at least one partner strictly better off (the agent chooses effort optimally in response to the sharing rule).

Matching patterns: It will be helpful to introduce some matching terminology. Suppose the people in $G1$ and in $G2$ are ordered from least to most risk-averse: $\{r_1^j, r_2^j, ..., r_Z^j\}$, $j \in \{1, 2\}$. Then "positive-assortative matching" (PAM) refers to the case where the i^{th} least risk-averse person in $G1$ is matched with the i^{th} least risk-averse person in $G2$: $\mu(r_i^1) = r_i^2$, $i \in \{1, ..., Z\}$. On the other hand, "negative-assortative matching" (NAM) refers to the case where the i^{th} least risk-averse person in $G1$ is matched with the i^{th} *most* risk-averse person in $G2$: $\mu(r_i^1) = r_{Z-i+1}^2$, $i \in \{1, ..., Z\}$. To say that the unique equilibrium matching pattern is PAM, for example, is to mean that the only μ which can be stable under optimal within-pair sharing rules and projects is the match function which assigns agents to each other positive-assortatively in risk attitudes.

3 The Results

3.1 The First-Best

It will be useful to begin by solving the first-best problem, when the agent's effort is observable and contractible.

The first step to characterizing the equilibrium network of relationships is to characterize what happens in a given relationship. Suppose principal r_1 is matched with agent r_2, and that the returns of the risky project R are distributed according to some general density function f:

$$R|a = \gamma a + \varepsilon, \; \varepsilon \sim f(\varepsilon)$$

Assume the random variable R has a well-defined cumulant generating function[3].

Denote the agent's share of realized output R by $s(R)$. Then, the pair's equilibrium sharing rule, given that r_2 receives expected utility at least $-e^{-v}$ for some fixed level v, solves the following problem:

$$\max_{a,s(R)} \int_{-\infty}^{\infty} - e^{-r_1[R-s(R)]} f(R-\gamma a) dR \; s.t.$$

$$(IR) \int_{-\infty}^{\infty} -e^{-r_2[s(R)-c(a)]} f(R-\gamma a) dR \geq -e^{-v}$$

The equilibrium sharing rule and effort are described in the following lemma.

[3] Recall that the cumulant-generating function is the log of the moment-generating function.

Lemma 1 *The optimal first-best contract of a principal-agent pair* (r_1, r_2)*, where* $s(R)$ *denotes the agent's share, is:*

$$\begin{aligned} s^*_{FB}(R) &= \frac{r_1}{r_1+r_2}R + K^*_{FB}(r_1, r_2, v) \\ K^*_{FB}(r_1, r_2, v) &= \frac{1}{r_2}v + c(c'^{-1}(\gamma)) + \frac{1}{r_2}\log\left(\int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1+r_2}R} f(R-\gamma c'^{-1}(\gamma))dR\right) \\ a^*_{FB} &= c'^{-1}(\gamma) \end{aligned}$$

(The proof is in Appendix 2.)

The first-best contract has several notable features. First, equilibrium effort level is the same in any possible pair–this is because effort is contractible. Hence, any principal-agent pair chooses the effort which "maximizes the pie", and then efficiently shares the risk of that pie. The effort which "maximizes the pie" is the effort level which equates the marginal benefit of effort (the marginal impact on mean output, γ) with the marginal cost of effort exertion, $c'(\cdot)$.

Second, the equilibrium wage is linear. This is unsurprising–again, effort is contractible, so the sharing rule needs only to provide insurance, and not incentives. Moreover, the less risk-averse individual receives a share that is more heavily dependent on output realization.

Now that we have characterized the optimal sharing rule and equilibrium effort within a matched pair r_1 and r_2, we can solve for the equilibrium network of relationships. Intuitively, since we know that any possible matched pair chooses the same effort level, we expect negative-assortative matching to arise as the unique equilibrium. This is because we know from Schulhofer-Wohl and Chiappori and Reny that endogenous matching under pure *ex post* risk management results in unique negative-assortative matching (there is no moral hazard, and no scope for *ex ante* risk management–matched pairs are not able to choose what risk they face). Negative-assortative matching arises because the least risk-averse individuals are differentially willing to provide insurance, while the most risk-averse individuals are differentially willing to pay for it.

To formalize this intuition, we need to identify a method for characterizing the equilibrium match. A challenge is posed by the heterogeneity of risk-aversion in agents, which makes this a model of matching under nontransferable utility. That is, the amount of utility experienced by an agent with risk aversion r_1 from consuming one unit of output differs from the amount of utility an agent with risk aversion r_2 experiences from one unit of output. Thus, we cannot directly apply the Shapley and Shubik (1962) result on sufficient conditions for assortative matching in transferable utility games.

It will be helpful to review briefly that environment and result. Consider a population consisting of two groups of risk-neutral workers, where all workers have utility $u(c) = c$. Let a_1 denote the ability of workers in one group, and a_2 denote the ability of workers in the other group. The production function is given by $f(a_1, a_2)$, which can be thought of as: "the size of the pie generated by matched workers a_1 and a_2". Then, $\frac{d^2 f}{da_1 da_2} > 0$ is a sufficient condition for unique positive-assortative matching, and $\frac{d^2 f}{da_1 da_2} < 0$ is a sufficient condition for unique negative-assortative matching.

My approach here will be to identify the function in this model of nontransferable utility which is analogous to the Shapley and Shubik production function $f(a_1, a_2)$. In Proposition 2 below, I prove that *expected utility* is transferable in this model–instead of thinking about moving "*ex post*" units of output between agents, we should instead think about moving "*ex ante*" units of expected utility. I show that the sum of the certainty-equivalents $CE(r_1, r_2)$ of a given matched pair (r_1, r_2) is the analogy to the joint output production function in the transferable utility problem. The sum of the certainty-equivalents of a matched pair is "the size of the expected utility pie generated by matched agents r_1 and r_2", and sufficient conditions for positive-assortative and negative-assortative matching correspond to conditions for the supermodularity and submodularity of $CE(r_1, r_2)$ in r_1, r_2.

More technically, expected utility is transferable in this model because the expected utility Pareto possibility frontier for a pair (r_1, r_2) is a line with slope -1 under some monotonic transformation.

Proposition 2 *Expected utility is transferable in this model.*

Proof. Using the optimal sharing rule and equilibrium effort from Lemma 1, we can write the expressions for the certainty-equivalent of a principal r_1 and of an agent r_2 who are matched with each other:

$$CE_{r_1} = -\frac{v}{r_2} - \left(\frac{1}{r_1} + \frac{1}{r_2}\right) \log E\left[e^{-\frac{r_1 r_2}{r_1+r_2}R} | a^* = c'^{-1}(\gamma)\right] - c(c'^{-1}(\gamma))$$

$$CE_{r_2} = \frac{v}{r_2}$$

Hence, it is clear the cost to the principal r_1 of increasing the certainty-equivalent of her agent r_2 by one unit is exactly one unit (and vice versa). That is, expected utility is transferable in the model. ∎

This tells us that the sum of certainty-equivalents for a matched principal and agent in this model can be thought of as the function which is analogous to the joint output function in the Shapley and Shubik transferable utility setting:

$$CE(r_1, r_2) = -\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \log E\left[e^{-\frac{r_1 r_2}{r_1+r_2}R} | a^* = c'^{-1}(\gamma)\right] - c(c'^{-1}(\gamma))$$

It will be helpful to make two observations at this point. First, we can define the *representative risk aversion* of a matched pair (r_1, r_2):

$$\hat{H}(r_1, r_2) = \frac{r_1 r_2}{r_1 + r_2}$$

Importantly, we can see that the sum of certainty-equivalents of a matched pair depends *only* on representative risk aversion. In other words, a matched pair (r_1, r_2) acts as a single individual with CARA utility and absolute risk aversion $\hat{H}(r_1, r_2)$.

And correspondingly, we can define the reciprocal of representative risk aversion, the *representative risk tolerance*:

$$\begin{aligned}\widetilde{H}(r_1, r_2) &= \frac{1}{\hat{H}(r_1, r_2)} \\ &= \frac{1}{r_1} + \frac{1}{r_2}\end{aligned}$$

Thus, the sum of certainty-equivalents written as a function of representative risk tolerance is:

$$CE(\widetilde{H}) = -\widetilde{H} \log E\left[e^{-\frac{1}{\widetilde{H}}R} | a^* = c'^{-1}(\gamma)\right] - c(c'^{-1}(\gamma))$$

Intuitively, the joint expected utility pie of a matched pair is some transformation of the representative individual's expected utility from facing the stream of returns R, minus the cost of effort, where effort in the first-best is independent of any risk types.

Furthermore, the transformation of the representative individual's expected utility is a special transformation:

$$K_{R|a^*}\left(t = -\frac{1}{\widetilde{H}}\right) = \log E\left[e^{-\frac{1}{\widetilde{H}}R} | a^* = c'^{-1}(\gamma)\right]$$

where $K_{R|a^*}\left(t = -\frac{1}{\widetilde{H}}\right)$ is the cumulant-generating function (log of the moment-generating function) of the random variable $R|a^*$ evaluated at the negative of the representative risk tolerance.

How does this contribute to our understanding of equilibrium matching in the economy? We know from the Shapley and Shubik assortative matching conditions that a sufficient condition for unique PAM in this setting is supermodularity of $CE(r_1, r_2)$ in r_1, r_2, and a sufficient condition for unique NAM in this setting is submodularity of $CE(r_1, r_2)$ in r_1, r_2.

Moreover, we know that:

$$\begin{aligned}\frac{d^2 CE(r_1, r_2)}{dr_1 dr_2} &= \frac{dCE}{d\widetilde{H}} \frac{d^2 \widetilde{H}}{dr_1 dr_2} + \frac{d^2 CE}{d\widetilde{H}^2} \frac{d\widetilde{H}}{dr_1} \frac{d\widetilde{H}}{dr_2} \\ &= \left(\frac{1}{r_1 r_2}\right)^2 \frac{d^2 CE}{d\widetilde{H}^2}\end{aligned}$$

Hence, a sufficient condition for unique PAM is convexity of $CE(\widetilde{H})$ in $\widetilde{H}$, while concavity of $CE(\widetilde{H})$ in $\widetilde{H}$ ensures unique NAM.

Checking the second derivative of $CE(\widetilde{H})$ in $\widetilde{H}$ yields the first-best matching result.

Proposition 3 *In the first-best model, when effort is observable and contractible, the unique equilibrium matching pattern in risk atititude of principals and agents is negative-assortative.*

Proof. The second derivative of $CE(\widetilde{H})$ in $\widetilde{H}$ is straightforward to find.

$$CE(\widetilde{H}) = -\widetilde{H}K_{R|a^*}\left(t = -\frac{1}{\widetilde{H}}\right) - c(c'^{-1}(\gamma))$$

$$\frac{dCE(\widetilde{H})}{d\widetilde{H}} = -K_{R|a^*}\left(t = -\frac{1}{\widetilde{H}}\right) - \frac{1}{\widetilde{H}}K'_{R|a^*}\left(t = -\frac{1}{\widetilde{H}}\right)$$

$$\frac{d^2CE(\widetilde{H})}{d\widetilde{H}^2} = -\frac{1}{\widetilde{H}^3}K''_{R|a^*}\left(t = -\frac{1}{\widetilde{H}}\right) < 0$$

since the cumulant-generating function is convex in t.

Hence, the unique equlibrium match in the first-best is negative-assortative. ■

Now that we understand what happens in the first-best, we can investigate the second-best.

3.2 The Second-Best

Now, suppose that effort is not observable and not contractible. In this case, the sharing rule of a given principal-agent pair must provide incentives as well as insurance.

To gain traction on this problem, I impose a specific functional form assumption on the distribution of output.

Recall that output given effort a is described by $R|a = \gamma a + \varepsilon$. In the previous subsection, it was assumed that $\varepsilon \sim f_\varepsilon$, a general density function with support on the real line and well-defined cumulant-generating function.

In this subsection, assume that f_ε is a Laplace distribution with mean 0 and exogenously-given variance $V > 0$, where V captures the riskiness of the environment.[4] (See Appendix 1 for more details on the Laplace distribution.)

The key features of this distribution for the setting of this paper are symmetry. The distribution resembles the normal distribution, but has fatter tails, and the density function is non-differentiable at the mean. Loosely, the fatter tails allow us to avoid the Mirrlees critique of linear contracts–a realized return in the tail of a Laplace distribution is not infinitely precise about effort exerted.

Then, the equilibrium sharing rule of a principal-agent pair (r_1, r_2), where the agent r_2 is ensured expected utility at least $-e^{-v}$, solves:

$$\max_{s(R)} \int_{-\infty}^{\gamma a} -e^{-r_1(R-s(R))}\frac{1}{2V}e^{\frac{1}{V}[R-\gamma a]}dR + \int_{\gamma a}^{\infty} -e^{-r_1(R-s(R))}\frac{1}{2V}e^{-\frac{1}{V}[R-\gamma a]}dR$$

such that:

$$(IR): \int_{-\infty}^{\gamma a} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{\frac{1}{V}[R-\gamma a]}dR +$$

$$+\int_{\gamma a}^{\infty} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{-\frac{1}{V}[R-\gamma a]}dR \geq -e^{-v}$$

[4] Assume $V < \frac{1}{\max(r_1)} + \frac{1}{\max(r_2)}$, for the problem to be well-defined.

$$(IC) : a \in \arg \max_{a \in (0,\infty)} \int_{-\infty}^{\gamma a} -e^{-r_2[s(R)-c(a)]} \frac{1}{2V} e^{\frac{1}{V}[R-\gamma a]} dR +$$

$$+ \int_{\gamma a}^{\infty} -e^{-r_2[s(R)-c(a)]} \frac{1}{2V} e^{-\frac{1}{V}[R-\gamma a]} dR$$

The complete equilibrium analysis of this problem can be found in Appendix 1, but I will provide a sketch of the solution here, to provide an understanding of the equilibrium behavior of a matched partnership.

First, we need to address the constraints. The IR constraint clearly binds in equilibrium. The IC constraint presents more of a challenge. It would be useful to be able to replace the global IC constraint with its first-order condition, but none of the existing sufficient conditions for the validity of the first-order approach apply to this model. Rogerson (1985) shows that sufficient conditions for a standard moral hazard model in which the principal may also be risk-averse are: (a) monotone likelihood ratio, and (b) convexity of the distribution function. Condition (a) holds in my model (the likelihood ratio here is a piecewise constant, $-c$ below the mean and c above the mean), but (b) fails–very few standard distribution functions satisfy CDFC. Jewitt (1988) identifies sufficient conditions that weaken CDFC, but for a model with a risk-neutral principal. Additionally, utility is assumed to be additively separable in consumption and effort, whereas it is multiplicatively separable here. A variety of more recent contributions identify sets of conditions that weaken CDFC slightly, at the cost of strenghtening other conditions, but none weakens CDFC enough for the Laplace distribution.

So, the validity of the first-order approach must be proved from first principles[5]. Since effort a is chosen from an open set, the optimum will be interior, if it exists. Hence, the first-order condition of the global IC constraint is a necessary, though perhaps not sufficient, condition for the optimum.

This means that the first-order problem (the problem with the global IC condition replaced by its first-order condition) is a relaxed problem, so that the actual optimum must be a solution of the first-order problem, if it exists, but a solution of the first-order problem is not necessarily the optimum.

Solving the first-order problem yields the following wage schedule:

$$s(R_p < \gamma\widehat{a}) = \frac{r_1}{r_1+r_2}R - \frac{r_1}{r_1+r_2}\gamma\widehat{a} + c(\widehat{a}) + \frac{1}{r_2}v - \frac{1}{r_2}\log\left[\left(1 - \frac{r_1 r_2}{r_1+r_2}V\right)\left(1 + \frac{r_2 c'(\widehat{a})V}{\gamma}\right)\right]$$
$$s(R_p > \gamma\widehat{a}) = \frac{r_1}{r_1+r_2}R_p - \frac{r_1}{r_1+r_2}\gamma\widehat{a} + c(\widehat{a}) + \frac{1}{r_2}v - \frac{1}{r_2}\log\left[\left(1 + \frac{r_1 r_2}{r_1+r_2}V\right)\left(1 - \frac{r_2 c'(\widehat{a})V}{\gamma}\right)\right]$$

where r_1 is the risk attitude of the principal, r_2 is the risk attitude of the agent, and $\widehat{a}$ is the level of effort "anticipated" by the principal. In equilibrium, the optimal effort chosen by the agent in response to the wage schedule $s(R_p|\widehat{a})$ should be $a^* = \widehat{a}$: the principal has no incentive to pay for a higher level of effort than she knows will actually be exerted, and the agent has no incentive

[5] Again, a rigorous proof can be found in Wang (2012b).

to exert more effort than he is compensated for.

What is equilibrium effort given this compensation scheme? It can be shown that $a = \widehat{a}$ is a stationary point of agent r_2's expected utility from exerting effort a given wage schedule $s(R|\widehat{a})$, for every possible $\widehat{a}$. However, for $\widehat{a} > \widehat{a}_t$, where $\widehat{a}_t$ is some threshold, there will be a second stationary point at $a < \widehat{a}$–because the wage schedule is discontinuous for $\widehat{a} \neq c'^{-1}\left(\frac{r_1}{r_1+r_2}\gamma\right)$, if the principal tries to induce a "too-high" level of effort, the agent will profitably deviate to a *discretely* lower level of effort.

More concisely, for $\widehat{a} \leq \widehat{a}_t$, the unique maximizing level of effort exerted by the agent is $a^* = \widehat{a}$; for $\widehat{a} > \widehat{a}_t$, the unique maximizing level of effort exerted by the agent is $a < \widehat{a}$.

Therefore, the equilibrium $\widehat{a}$ set by the principal is $\widehat{a}^* = \widehat{a}_t$, where this threshold is characterized by:

$$\left(c'(\widehat{a}_t) - \frac{r_1}{r_1 + r_2}\gamma\right)\left(c'(\widehat{a}_t) + \frac{\gamma}{r_2 V}\right) = \frac{1}{r_2}c''(\widehat{a}_t) > 0$$

Because the agent's expected utility from exerting effort a given wage $s(R|\widehat{a}^*)$ is strictly concave in a, it must be that $s(R|\widehat{a}^*)$ is in fact the optimum.

Observe that setting $\widehat{a}_t = c'^{-1}\left(\frac{r_1}{r_1+r_2}\gamma\right)$ causes the left-hand side of the equation to be 0, while the right-hand side is positive. Since the left-hand side is strictly increasing in $\widehat{a}_t$ (the cost function $c(a)$ is strictly convex), it must be that $\widehat{a}_t > c'^{-1}\left(\frac{r_1}{r_1+r_2}\gamma\right)$.

Therefore, the wage schedule within a principal-agent pair (r_1, r_2) is piecewise linear: at the anticipated mean level of output, $\gamma\widehat{a}$, there is a jump in the wage–realized output levels greater than the mean $\gamma\widehat{a}^*$ are rewarded at a discretely higher level than output levels that are below the mean. At output levels away from the anticipated mean, the wage is linear with slope $\frac{r_1}{r_1+r_2}$. Hence, the equilibrium wage can be cleanly decomposed into insurance provision and incentive provision. The jump in the wage at the mean provides incentives (since the likelihood ratio is a piecewise constant, in some sense knowing whether output is above or below the mean is differentially more informative about effort exertion), and the linearity away from the mean captures risk-sharing.

Using this characterization of equilibrium sharing rule and effort in a given principal-agent pair (r_1, r_2), we can solve for conditions for unique assortative matching, under a functional form assumption on cost of effort: $c(a) = \eta a^M$, $M > 1$ for convexity.

We use the same trick as in the first-best: expected utility is transferable in the second-best as well.

Proposition 4 *Expected utility is transferable in the second-best.*

Proof. Using the equilibrium sharing rule of a given pair and the characterization of equilibrium effort, we can write the certainty-equivalent of principal r_1 and agent r_2 when matched:

$$\begin{aligned} CE_{r_1} &= \gamma\widehat{a}_{12} - c(\widehat{a}_{12}) - \frac{v}{r_2} \\ &\quad -\frac{1}{r_1}\log\left(\frac{1}{2}\left[\frac{1}{\left[1+\frac{r_2 c'(\widehat{a}_{12})V}{\gamma}\right]^{\frac{r_1}{r_2}}\left[1-\frac{r_1 r_2}{r_1+r_2}V\right]^{1+\frac{r_1}{r_2}}} + \frac{1}{\left[1-\frac{r_2 c'(\widehat{a}_{12})V}{\gamma}\right]^{\frac{r_1}{r_2}}\left[1+\frac{r_1 r_2}{r_1+r_2}V\right]^{1+\frac{r_1}{r_2}}}\right]\right) \end{aligned}$$

$$CE_{r_2} = \frac{v}{r_2}$$

Hence, it is clear that there is a one-to-one tradeoff in the certainty-equivalents of r_1 and r_2. Thus, expected utility is transferable in this model. ■

Therefore, a sufficient condition for unique positive-assortative matching is supermodularity of the pairwise sum of certainty-equivalents, and a sufficient condition for unique negative-assortative matching is submodularity of the pairwise sum:

$$\begin{aligned} CE(r_1, r_2) &= \gamma\widehat{a}_{12} - c(\widehat{a}_{12}) \\ &\quad -\frac{1}{r_1}\log\left(\frac{1}{2}\left[\frac{1}{\left[1+\frac{r_2 c'(\widehat{a}_{12})V}{\gamma}\right]^{\frac{r_1}{r_2}}\left[1-\frac{r_1 r_2}{r_1+r_2}V\right]^{1+\frac{r_1}{r_2}}} + \frac{1}{\left[1-\frac{r_2 c'(\widehat{a}_{12})V}{\gamma}\right]^{\frac{r_1}{r_2}}\left[1+\frac{r_1 r_2}{r_1+r_2}V\right]^{1+\frac{r_1}{r_2}}}\right]\right) \end{aligned}$$

The first two terms of this sum can be thought of as the part of the expected utility pie coming from productivity (effort exertion and corresponding expected output), while the third term can be thought of as the part of the expected utility pie coming from risk-sharing.

The challenge of identifying conditions for assortative matching in this model is the *one-sided* moral hazard. Although expected utility is transferable, r_1 and r_2 do not enter symmetrically into the sum of certainty-equivalents of the matched pair. Consequently, the matching conditions will *not* be distribution-free as they are in the absence of moral hazard (for example, in the first-best, or in the case of endogenous matching under the trade-off of *ex ante* and *ex post* risk management, as in Wang (2012a)).

Finding conditions for the supermodularity and submodularity of $CE(r_1, r_2)$ in r_1, r_2 yields the following matching results.

Proposition 5 *Let $c(a) = \eta a^M$, $M > 1$.*

1. NAM is the unique eqm matching pattern for $M \in [1, M_1] \cup [M_4, \infty)$, where $M_1 \leq M_4$.

a. M_1 is increasing in $\underline{r}_1$ (the least risk-averse principal's risk aversion) and in $\bar{r}_1$ (the most risk-averse principal's risk aversion), and decreasing in $\underline{r}_2, \bar{r}_2$. Furthermore, M_1 is increasing in V, and decreasing in γ.

b. M_4 is decreasing in $\underline{r}_1$ and $\bar{r}_1$, and increasing in $\underline{r}_2$ and $\bar{r}_2$. Furthermore, M_4 is decreasing in V, and increasing in γ.

2. PAM is the unique eqm matching pattern for $M \in [M_2, M_3]$, where $M_1 \leq M_2$ and $M_3 \leq M_4$. (It may be that the interval $[M_2, M_3]$ is empty.)

a. M_2 is increasing in $\underline{r}_1$ and $\bar{r}_1$, and decreasing in $\underline{r}_2$ and $\bar{r}_2$. Furthermore, M_2 is increasing in V, and decreasing in γ.

b. M_3 is decreasing in $\underline{r}_1$ and $\bar{r}_1$, and increasing in $\underline{r}_2$ and $\bar{r}_2$. Furthermore, M_3 is decreasing in V, and increasing in γ.

(The proof is relegated to the Appendix.)

This result highlights the key determinants of the equilibrium matching pattern: the riskiness of the environment V, the across-group and within-group heterogeneity in risk attitude, captured by the endpoints of the supports of the risk type distributions of principals and agents, and the marginal impact of effort on mean output, γ.

In words, the takeaways from the main matching result are the following. First, positive-assortative matching (PAM) is more likely for moderately convex cost of effort functions, while negative-assortative matching (NAM) is more likely for cost of effort functions which are close to linear or extremely convex.

What delineates the boundaries of "close to linear", "moderately" convex, and "extremely" convex? The comparative statics on the bounds tell us the following:

1. Negative-assortative matching (NAM) is more likely to arise when principals are distinctly more risk-averse than agents, while positive-assortative matching (PAM) is more likely to arise when principals are distinctly less risk-averse than agents.

2. NAM is more likely to arise when the environment is very risky (V is large), while PAM is more likely to arise when the environment is very safe (V is low).

3. NAM is more likely to arise when the marginal benefit of effort (for mean ouptut) is low (γ is small), while PAM is more likely to arise when the margnal benefit of effort is high (γ is large).

What is the intuition behind these comparative statics? Consider a relatively safe environment where the cost of effort function is moderately convex and the marginal benefit of effort is large. Effort exertion across different risk attitudes is most heterogeneous when the cost of effort function is moderately convex. Moreover, when the environment is relatively safe (that is, V is relatively low), the "need" for insurance is small and output is a fairly precise signal of effort. Hence, rewarding effort based on realized output is effective, less risk-averse agents are substantially cheaper to incentivize than more risk-averse agents, and more risk-averse agents are not willing to pay particularly high risk premia. All of these forces push the incentive provision effect to outwiegh the insurance provision effect, which favors PAM over NAM. If in addition principals are less risk-averse than agents, then the tradeoff between incentive provision and insurance provision is particularly stark (since if a principal were more risk-averse than the agent, the principal would be happy to provide incentives for the agent, as this would be a method of self-insurance).

Hence, in this environment, the least risk-averse principal *experiences the biggest difference in utility* between being paired with the least risk-averse agent versus a more risk-averse agent. Thus, the least risk-averse principal will outbid the other principals for the least risk-averse agent, and once the least risk-averse principal and the least risk-averse agent are removed from the pool of candidates, the least risk-averse principal of those remaining will outbid the other principals for the least risk-averse agent remaining, and so on, and the equilibrium matching pattern will be positive-assortative.

On the other hand, when agents have close to linear or extremely convex cost of effort, the difference between effort exertion across agents of different risk types is small–either all the agents exert very high effort, or all the agents exert very low effort. If the environment is also risky, that is, V is high, then individuals, especially more risk-averse individuals, will be willing to pay a high price for insurance. Moreover, output is a noisy signal of actual effort exertion. Hence, the insurance provision effect will tend to outweigh the incentive provision effect. If in addition principals are more risk-averse than agents, then the incentive provision is aligned with insurance provision: a more risk-averse principal prefers a less risk-averse agent, because the principal desires insurance for herself, and this will naturally provide incentives to the agent. This means that the most risk-averse principal *experiences the biggest difference in utility* between being paired with the least risk-averse agent versus a more risk-averse agent. Thus, the most risk-averse principal will outbid the other principals for the least risk-averse agent, and once they are matched, the most risk-averse principal remaining will outbid the others for the most risk-averse agent remaining, and so forth, and the equilibrium matching pattern will be negative-assortative.

An interesting but informal insight that emerges from the analysis in this framework is that, in contrast with the standard view, there is a strong case for the more risk-averse individuals to be principals, and the less risk-averse individuals to be agents: a more risk-averse principal is happy to incentivize a less risk-averse agent, because she wants insurance, and the less risk-averse agent doesn't mind the riskiness of the incentives.

Finally, a word on efficiency.

Proposition 6 *The equilibrium maximizes the sum of certainty-equivalents, and is Pareto efficient.*

The equilibrium maximizes the sum of certainty-equivalents, since the conditions for PAM and NAM were derived by finding conditions for the supermodularity and submodularity of the pairwise sum of certainty-equivalents. Since the sum of certainty-equivalents is a social welfare function, and the equilibrium maximizes this sum, it must be Pareto efficient.

While the natural measure of welfare for a partnership and the individuals within that partnership is the pairwise certainty-equivalent, the unweighted sum of certainty-equivalents across pairs is not the right way of thinking about economy-wide welfare. A more risk-averse individual needs to be guaranteed a smaller amount than a less risk-averse individual to be made indifferent between accepting that amount with certainty and partaking in her risky equilibrium, but there is no reason society should value her less because of that. Hence, policy in this framework will be considered to improve aggregate welfare if it is Pareto-improving. (Alternatively, using a Rawlsian social welfare function which weights the utility of more risk-averse individuals would also be a reasonable approach.)

4 The Policy Example

There are a variety of natural policies a government might wish to implement in this setting. In this hypothetical policy example, I will discuss a very simple approach to thinking about the welfare impacts of introducing formal insurance using this framework, and show how accounting for endogeneity is essential to the evaluation and design of this policy.

In particular, suppose that the status quo environment is very risky, that is, V is very large, and a government wishes to reduce the risk burden shouldered by risk-averse citizens. The government takes steps to reduce V by introducing formal insurance (modeling the introduction of formal insurance as a risk-reduction measure has precedent in the literature, for example in Attanasio and Rios-Rull (2000). The matching results from Proposition 5 tell us that such a decrease in V may trigger an endogenous network response–in a very risky environment, the least risk-averse principals hire the most risk-averse agents, since a high V means that the the gains from trade from risk-sharing are high, and moreover that output is a very noisy signal of effort. However, a decrease in V means that there is less risk that needs to be shared, and additionally that output is a much more precise signal of effort. Hence, the least risk-averse principals may switch to hiring the least risk-averse agents instead.

It will be helpful to work through a specific numeric example[6]. Let the parameters of the status quo be: $\gamma = 11.5, \eta = 0.5$,and $M = 4$, so that:

$$
\begin{aligned}
R &= 11.5a + \varepsilon,\ \varepsilon \sim Laplace(0, V) \\
c(a) &= \frac{1}{2}a^4
\end{aligned}
$$

Suppose there are three principals and three agents: the principals are $\{r_1^1 = 0.3, r_2^1 = 0.8, r_3^1 = 1.5\}$, while the agents are $\{r_1^2 = 2, r_2^2 = 2.6, r_3^2 = 3\}$. That is, principals are distinctly less risk-averse than agents, and there is more heterogeneity in risk type amongst principals than amongst agents.

Suppose that initially, the level of risk in the economy is high. Specifically, the variance of returns of the project undertaken by all pairs is $V = 0.9$ (e.g., the terrain is such that rice paddy is the crop that all landlord-farmer pairs grow, and the standard deviation of profits from rice paddy is determined by V). Then the equilibrium match is negative-assortative, and the agent in each matched pair exerts the following effort:

[6] The specific numbers are not important, as the results from Proposition 5 are comparative statics, and thus it is the relative comparisons that matter.

$(r_3^1 = 1.5, r_1^2 = 2) : a_{r_1^2=2} = 1.34$
$(r_2^1 = 0.8, r_2^2 = 2.6) : a_{r_2^2=2.6} = 1.15$
$(r_1^1 = 0.3, r_3^2 = 3) : a_{r_3^2=3} = 0.87$

The least risk-averse agent exerts the highest level of effort, because she has an effectively lower marginal cost of effort, and, more importantly, works for the most risk-averse principal. Hence, the least risk-averse principal has very sharp incentives: she essentially insures the most risk-averse principal, and works hard herself, and this is a mutually-satisfying agreement.

The most risk-averse agent exerts the lowest level of effort, because she works for the least risk-averse principal, who provides her with informal insurance. The least risk-averse principal's land plot is therefore the least productive, but he is paid a risk premium by his agent.

The certainty-equivalents in each negatively-assorted partnership are:

$CE(r_3^1 = 1.5, r_1^2 = 2) = 13.08$
$CE(r_2^1 = 0.8, r_2^2 = 2.6) = 11.79$
$CE(r_1^1 = 0.3, r_3^2 = 3) = 9.53$

The happiest partnership is between the most risk-averse principal and the least risk-averse agent, because of the alignment between the principal's desire for insurance, and the agent's incentivization from a scheme which insures the principal. The unhappiest partnership is between the least risk-averse principal and the most risk-averse agent–even though there are gains from trade from risk-sharing, the loss of productivity resulting from the agent being insured reduces the joint expected utility pie.

Now suppose that the introduction of formal insurance reduces the riskiness of the environment, so that effectively the variance of the returns of the risky project falls to $V = 0.07$ (and the other parameters remain unchanged). This causes the principals and agents to re-sort: in this environment, the unique equilibrium match is positive-assortative in risk types.

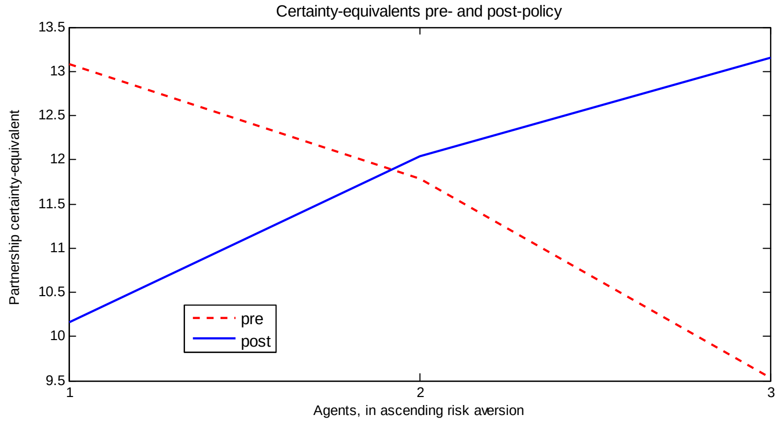
Then, the agent in each positively-assorted couple exerts effort:

$(r_1^1 = 0.3, r_1^2 = 2) : a_{r_1^2=2} = 0.914$
$(r_2^1 = 0.8, r_2^2 = 2.6) : a_{r_2^2=2.6} = 1.11$
$(r_3^1 = 1.5, r_3^2 = 3) : a_{r_3^2=3} = 1.25$

And the certainty-equivalent of each positively-assorted couple is:

$CE(r_1^1 = 0.3, r_1^2 = 2) = 10.16$
$CE(r_2^1 = 0.8, r_2^2 = 2.6) = 12.04$
$CE(r_3^1 = 1.5, r_3^2 = 3) = 13.15$

The certainty-equivalents of each partnership pre- and post-policy are plotted below. The agents are arranged along the x-axis in increasing risk aversion.



We can see a stark difference in the distribution of welfare pre- and post-policy. Pre-policy, in the risky environment, the least risk-averse agents are the best off, while they are the worst off following the introduction of formal insurance. Why is this?

Analysis of the impact of the introduction of formal insurance within this endogenous matching framework enables us to see precisely and quite concretely the often-discussed "crowding-out" effect. Following a decrease in V, insurance provision plays much less of a role in matching than does incentive provision, leading principals to hire agents with similar risk attitudes rather than different risk attitudes. This means that less insurance is provided informally in equilibrium.

Moreover, when V was high, the least risk-averse agents essentially insured the most risk-averse principals: the most risk-averse principals would offer their agents a wage scheme with high-powered incentives. This meant that the principal's income depended very little on the realized return of output, while the agent's income depended highly on realized return. But the agent, being less risk-averse, was happy under this wage scheme and worked hard. This benefited both the principal and the agent: the principal was well-insured and expected return was high, while the agent was close to being a residual claimant to her effort exertion.

Following the introduction of formal insurance, however, the most risk-averse principal's desire for informal insurance is heavily dampened. She now prefers to take more risk, in the sense that she wants her own income stream to depend more on realized return of output. She therefore prefers to hire an agent who is also quite risk-averse.

Thus, the less risk-averse agent's role in providing informal insurance is "crowded out" by the introduction of formal insurance, and he is worse off, despite the drastic decrease in riskiness of the environment.

Although the approach to modeling the introduction of formal insurance is certainly over-simplistic in this example, the results are striking and capture the much-discussed "crowding out effect" concretely and rigorously. This analysis provides a strong argument for rigorously accounting for the impact on informal insurance relationships when introducing formal insurance.

5 Conclusion

A large literature has explored the consequences of balancing incentive provision with insurance provision for the contractual arrangement between a given principal and agent who work together to produce some output, where otuput depends on unobservable and noncontractible inputs put in by the agent. In this paper, I focus on the formation of the contracting relationships themselves. I argue that individuals in developing economies are typically forced to use their interpersonal relationships, which may already be serving several purposes, to address in addition the needs typically met by formal institutions in developed economies, such as risk management. I think of the equilibrium network of contracting relationships arising in this way as informal insurance.

I find conditions on the environment for assortative matching, in particular, on the level of riskiness in the environment, the marginal benefit of effort for mean output, the convexity of the cost of effort function, and the risk type distributions of principals and agents. I show that environments where principals are distinctly more risk-averse than agents, with high levels of risk, with low marginal benefit of effort, and with cost of effort functions which are close to linear or extremely convex are amenable to unique negative-assortative matching, while environments where principals are distinctly less risk-averse than agents, with low levels of risk, with high marginal benefit of effort, and with cost of effort functions which are moderately convex are more conducive to positive-assortative matching. Loosely, the tradeoff that drives the equilibrium matching pattern is the costs and benefits of insurance provision versus the costs and benefits of insurance provision, for each possible pairing of risk types.

I then discuss applications of this analysis for policymaking. First, proper evaluation of policies which affect the parameters of this environment–for example, a policy which subsidizes inputs through an innovative technology and reduces the convexity of the cost of effort function, or a policy which introduces formal insurance and reduces the aggregate risk of the environment–requires accounting for the endogenous network response. A key point is that the multidimensionality of the relationships in the network is a consequence of missing formal institution, and hence partnerships between principals and agents, which are ostensibly formed for productivity purposes, may respond to changes in the risk environment, because embedded implicitly into these relationships is informal insurance. In particular, I showed that the introduction of formal insurance crowds out informal insurance, and may leave those individuals who acted as informal insurers worse off.

Second, this analysis is important for making accurate inferences about the environment from observations of equilibrium contracts. In particular, this analysis highlights the importance of collecting data about contracting partners, not just the sharing rules.

Much work remains to be done. A natural next step would be to allow principal-agent pairs to choose the riskiness of the income stream they face, instead of keeping the riskiness of the environment exogenous and the same for all possible pairs. It would also be interesting to allow for richer patterns of group formation. For example, tenant farmers working for different landlords might agree to share risk as a group. How would this affect the equilibrium network of relationships and contracts?

This analysis also yielded an insight into why more risk-averse individuals may be better suited as principals, while less risk-averse individuals may be better suited as agents, contrary to the standard perspective. A more risk-averse principal and a less risk-averse agent can be seen to work well together in a model of endogenous matching under risk and moral hazard, because the more risk-averse principal insures herself by providing incentives to her less risk-averse employee. Hence, in these relationships, insurance provision seems aligned with incentive provision. This suggests a closer study of risk attitudes and principal-agent roles in developing economies.

Thinking about informal institutions as the interpersonal relationships which emerge in equilibrium to address the needs usually served by formal institutions in developed economies gives us a deeper understanding of the operation of informal institutions in the status quo, and how they would be affected by changes in the formal institutional structure. Furthering this understanding is important for strong development policymaking.

6 References

1. Akanda, M., H. Isoda, and S. Ito. "Problem of Sharecrop Tenancy System in Rice Farming in Bangladesh: A Case Study on Alinapara Village in Sherpur District." Journal of Int. Farm Management (4) 2008 23-30.

2. Alderman, H. and C. Paxson. "Do the Poor Insure? A Synthesis of the Literature on Risk and Consumption in Developing Countries." Economics in a Changing World, vol. 4 1992 49-78.

3. Allen, D. and D. Lueck. "Contract Choice in Modern Agriculture: Cash Rent versus Cropshare." Journal of Law and Economics (35) 1992 397-426.

4. Attanasio, O. and J. Rios-Rull. "Consumption Smoothing in Island Economies: Can Public Insurance Reduce Welfare?" European Economic Review (44) 2000 1225-1258.

5. Bardhan, P. "Interlocking Factor Markets and Agrarian Development: A Review of Issues." Oxford Econ. Papers (32) 1980 82-98.

6. Becker, G.S. "A Theory of Marriage: Part I." Journal of Political Economy (81) 1973 813-846.

7. Bellemare, M. "Sharecropping, Insecure Land Rights and Land Titling Policies: A Case Study of Lac Alaotra, Madagascar." Development Policy Review (27) 2009 87-106.

8. Chiappori, P-A. and P. Reny. "Matching to share Risk." Working Paper 2006.

9. Dercon, S. "Risk, Crop Choice, and Savings: Evidence from Tanzania." Econ. Dev. and Cultural Change (44) 1996 483-513.

10. Eswaran, M. and A. Kotwal. "A Theory of Contractual Structure in Agriculture." American Economic Review, 75(3) 1985 352-67.

11. Fafchamps, M. "Risk-sharing Between Households." Handbook of Social Economics 2008.

12. Franco, A., M. Mitchell, and G. Vereshchagina. "Incentives and the Structure of Teams." Journal of Economic Theory (146) 2011 2307-2332.

13. Ghatak, M. and A. Karaivonov. "Contractual Structure in Agriculture with Endogenous Matching." CAGE Working Paper 2013.

14. Holmstrom, B. and P. Milgrom. "Aggregation and Linearity." Econometrica (55) 1987 303-328.

15. Jewitt, I. "Justifying the First-Order Approach to Principal-Agent Problems." Econometrica (56) 1988 1177-1190.

16. Jewitt, I. "Risk Aversion and the Choice Between Risky Prospects: the Preservation of Comparative Statics Results." Review of Economic Studies (54) 1987 73-85.

17. Legros, P. and A. Newman. "Beauty is a Beast, Frog is a Prince: Assortative Matching with Nontrasferabilities (75) 2007 1073-1102.

18. Mirrlees, J. "The Implications of Moral Hazard for Optimal Insurance." Mimeo 1979.

19. Morduch, J. "Income Smoothing and Consumption Smoothing." J. Econ. Perspectives (9) 1995 103-114.

20. Rogerson, W. "The First-Order Approach to Principal-Agent Problems." Econometrica (53) 1985 1357-1367.

21. Schulhofer-Wohl, S. "Negative-assortative Matching of Risk-Averse Agents with Transferable Expected Utility." Econ. Letters (92) 2006 383-388.

22. Serfes, K. "Endogenous Matching in a Market with Heterogeneous Principals and Agents." Int J Game Theory (36) 2008 587-619.

23. Shapley, L. and M. Shubik. "The assignment game I: The Core." Intl. J. Game Theory (1) 1971 111-130.

24. Stiglitz 1974. "Incentives and Risk Sharing in Sharecropping." Review of Economic Studies (41) 1974 219-255.

25. Wang, X. "Endogenous Insurance and Informal Relationships." Working Paper 2012a.

26. Wang, X. "A Note on Moral Hazard and Linear Compensation Schemes." Working Paper 2012b.

7 Appendices

7.1 A1: The One-sided CARA-Laplace Moral Hazard Model

In this Appendix, I construct a model of one-sided moral hazard where both the principal and the agent are risk-averse (and may differ in the degree of their risk aversion). The principal and the agent have CARA utility, and the productive output of the pair depends noisiliy on the effort exerted by the agent, which is costly for the agent. The novelty is that the noise follows a Laplace distribution. I show that the unique equilibrium wage scheme in this model is piecewise linear. This is in contrast to the typical usage of a CARA-Normal framework with a linear wage justified by Holmstrom and Milgrom (1987) reasoning to counter the difficulty of characterizing the equilibrium of one-sided moral hazard models with risk-averse principals and agents, and a cumulative distribution function of returns which isn't globally convex. (Recall the standard monotone likelihood ratio and convexity of distribution function conditions, and slight variants, from Mirrlees (1979), Rogerson (1983), and Jewitt (1988). Convexity of the cumulative distribution function is usually the most difficult condition to satisfy to justify the first-order approach.)

In addition to the value of constructing a model in which the unique equilibrium can be characterized, and in particular has the convenient form of piecewise linearity, this model is well-suited to answering the specific question posed by this paper. I show that the equilibrium wage scheme is linear away from the mean, with the optimal risk-sharing slope–this captures insurance provision. And, the jump at the mean captures incentive provision: the agent is compensated linearly in output, and gets a bonus if output exceeds a threshold, where that threshold is the expected output given (correctly) anticipated effort.

7.1.1 The Model

The framework consists of the following elements:

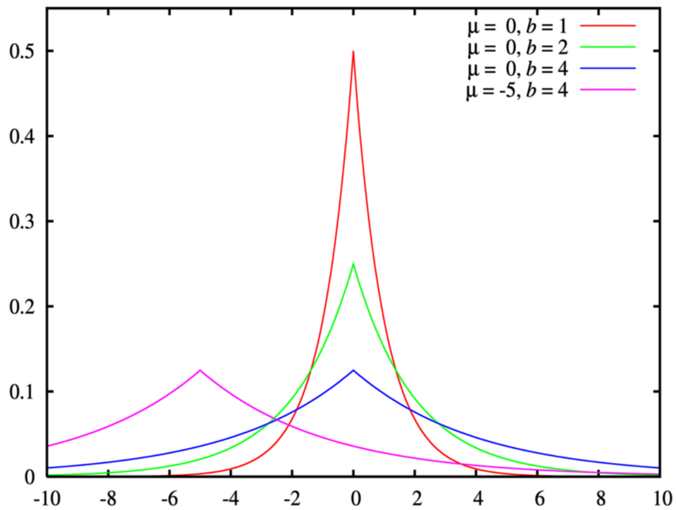
The principal and the agent: both the principal r_1 and the agent r_2 are risk-averse with CARA utility $u(x) = -e^{-rx}$, $r > 0$. The principal owns one unit of physical capital but has infinite marginal cost of effort, while the agent owns no physical capital, but has a finite marginal cost of effort. The cost of effort for all agents is $c(a)$, $c(a) > 0$, $c'(a) > 0$, $c''(a) > 0$. The agent has an exogenously-given outside option denoted by $-e^{-v}$.

The risky environment: A principal-agent partnership can only produce positive output if one unit of physical capital is combined with human capital. Output is given by $R|a = \gamma a + \varepsilon$, where the riskiness of the environment is captured by $\varepsilon \sim Laplace(0, V)$, which has support $(-\infty, \infty)$. Hence, the effort of an agent, a, increases the mean but doesn't affect the variance of returns.

The Laplace distribution is a continuous probability distribution with location and scale parameters $\mu \in \mathbb{R}$ and $b \in \mathbb{R}^+$, respectively. The pdf of a random variable $X \sim Laplace(\mu, b)$ is:

$$f(x|\mu, b) = \begin{cases} \frac{1}{2b} e^{-\frac{(\mu - x)}{b}}, & x < \mu \\ \frac{1}{2b} e^{-\frac{(x-\mu)}{b}}, & x \geq \mu \end{cases}$$

Pictorially, the pdf is:



Note that this resembles two back-to-back exponential distributions. In fact, if $Y \sim Laplace(0, V)$, then $|Y| \sim \exp(\frac{1}{V})$.

Information and commitment: The principal and the agent know each other's risk types. However, the agent's effort is not observable and not contractible.

A given principal and agent pair (r_1, r_2) observes the realized output of their partnership, and is able to commit to a return-contingent sharing rule $s(R)$, where R is realized output.

The equilibrium: An equilibrium consists of a wage $s(R)$ set by the principal satisfying the constraints of the problem, such that any other choice of feasible sharing rule by the principal would leave her worse off, as well as an effort level a chosen by the agent such that any other choice of effort would leave the agent worse off.

7.1.2 The Solution

The principal r_1 chooses wage $s(R)$ for the agent r_2 by solving the following problem:

$$\max_{s(R)} \int_{-\infty}^{\gamma a} -e^{-r_1(R-s(R))}\frac{1}{2V}e^{\frac{1}{V}[R-\gamma a]}dR + \int_{\gamma a}^{\infty} -e^{-r_1(R-s(R))}\frac{1}{2V}e^{-\frac{1}{V}[R-\gamma a]}dR$$

such that:

$$(IR): \int_{-\infty}^{\gamma a} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{\frac{1}{V}[R-\gamma a]}dR +$$
$$+\int_{\gamma a}^{\infty} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{-\frac{1}{V}[R-\gamma a]}dR \geq -e^{-v}$$

$$(IC): a \in \arg\max_{a\in(0,\infty)} \int_{-\infty}^{\gamma a} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{\frac{1}{V}[R-\gamma a]}dR +$$
$$+\int_{\gamma a}^{\infty} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{-\frac{1}{V}[R-\gamma a]}dR$$

It is clear that the IR constraint binds in equilibrium.

Since effort a is chosen from an open interval, all global maxima are stationary points, if they exist. Hence, the first-order problem (the problem wherein the global IC constraint is replaced by its first-order condition) is a relaxed problem: an optimum must be a stationary point, but a stationary point is not necessarily an optimum.

Differentiating the global IC constraint with respect to a yields the first-order condition of the global IC constraint:

$$\left[r_2 c'(a) - \frac{\gamma}{V}\right] \int_{-\infty}^{\gamma a} -e^{-r_2[s(R)-c(a)]} \frac{1}{2V} e^{\frac{1}{V}[R-\gamma a]} dR + \left[r_2 c'(a) + \frac{\gamma}{V}\right] \int_{\gamma a}^{\infty} -e^{-r_2[s(R)-c(a)]} \frac{1}{2V} e^{-\frac{1}{V}[R-\gamma a]} dR = 0$$

Note that the compensation schedule $s(R)$ specifically *does not* depend on a, so differentiating the agent's expected utility from exerting effort a given $s(R)$ with respect to a does not require any assumptions about $s(R)$.

Solving the first-order problem yields the following wage schedule:

$$\begin{aligned} s(R &< \gamma\widehat{a}) = \frac{r_1}{r_1+r_2}R - \frac{r_1}{r_1+r_2}\gamma\widehat{a} + c(\widehat{a}) + \frac{1}{r_2}v - \frac{1}{r_2}\log\left[\left(1 - \frac{r_1 r_2}{r_1+r_2}V\right)\left(1 + \frac{r_2 c'(\widehat{a})V}{\gamma}\right)\right] \\ s(R &> \gamma\widehat{a}) = \frac{r_1}{r_1+r_2}R - \frac{r_1}{r_1+r_2}\gamma\widehat{a} + c(\widehat{a}) + \frac{1}{r_2}v - \frac{1}{r_2}\log\left[\left(1 + \frac{r_1 r_2}{r_1+r_2}V\right)\left(1 - \frac{r_2 c'(\widehat{a})V}{\gamma}\right)\right] \end{aligned}$$

where $\widehat{a}$ is the effort level "anticipated" by the principal.

Observe that if $\widehat{a} = c'^{-1}\left(\frac{r_1}{r_1+r_2}\gamma\right)$, then this compensation schedule is fully linear with slope $\frac{r_1}{r_1+r_2}$; if $\widehat{a} > c'^{-1}\left(\frac{r_1}{r_1+r_2}\gamma\right)$, then this compensation schedule is piecewise linear with a discrete jump at $\gamma\widehat{a}$, and slope $\frac{r_1}{r_1+r_2}$ everywhere else, that is, at output levels $R \neq \gamma\widehat{a}$.

What $\widehat{a}$ does the principal choose in equilibrium? Note that in equilibrium it must be that $a^* = \widehat{a}$: that is, the principal would never pay for a level of effort higher than the one she anticipates, and an agent would never exert more effort than she is compensated for.

Now observe that for all $\widehat{a} < \widehat{a}_t$, for some threshold $\widehat{a}_t > 0$, the unique stationary point of the agent r_2's expected utility given effort exertion a and compensation scheme $s(R|\widehat{a})$ is $a = \widehat{a}$. Moreover, r_2's expected utility given a and $s(R|\widehat{a})$ is strictly concave in a, when $\widehat{a} < \widehat{a}_t$.

However, once $\widehat{a} > \widehat{a}_t$, where again, this $\widehat{a}_t$ is a threshold which will be rigorously characterized shortly, there are two stationary points of r_2's expected utility given effort exertion a and compensation scheme $s(R|\widehat{a})$: $a_1 = \widehat{a}$ continues to be a stationary point, but $a_2 < \widehat{a}$ is also a stationary point, and it can be seen that a_2 is the unique maximizer. The intuition is that, if the principal tries to induce "too much" effort, the agent will find it profitable to discretely deviate downwards.

Therefore, the equilibrium $\widehat{a}$ is $\widehat{a} = \widehat{a}_t$, where $\widehat{a}_t$ is the value of $\widehat{a}$ such that the second derivative of r_2's expected utility given effort exertion a and compensation scheme $s(R|\widehat{a})$ at $a = \widehat{a}$ is precisely 0.

This yields the following expression characterizing $\widehat{a}^* = \widehat{a}_t$:

$$\left(c'(\widehat{a}_t) - \frac{r_1}{r_1+r_2}\gamma\right)\left(c'(\widehat{a}_t) + \frac{\gamma}{r_2 V}\right) = \frac{1}{r_2}c''(\widehat{a}_t) > 0$$

where it can clearly be seen that $\widehat{a}_t > c'^{-1}\left(\frac{r_1}{r_1+r_2}\gamma\right)$, since $c(a)$ is increasing and convex, and the left-hand side is increasing in a.

But we know the agent's expected utility from exerting effort a given compensation schedule $s(R|\widehat{a}^*)$ is strictly concave in a.

Therefore, the unique solution we found to the first-order problem is indeed the unique optimum.

This characerizes the equilibrium.

7.1.3 Incentive Provision and Insurance Provision

The piecewise linearity of the equilibrium wage schedule which emerges in this framework is a nice property for two key reasons. First, it's both tractable and realistic. Second, it neatly separates incentive provision from insurance provision: the linearity with slope $\frac{r_1}{r_1+r_2}$ at output levels above and below the mean captures efficient risk-sharing, while the discrete jump at the mean captures incentive provision. Because the likelihood ratio is a piecewise constant with discontinuity at the mean output level, the key information about effort contained in output realization is whether the realized output is above or below $R = \gamma\widehat{a}$. Conditional on knowing that realized output is below (above) mean output, however, no output level below (above) the mean is more informative about effort than another output level below (above) the mean. Furthermore, because the likelihood ratio assumes two "symmetric" values, $-Q$ and Q, the slope is the same for output levels above and below the mean.

To see more clearly how the piecewise linear wage scheme of the second-best solution cleanly separates incentive and insurance provision, it will be helpful to solve two cases: (a) the contracting principal between a risk-neutral principal and a risk-averse agent, where effort is not contractible, and (b) the contracting problem between a risk-averse principal and a risk-averse agent, where effort is contractible. The solution to the first case will be a step function, with one fixed wage for low output and a discretely higher fixed wage for high output, and the solution to the second case will be a perfectly linear wage with slope $\frac{r_1}{r_1+r_2}$, to capture efficient risk-sharing.

Case (a): the contracting problem between a risk-neutral principal and a risk-averse agent; effort not contractible The principal has utility $u(x) = x$, while the agent has utility $u(x) = -e^{-r_2 x}$. Effort is not contractible. Hence, the principal chooses wage $s(R)$ for the agent r_2 by solving the following problem:

$$\max_{s(R)} \int_{-\infty}^{\gamma a} (R - s(R))\frac{1}{2V}e^{\frac{1}{V}[R-\gamma a]}dR + \int_{\gamma a}^{\infty} (R - s(R))\frac{1}{2V}e^{-\frac{1}{V}[R-\gamma a]}dR$$

such that:

$$(IR): \int_{-\infty}^{\gamma a} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{\frac{1}{V}[R-\gamma a]}dR +$$

$$+\int_{\gamma a}^{\infty} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{-\frac{1}{V}[R-\gamma a]}dR \geq -e^{-v}$$

$$(IC): a \in \arg\max_{a\in(0,\infty)} \int_{-\infty}^{\gamma a} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{\frac{1}{V}[R-\gamma a]}dR+$$

$$+\int_{\gamma a}^{\infty} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{-\frac{1}{V}[R-\gamma a]}dR$$

Replacing the global IC constraint with the first-order condition and writing the Lagrangean (letting λ be the multiplier on the binding IR constraint, and μ be the multiplier on the binding first-order condition):

$$\int_{-\infty}^{\gamma a}(R-s(R))\frac{1}{2V}e^{\frac{1}{V}[R-\gamma a]}dR+\int_{\gamma a}^{\infty}(R-s(R))\frac{1}{2V}e^{-\frac{1}{V}[R-\gamma a]}dR+$$
$$+\lambda\left[\int_{-\infty}^{\gamma a} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{\frac{1}{V}[R-\gamma a]}dR+\int_{\gamma a}^{\infty} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{-\frac{1}{V}[R-\gamma a]}dR+e^{-v}\right]$$
$$+\mu\left[\left[r_2c'(a)-\frac{\gamma}{V}\right]\int_{-\infty}^{\gamma a} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{\frac{1}{V}[R-\gamma a]}dR+\left[r_2c'(a)+\frac{\gamma}{V}\right]\int_{\gamma a}^{\infty} -e^{-r_2[s(R_p)-c(a)]}\frac{1}{2V}e^{-\frac{1}{V}[R-\gamma a]}dR\right]$$

Then differentiating pointwise with respect to $s(R)$ for $R<\gamma a$ yields:

$$\begin{aligned}
-1+\lambda r_2e^{-r_2[s(R)-c(a)]}+\mu\left[r_2c'(a)-\frac{\gamma}{V}\right]r_2e^{-r_2[s(R_p)-c(a)]} &= 0 \Rightarrow \\
r_2\left(\lambda+\mu\left[r_2c'(a)-\frac{\gamma}{V}\right]\right)e^{-r_2[s(R_p)-c(a)]} &= 1 \\
\frac{1}{r_2}\log\left(r_2\left(\lambda+\mu\left[r_2c'(a)-\frac{\gamma}{V}\right]\right)\right)+c(a) &= s(R|R<\gamma a)
\end{aligned}$$

Hence, $s(R)$ for $R<\gamma a$ is:

$$s(R|R<\gamma a)=\frac{1}{r_2}\log\left(r_2\left(\lambda+\mu\left[r_2c'(a)-\frac{\gamma}{V}\right]\right)\right)+c(a)$$

And, differentiating pointwise with respect to $s(R)$ for $R>\gamma a$ yields:

$$s(R|R>\gamma a)=\frac{1}{r_2}\log\left(r_2\left(\lambda+\mu\left[r_2c'(a)+\frac{\gamma}{V}\right]\right)\right)+c(a)$$

Hence, the equilibrium wage contract for the agent r_2 is a wage fixed at a certain level for $R<\gamma\widehat{a}$, and a wage fixed at a discretely higher level for $R>\gamma\widehat{a}$, where $\widehat{a}$ is the effort level anticipated by the principal. So, the wage looks like, "flat, jump at the anticipated mean level of output, and flat again".

Case (b): the contracting problem between a risk-averse principal and a risk-avesre agent; effort contractible Now, both the principal and the agent are risk-averse with CARA utility, but effort is contractible. The principal r_1 solves the following problem:

$$\max_{a,s(R)} \int_{-\infty}^{\gamma a} -e^{-r_1(R-s(R))}\frac{1}{2V}e^{\frac{1}{V}[R-\gamma a]}dR + \int_{\gamma a}^{\infty} -e^{-r_1(R-s(R))}\frac{1}{2V}e^{-\frac{1}{V}[R-\gamma a]}dR$$

such that:

$$(IR): \int_{-\infty}^{\gamma a} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{\frac{1}{V}[R-\gamma a]}dR+$$

$$+\int_{\gamma a}^{\infty} -e^{-r_2[s(R)-c(a)]}\frac{1}{2V}e^{-\frac{1}{V}[R-\gamma a]}dR \geq -e^{-v}$$

Letting λ denote the Lagrange multiplier on the IR constraint, differentiating pointwise with respect to $s(R)$ for $R < \gamma a$ yields:

$$-r_1 e^{-r_1(R-s(R))} + \lambda r_2 e^{-r_2[s(R)-c(a)]} = 0$$

And differentiating pointwise with respect to $s(R)$ for $R > \gamma a$ yields:

$$-r_1 e^{-r_1(R-s(R))} + \lambda r_2 e^{-r_2[s(R)-c(a)]} = 0$$

the same condition.

Hence:

$$s(R|\widehat{a}) = \frac{r_1}{r_1+r_2}R + \frac{r_2}{r_1+r_2}c(\widehat{a})\frac{1}{r_1+r_2}\log\left(\lambda\frac{r_2}{r_1}\right)$$

a linear wage schedule with slope $\frac{r_1}{r_1+r_2}$.

The key takeaway from both of these cases is that, in this model, it is possible to see the effect of both the principal and the agent being risk-averse on the equilibrium wage, *and* the effect of effort not being observable or contractible. The equilibrium wage schedule that emerges when both of these components are combined is the wage schedule which is linear at output levels away from the mean (as in Case (b)), and where there is a discontinuous jump at the mean (as in Case (a)).

7.2 A2: The Equilibrium for a Given Pair in the First-Best

First, suppose principals have a more general increasing, strictly concave utility function given by $U(x;p)$, and agents have a more general increasing, strictly concave utility function given by $V(x;q)$.

Then, observe that the constraint set is convex. That is, for a given a, if $s_1(R)$ satisfies IR, and $s_2(R)$ satisfies IR, then $\alpha s_1(R) + (1-\alpha)s_2(R)$ also satisfies IR.

If $s_1(R)$ satisfies IR, and $s_2(R)$ satisfies IR, then:

$$\int_{-\infty}^{\infty} V(s_1(R) - c(a))f(R-\gamma a)dR \geq v$$
$$\int_{-\infty}^{\infty} V(s_2(R) - c(a))f(R-\gamma a)dR \geq v$$

Then: use the concavity of $V(\cdot)$:

$$\begin{aligned}
\int_{-\infty}^{\infty} V(\alpha s_1(R) + (1-\alpha)s_2(R) - c(a))f(R-\gamma a)dR &= \\
\int_{-\infty}^{\infty} V[\alpha(s_1(R) - c(a)) + (1-\alpha)(s_2(R) - c(a))]f(R-\gamma a)dR &> \\
\int_{-\infty}^{\infty} \alpha V(s_1(R) - c(a))f(R-\gamma a)dR + \int_{-\infty}^{\infty} (1-\alpha)V(s_2(R) - c(a))f(R-\gamma a)dR &\geq \\
\alpha v + (1-\alpha)v &= v
\end{aligned}$$

And, the objective function is strictly quasiconcave: the objective function is:

$$\int_{-\infty}^{\infty} U(R - s(R))f(R-\gamma a)dR$$

Fix a. Then, pointwise differentiate twice wrt $s(R)$ for each R:

$$\begin{aligned}
f' &: -U'(R - s(R))f(R-\gamma a) \\
f'' &: U''(R - s(R))f(R-\gamma a) < 0
\end{aligned}$$

since $U(\cdot)$ is concave.

So, there should be a unique global constrained maximizer.

Now, let's solve for the first-best equilibrium when principals and agents explicitly have CARA utility, and differ in their Arrow-Pratt coefficient of risk aversion, r.

The first-order conditions of the optimization problem are:

$$
\begin{aligned}
\frac{r_1}{r_2} e^{-r_1 R + (r_1+r_2)s(R) - r_2 c(a)} &= \lambda \;\; \textit{for each } R \\
\frac{c'(a)}{\gamma} \frac{\int_{-\infty}^{\infty} r_2 e^{-r_2[s(R)-c(a)]} f(R-\gamma a) dR_p}{\int_{-\infty}^{\infty} e^{-r_1[R-s(R)]} f'(R-\gamma a) dR_p} - \frac{\int_{-\infty}^{\infty} e^{-r_2[s(R)-c(a)]} f'(R-\gamma a) dR}{\int_{-\infty}^{\infty} e^{-r_1[R-s(R)]} f'(R-\gamma a) dR} &= \frac{1}{\lambda} \\
\int_{-\infty}^{\infty} -e^{-r_2[s(R)-c(a)]} f(R-\gamma a) dR &= -e^{-v}
\end{aligned}
$$

We know that in the first-best, under the general $U(x;p), V(x;q)$ utility functions, the slope of the equilibrium wage will be:

$$
s'_{FB}(R) = \frac{1}{1 + \left(-\frac{U'(R-s(R))}{U''(R-s(R))}\right)\left(-\frac{V''(s(R)-c(a))}{V'(s(R)-c(a))}\right)}
$$

When the utility functions are CARA, this expression is equal to $\frac{r_1}{r_1+r_2}$. So, under CARA utility, the equilibrium wage of the agent is linear with slope $\frac{r_1}{r_1+r_2}$: this means that $s^*_{FB}(R_p) = \frac{r_1}{r_1+r_2} R_p + K$, where K is some constant depending on parameters. This is intuitive: effort is contractible, so there is no need to provide incentives. Hence the sharing rule efficiently shares risk; this means the schedule is linear. Moreover, the less risk-averse individual will have the share that is more dependent on output realization.

Thus, our conditions are:

$$
\begin{aligned}
\frac{r_1}{r_2} e^{(r_1+r_2)K - r_2 c(a)} &= \lambda \\
\frac{c'(a) r_2 e^{-(r_1+r_2)K} e^{r_2 c(a)}}{\gamma} \frac{\int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1+r_2}R} f(R-\gamma a) dR}{\int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1+r_2}R} f'(R-\gamma a) dR} - e^{-(r_1+r_2)K} e^{r_2 c(a)} &= \frac{1}{\lambda} \\
e^{-r_2(K-c(a))} \int_{-\infty}^{\infty} -e^{-\frac{r_1 r_2}{r_1+r_2}R} f(R-\gamma a) dR &= -e^{-v}
\end{aligned}
$$

Observe that we can integrate by parts:

$$
\begin{aligned}
\int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1+r_2}R} f'(R-\gamma a) dR &= e^{-\frac{r_1 r_2}{r_1+r_2}R} f(R-\gamma a)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \left(-\frac{r_1 r_2}{r_1+r_2}\right) e^{-\frac{r_1 r_2}{r_1+r_2}R} f(R-\gamma a) dR \\
&= \frac{r_1 r_2}{r_1+r_2} \int_{-\infty}^{\infty} -e^{-\frac{r_1 r_2}{r_1+r_2}R} f(R-\gamma a) dR
\end{aligned}
$$

Hence:

$$
\frac{\int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1+r_2}R} f(R-\gamma a) dR}{\int_{-\infty}^{\infty} e^{-\frac{r_1 r_2}{r_1+r_2}R} f'(R-\gamma a) dR} = \frac{r_1+r_2}{r_1 r_2}
$$

Then the first two conditions imply:

$$\begin{aligned}
\frac{r_2}{r_1}e^{-(r_1+r_2)K+r_2c(a)} &= \frac{c'(a)r_2e^{-(r_1+r_2)K}e^{r_2c(a)}}{\gamma}\left(\frac{r_1+r_2}{r_1r_2}\right) - e^{-(r_1+r_2)K}e^{r_2c(a)} \Rightarrow \\
a^*_{FB} &= c'^{-1}(\gamma)
\end{aligned}$$

Hence, first-best effort is independent of risk type. Any employer r_1 who hires an employee r_2 will require her to exert effort $a^*_{FB} = c'^{-1}(\gamma)$. That is, first-best effort equates marginal benefit of effort (marginal impact of effort on mean output) with marginal cost.

Finally, use K to satisfy r_2's IR constraint:

$$e^{-r_2(K-c(c'^{-1}(\gamma)))}\int_{-\infty}^{\infty} -e^{-\frac{r_1r_2}{r_1+r_2}R}f(R-\gamma c'^{-1}(\gamma))dR = -e^{-v} \Rightarrow$$

$$K^*_{FB}(r_1,r_2,v) = \frac{1}{r_2}v + c(c'^{-1}(\gamma)) + \frac{1}{r_2}\log\left(\int_{-\infty}^{\infty} e^{-\frac{r_1r_2}{r_1+r_2}R}f(R-\gamma c'^{-1}(\gamma))dR\right)$$

Hence, the optimal first-best contract is:

$$\begin{aligned}
s^*_{FB}(R) &= \frac{r_1}{r_1+r_2}R + K^*_{FB}(r_1,r_2,v) \\
K^*_{FB}(r_1,r_2,v) &= \frac{1}{r_2}v + c(c'^{-1}(\gamma)) + \frac{1}{r_2}\log\left(\int_{-\infty}^{\infty} e^{-\frac{r_1r_2}{r_1+r_2}R}f(R-\gamma c'^{-1}(\gamma))dR\right) \\
a^*_{FB} &= c'^{-1}(\gamma)
\end{aligned}$$

8 A3: The Second-Best: Proof of the Main Result

We know that the sum of certainty-equivalents in a pair (r_1, r_2) is:

$$\begin{aligned}
CE(r_1,r_2) &= \gamma\widehat{a}_t - c(\widehat{a}_t) \\
&\quad -\frac{1}{r_1}\log\left(\frac{1}{2}\left[\frac{1}{\left[1+\frac{r_2c'(\widehat{a}_t)V}{\gamma}\right]^{\frac{r_1}{r_2}}\left[1-\frac{r_1r_2}{r_1+r_2}V\right]^{1+\frac{r_1}{r_2}}} + \frac{1}{\left[1-\frac{r_2c'(\widehat{a}_t)V}{\gamma}\right]^{\frac{r_1}{r_2}}\left[1+\frac{r_1r_2}{r_1+r_2}V\right]^{1+\frac{r_1}{r_2}}}\right]\right)
\end{aligned}$$

where $\widehat{a}_t$ is characterized by:

$$\left(c'(\widehat{a}_t) - \frac{r_1}{r_1+r_2}\gamma\right)\left(c'(\widehat{a}_t) + \frac{\gamma}{r_2V}\right) = \frac{1}{r_2}c''(\widehat{a}_t) > 0$$

We can easily see that $\widehat{a}_t > c'^{-1}\left(\frac{r_1}{r_1+r_2}\gamma\right)$.

Replacing the functional form for cost of effort into the expressions:

$$
\begin{aligned}
c(a) &= \eta a^M, M > 1 \\
c'(a) &= \eta M a^{M-1} \\
c''(a) &= \eta M(M-1)a^{M-2}
\end{aligned}
$$

Then the expressions characterizing the pairwise sum and the threshold $\widehat{a}_t$ become:

$$
\begin{aligned}
CE(r_1, r_2) &= \gamma \widehat{a}_t - c(\widehat{a}_t) \\
&\quad - \frac{1}{r_1} \log \left(\frac{1}{2} \left[\frac{1}{\left[1 + \frac{r_2 \eta M \widehat{a}_t^{M-1} V}{\gamma}\right]^{\frac{r_1}{r_2}} \left[1 - \frac{r_1 r_2}{r_1 + r_2} V\right]^{1 + \frac{r_1}{r_2}}} + \frac{1}{\left[1 - \frac{r_2 \eta M \widehat{a}_t^{M-1} V}{\gamma}\right]^{\frac{r_1}{r_2}} \left[1 + \frac{r_1 r_2}{r_1 + r_2} V\right]^{1 + \frac{r_1}{r_2}}} \right] \right)
\end{aligned}
$$

and:

$$
\left(\eta M \widehat{a}_t^{M-1} - \frac{r_1}{r_1 + r_2} \gamma \right) \left(\eta M \widehat{a}_t^{M-1} + \frac{\gamma}{r_2 V} \right) = \frac{1}{r_2} \eta M(M-1) \widehat{a}_t^{M-2}
$$

Finding the conditions for supermodularity and submodularity produces the comparative statics.