

BA 932 Session 8 Real Effects of Accounting Disclosure

Xu Jiang

April 11th, 2023

What we have done so far regarding disclosure

- Disclosure models are about an informed party (the firm/manager) disclosing to an uninformed party (the investors)
- There is conflict of interest between the informed party and the uninformed party
- We talked about ex-post disclosure models and ex-ante disclosure models with a focus on conservatism
- Each has different underlying assumptions, which may be valid in different settings

This class - real effects of accounting disclosure

- Incorporate manager's real decisions; both the manager's real decisions and disclosure affect market perception and therefore short-term prices; most papers are about ex-ante disclosure models with a focus on disclosure precision
- Two necessary condition to generate investment distortions (relative to first-best):
 - The manager and the market have different objective functions
 - The manager knows something that the market does not know and there is no credible way to perfectly communicate such information
 - Law of iterated expectations will not apply, generating investment distortions

How do real effects literature introduce different objective functions

- Similar to many papers we see before, they assume that manager has to sell the firm (at least partially) after disclosure but before all uncertainties are realized.
- Managers therefore care about short-term stock price - also referred to as “managerial myopia”.
- Such incentive of the manager can also come from some investors being short-term and only caring about short-term prices -see Kanodia and Saprà (2016) for a nice discussion of this issue

What is managerial myopia?

- Managers take myopic actions (e.g. manage earnings, cut R&D expenses, choose risky projects) to maximize their payoffs at the expense of firm value.
- Implicitly, this implies the existence of an efficient benchmark such that the actions that are optimal for the managers deviate from this efficient benchmark.
- Thus, a necessary but not sufficient condition for managerial myopia is the existence of frictions.
- In general, two conditions are needed for managerial myopia to make a difference: information asymmetry and managerial focus on short-term price (rather than on long-term value). Absence of either condition would make managerial myopia not an issue.

The basic managerial myopia setup

- Three dates $t = 0, 1, 2$.
- The manager has some private signal θ that, combined with his action a , will affect the firm's terminal value, denoted as $V(a(\theta), \theta, \varepsilon)$ where ε represents some random shock not known to anybody in advance. Note that a can be a function of θ .
- At date 2 all uncertainties will be resolved and thus $P_2 = V(a(\theta), \theta, \varepsilon)$.
- At date 1 price $P_1 = E[V(a(\theta), \theta, \varepsilon) | I_1]$ where I_1 contains the market's information at date 1 which is usually a subset of θ , i.e. knowing I_1 is usually insufficient to infer θ .
- The manager chooses a to maximize a weighted average of prices, $\alpha E[P_1 | I_0] + (1 - \alpha) E[P_2 | I_0]$ where $\alpha \in (0, 1)$ and I_0 contains the manager's information at date 0 and $I_0 = \{\theta, a(\theta)\}$.
- First best: a_{FB} maximizes $E[V(a(\theta), \theta, \varepsilon) | a, \theta]$, i.e. $\frac{\partial}{\partial a} E[V(a(\theta), \theta, \varepsilon) | a, \theta] |_{a=a_{FB}} = 0$.

Myopia is not an issue if there is no informational friction

- When θ is known by the market, the market will also learn $a(\theta)$, resulting in $I_1 = \{\theta, a(\theta)\}$.
- Thus $E[P_1|I_0] = E[E[V(a(\theta), \theta, \varepsilon)|I_1]|I_0] = E[V(a(\theta), \theta, \varepsilon)|a, \theta]$ and $E[P_2|I_0] = E[V(a(\theta), \theta, \varepsilon)|a, \theta]$.
- Therefore optimal a maximizes $\alpha E[P_1|I_0] + (1 - \alpha)E[P_2|I_0] = E[V(a(\theta), \theta, \varepsilon)|a, \theta]$ so $a^* = a_{FB}$ and myopia is not an issue.
- Intuition: when there is no informational friction, expectation of the market is the same as the expectation of the manager as they have the same information set, resulting in $E[P_1|I_0] = E[P_2|I_0]$.

Myopia is not an issue if alpha is zero, even when there is informational friction at date 1

- When $\alpha = 0$, manager chooses a to maximize $E[P_2|I_0] = E[V(a(\theta), \theta, \varepsilon)|a, \theta]$ so $a^* = a_{FB}$ and myopia is not an issue.
- Intuition: when market only cares about terminal payoffs, P_1 does not matter and manager only cares about long-term.

Myopia will be an issue when alpha is greater than zero and there is informational friction at date 1

- In this case $E[P_1|I_0] = E[E[V(a(\theta), \theta, \varepsilon)|I_1]|I_0] \neq E[V(a(\theta), \theta, \varepsilon)|I_0]$ as law of iterated expectations does not apply when $I_1 \subset I_0$, i.e. manager knows more than the market. Law of iterated expectations will apply if $I_1 \supseteq I_0$, i.e. the market knows at least as much as the manager.
- Therefore optimal a maximizes $\alpha E[P_1|I_0] + (1 - \alpha)E[P_2|I_0] \neq E[V(a(\theta), \theta, \varepsilon)|a, \theta]$ so $a^* \neq a_{FB}$ in general and myopia becomes an issue.
- Will making I_1 more informative (i.e. more transparency) help? Not unless I_1 directly reveals θ . Intuition: in an economy with multiple frictions, removing one friction does not necessarily make things better off.
- We will see this in Kanodia et al. (2005).

Stein (1989): First paper to model myopia

- θ as current true earnings that is unobservable to the market, $a(\theta)$ as the amount of earnings management to increase current earnings but reduce long-term firm value and is thus inefficient.
- $V(a(\theta), \theta, \varepsilon) = \theta + \beta\theta - c(a(\theta)) + \varepsilon$ where θ is current true earnings and $\beta\theta$ is long-term firm value as a function of current true earnings. Note that any earnings management $a(\theta)$ reduces long-term firm value. Thus, in a first-best situation $a(\theta) = 0$.

Stein (1989): First paper to model myopia

- $P_1 = E[V(a(\theta), \theta, \varepsilon) | I_1]$ where $I_1 = r = \theta + a(\theta)$;
 $P_2 = E[V(a(\theta), \theta, \varepsilon) | I_2]$ where $I_2 = \theta$.
- The manager chooses $a(\theta)$ to maximize $E[\alpha P_1 + (1 - \alpha) P_2 | \theta]$. The market conjectures that the manager will choose \hat{a} and will set $P_1 = r - \hat{a} + \beta(r - \hat{a}) - c(\hat{a})$. The manager therefore chooses $a(\theta)$ to maximize
$$\alpha[r - \hat{a} + \beta(r - \hat{a}) - c(\hat{a}) | \theta, a] + (1 - \alpha)[\theta + \beta\theta - c(a(\theta)) | \theta, a].$$
First order condition results in $\alpha(1 + \beta) = (1 - \alpha)c'(a(\theta))$.
Therefore the higher α (i.e. the more myopic the manager is), the higher the amount of earnings management. When $\alpha = 0$, $a(\theta) = 0$ and we are back at the first-best.

- It provides a justification for why periodic performance reports are needed.
- As in a typical real effects setting, the manager/current shareholders sells the firm to future shareholders.
- In the absence of information asymmetry, no disclosure should be provided if assuming that the future shareholders are less risk averse -> future shareholders should bear more risk, which is destroyed by disclosure
- In the presence of information asymmetry, disclosure may be beneficial as it disciplines the manager's investment behavior -> you do not want to invest inefficiently as it will show up in disclosed reports, leading to a lower market price

- Three dates $t = 0, 1, 2$.
 - Date 0: a firm owned by current generation of investors observe the expected profitability of a risky project $\tilde{\mu}$ and choose the amount k to invest in the project k . k is perfectly observable. The terminal cash flow $\tilde{\theta} = k(\tilde{\mu} + \tilde{\gamma})$. $\tilde{\mu}$ distributed on $[\mu_H, \mu_L]$ with density f and $\tilde{\theta} \sim N(0, \sigma)$.
 - Date 1: an accounting report is published as $\tilde{y} = \tilde{\theta} + k\tilde{\varepsilon} = k(\tilde{\mu} + \tilde{\gamma} + \tilde{\varepsilon})$ where $\tilde{\varepsilon} \sim N(0, \sigma_\varepsilon^2)$. The firm is then sold to a new generation of investors at a price \tilde{P} (random from date 0 perspective).
 - Date 2: terminal cash flow $\tilde{\theta}$ is realized. Current generation of investors consume $\tilde{P} - k$ while new generation of investors consume $\tilde{\theta} - \tilde{P}$.

KL- more details

- Current generation of investors with CARA utility function and risk aversion coefficient $\frac{1}{2}\rho_e$
- New generation of investors with CARA utility function aggregate risk aversion $\lambda = \frac{1}{\sum_i \rho_i}$
- Assume that $\lambda < \frac{1}{2}\rho_e$, i.e., new generation of investors are less risk-averse (not unreasonable if thinking of current investors as VCs and new investors as the broad market)
- Define $\beta \equiv \frac{\sigma}{\sigma + \sigma_\varepsilon^2}$ as the measure of disclosure quality. No disclosure is equivalent to $\beta = 0$ and perfect disclosure (i.e., no measurement noise) is equivalent to $\beta = 1$. Since β monotonically decreases with σ_ε^2 , from now on we will focus on variations of β as variations of disclosure quality.

KL - investors' payoff function

- CARA utility combined with normal distribution implies that
 - Current shareholders choose k and β at date 0 to maximize $-k + E(\tilde{P}|\mu) - \frac{1}{2}\rho_e \text{var}(\tilde{P}|\mu)$.
 - New generation of investors set $P(y, k, \beta) = E(\tilde{\theta}|y, k, \beta) - \lambda \text{var}(\tilde{\theta}|y, k, \beta)$.

KL - First-best benchmark

- First best refers to the no friction case. In this model, the friction is the unobservability of $\tilde{\mu}$ to the new generation of investors. So, first best is equivalent to $\tilde{\mu}$ being observed by everybody.
- Since $\tilde{\mu}$ can be observed, there is no inference problem, \tilde{y} is used merely for another piece of information for statistical updating.
- Bayesian updating gives $E(\tilde{\theta}|y, k, \mu, \beta) = \beta y + (1 - \beta)k\mu$ and $var(\tilde{\theta}|y, k, \mu, \beta) = (1 - \beta)\sigma k^2$.
- Insert into $P(y, k, \mu, \beta) = E(\tilde{\theta}|y, k, \mu, \beta) - \lambda var(\tilde{\theta}|y, k, \mu, \beta)$ we have that $P(y, k, \mu, \beta) = \beta y + (1 - \beta)k\mu - \lambda(1 - \beta)\sigma k^2$.
- Insert into current shareholders' objective function $-k + E(\tilde{P}|\mu) - \frac{1}{2}\rho_e var(\tilde{P}|\mu)$ the objective function becomes $-k + k\mu - \sigma k^2[\beta\frac{1}{2}\rho_e + (1 - \beta)\lambda]$.

- First order condition gives

$$k_{FB} = \frac{\mu - 1}{2\sigma[\beta\frac{1}{2}\rho_e + (1 - \beta)\lambda]}$$

- We assume that $\frac{1}{2}\rho_e > \lambda$, i.e., current shareholders are more risk-averse than new generation of shareholders, resulting in $\frac{\partial k}{\partial \beta} < 0$, i.e., more precise accounting disclosure results in lower level of investment.

- Intuition: when there is no information asymmetry but only measurement noise, selling in the intermediate period involves the allocation of total risk $\tilde{\theta} - k$ between current shareholders and new shareholders. Since current shareholders are more risk averse than new generation of investors, optimal risk sharing implies new generation of investors will bear as much risk as possible. Disclosure, however, destroys such risk-bearing opportunity by reducing the risk the new generation of investors will bear.
- Both generation of investors will be better off without any disclosure.

KL - Solving for the main model

- When μ is not observable by new investors, they have to make inferences of μ based on y , k and β .
- Current investors will signal μ by their choice of k and β .
- This is a typical signalling model when informed party signal their type through their choices.
- However, such choice is not credible if there is no y . (Why?) Thus, y serves a disciplinary role.

KL - Solving for the signaling equilibrium

- Assume that there is a fully-revealing signalling equilibrium, i.e., type μ is completely revealed through the choice of $\{k(\mu), \beta(\mu)\}$.
- The expected payoff of type μ current investor who chooses $\{k(\mu_r), \beta(\mu_r)\}$ (i.e., made the choice that signal a type of μ_r) will be

$$W(k, \beta, \mu_r; \mu) = -k + \beta k \mu + (1 - \beta) k \mu_r - \lambda(1 - \beta) \sigma k^2 - \frac{1}{2} \rho_e \beta \sigma k^2$$

- A necessary condition for a fully revealing signalling equilibrium to exist is the satisfaction of single-crossing properties, which requires $W_{k\mu} > 0$. In this case $W_{k\mu} = \beta$, which is positive if $\beta > 0$.
- Incentive compatibility condition requires that $W(k(\mu), \beta(\mu), \mu; \mu) \geq W(k(\mu'), \beta(\mu'), \mu'; \mu) \quad \forall \mu, \mu'$.

KL - Solving for the signaling equilibrium

- Denote $V(\mu) \equiv W(k(\mu), \beta(\mu), \mu; \mu) = -k(\mu) + k(\mu)\mu - \sigma k^2(\mu)[\beta(\mu)\frac{1}{2}\rho_e + (1 - \beta(\mu))\lambda] = k(\mu)(\mu - 1) - \sigma k^2(\mu)(\lambda + \alpha\beta(\mu))$ where $\alpha = \frac{1}{2}\rho_e - \lambda > 0$.
- Then $V(\mu, \mu') \equiv W(k(\mu'), \beta(\mu'), \mu'; \mu) = -k(\mu') + \beta(\mu')k(\mu')\mu + (1 - \beta(\mu'))k(\mu')\mu' - \sigma k^2(\mu')(\lambda + \alpha\beta(\mu')) = V(\mu') - \beta(\mu')k(\mu')(\mu' - \mu)$.
- Thus, the incentive compatibility constraint can be rewritten as $V(\mu) \geq V(\mu') - \beta(\mu')k(\mu')(\mu' - \mu) \quad \forall \mu, \mu'$.
- This constraint holds if and only if (i) $V'(\mu) = \beta(\mu)k(\mu)$ and (ii) $\beta(\mu)k(\mu)$ increases in μ (Theorem 1).

KL - Proof of Theorem 1

- First prove the only if part: The incentive compatibility constraints result in

$$V(\mu) \geq V(\mu') - \beta(\mu')k(\mu')(\mu' - \mu)$$

and

$$V(\mu') \geq V(\mu) - \beta(\mu)k(\mu)(\mu - \mu')$$

so

$$\beta(\mu)k(\mu)(\mu' - \mu) \leq V(\mu') - V(\mu) \leq \beta(\mu')k(\mu')(\mu' - \mu).$$

- For any $\mu' > \mu$ we have $\beta(\mu)k(\mu) \leq \beta(\mu')k(\mu')$ so $\beta(\mu)k(\mu)$ is increasing in μ .

KL - Proof of Theorem 1

- Also, from above we have

$$\beta(\mu)k(\mu) \leq \frac{V(\mu') - V(\mu)}{\mu' - \mu} \leq \beta(\mu')k(\mu')$$

- Take the limit of $\mu' \rightarrow \mu$ results in $V'(\mu) = \beta(\mu)k(\mu)$. The proof of the only if part is complete.

KL - Proof of Theorem 1

- Now prove the if part. Integrating $V'(\mu) = \beta(\mu)k(\mu)$ up from μ_L results in

$$V(\mu) = V(\mu_L) + \int_{\mu_L}^{\mu} k(t)\beta(t)dt.$$

- Therefore

$$\begin{aligned} & V(\mu) - V(\mu, \mu') \\ = & [V(\mu) - V(\mu')] - [V(\mu, \mu') - V(\mu')] \\ = & \int_{\mu'}^{\mu} k(t)\beta(t)dt - k(\mu')\beta(\mu')(\mu - \mu') \\ = & \int_{\mu'}^{\mu} [k(t)\beta(t) - k(\mu')\beta(\mu')]dt. \end{aligned}$$

- If $\mu' < (>) \mu$, the integral is non-negative as $k(t)\beta(t) \geq (\leq) k(\mu')\beta(\mu')$ for any $t > (<) \mu'$. Therefore, $V(\mu) \geq V(\mu, \mu') \forall \mu, \mu'$.

KL - Solving for the signaling equilibrium

- The current investor is now choosing $\{\beta(\mu), k(\mu)\}$ to maximize $\int V(\mu)f(\mu)d\mu$ subject to (i) $V'(\mu) = \beta(\mu)k(\mu)$ and (ii) $\beta(\mu)k(\mu)$ increases in μ .
- Ignore the monotonicity constraint first and later check that the solution satisfies the constraint.
- Use optimal control theory, let Hamiltonian $H(k(\mu), \beta(\mu)) = V(\mu)f(\mu) + L(\mu)\beta(\mu)k(\mu)$ where $L(\mu)$ is the Lagrangian multiplier for constraint (i).

KL - Solving for the signaling equilibrium

- First order condition with respect to $\beta(\mu)$ and $k(\mu)$ will yield:

$$\{\mu - 1 - k(\mu)2\sigma(\lambda + \alpha\beta(\mu))\}f(\mu) + L(\mu)\beta(\mu) = 0,$$

and

$$-k^2(\mu)\sigma\alpha f(\mu) + L(\mu)k(\mu) = 0.$$

- Dividing the above two equations result in

$$\frac{\mu - 1 - k(\mu)2\sigma(\lambda + \alpha\beta(\mu))}{-k^2(\mu)\sigma\alpha} = \frac{\beta(\mu)}{k(\mu)}.$$

Solving the equation above gives

$$k(\mu) = \frac{\mu - 1}{\sigma(2\lambda + \alpha\beta(\mu))}.$$

KL - Comparing with first-best

- We have $k(\mu) = \frac{\mu-1}{\sigma(2\lambda+\alpha\beta(\mu))}$
- Note that $k^{FB} = \frac{\mu-1}{2\sigma(\lambda+\alpha\beta)}$
- we can see that so long as $\beta > 0$ there will be overinvestment.
- Intuition: high types overinvest to make it more difficult for low types to mimic them (almost universal phenomena among signalling models).

KL - Still need to verify that monotonicity condition is satisfied

- Inserting $k(\mu) = \frac{\mu-1}{\sigma(2\lambda+\alpha\beta(\mu))}$ into the

$$V(\mu) = k(\mu)(\mu - 1) - \sigma k^2(\mu)(\lambda + \alpha\beta(\mu))$$

$$\begin{aligned} V(\mu) &= k^2\sigma(2\lambda + \alpha\beta(\mu)) - \sigma k^2(\mu)(\lambda + \alpha\beta(\mu)) \\ &= \lambda\sigma k^2(\mu). \end{aligned}$$

- Taking derivative of $V(\mu)$ with respect to μ and using the IC constraint $V'(\mu) = \beta(\mu)k(\mu)$ we have $k'(\mu) = \frac{\beta(\mu)}{2\lambda\sigma}$. Also using equation $k(\mu) = \frac{\mu-1}{\sigma(2\lambda+\alpha\beta(\mu))}$ is equivalent to $k(\mu)((2\lambda + \alpha\beta(\mu))) = \frac{\mu-1}{\sigma}$. Taking derivative with respect to μ and using $k'(\mu) = \frac{\beta(\mu)}{2\lambda\sigma}$ will give us $\frac{d}{d\mu}\{\beta(\mu)k(\mu)\} = \frac{1-\beta(\mu)}{\sigma\alpha}$. Thus, the monotonicity constraint is satisfied. (Theorem 2)

- $\beta(\mu_L) = 0$ and $\beta(\mu) > 0 \forall \mu > \mu_L$. (Theorem 3)
- This is typical in the signalling models as there is no distortion for the lowest type (i.e., first-best solution) and all higher types have a less efficient second-best solution. Again, the intuition is that the highest type involve in such inefficient behavior to prevent the low types from mimicking.

KL - Proof of Theorem 3

- From $V(\mu) = V(\mu_L) + \int_{\mu_L}^{\mu} k(t)\beta(t)dt$ it is clear that $\beta(\mu_L)$ and $k(\mu_L)$ affect $V(\mu)$ only through $V(\mu_L)$ and the bigger $V(\mu_L)$ is, the bigger $V(\mu)$ is. Therefore, $\beta(\mu_L)$ and $k(\mu_L)$ has to be chosen to maximize $V(\mu_L)$. Since $V(\mu_L) = \lambda\sigma k^2(\mu_L)$ and $k(\mu_L) = \frac{\mu_L - 1}{\sigma(2\lambda + \alpha\beta(\mu_L))}$, $\beta(\mu_L) = 0$ and $k(\mu_L) = \frac{\mu_L - 1}{2\sigma\lambda}$ maximize $V(\mu_L)$.
- We now show that $\beta(\mu) > 0$ for any $\mu > \mu_L$. Consider any μ_1, μ_2 s.t. $\mu_L < \mu_1 < \mu_2$. IC constraint thus requires that

$$V(\mu_1) \geq V(\mu_2) - \beta(\mu_2)k(\mu_2)(\mu_2 - \mu_1).$$

In addition, for any μ ,

$$V(\mu) = \lambda\sigma k^2(\mu) = \frac{\lambda\sigma(\mu - 1)^2}{[\sigma(2\lambda + \alpha\beta(\mu_L))]^2}.$$

- If $\beta(\mu_2) = 0$, then

$$\begin{aligned} & V(\mu_2) - \beta(\mu_2)k(\mu_2)(\mu_2 - \mu_1) \\ = & V(\mu_2) = \frac{\lambda\sigma(\mu_2 - 1)^2}{[\sigma(2\lambda)]^2} \\ > & \frac{\lambda\sigma(\mu_1 - 1)^2}{[\sigma(2\lambda)]^2} \geq \frac{\lambda\sigma(\mu_1 - 1)^2}{[\sigma(2\lambda + \alpha\beta(\mu_1))]^2} = V(\mu_1), \end{aligned}$$

which is a contradiction. Therefore, $\beta(\mu) > 0$ for any $\mu > \mu_L$.

- Expanding $\frac{d}{d\mu}\{\beta(\mu)k(\mu)\} = \frac{1-\beta(\mu)}{\sigma\alpha}$ and using $k'(\mu) = \frac{\beta(\mu)}{2\lambda\sigma}$ we will have

$$2\beta'(\mu)k(\mu)\sigma\alpha\lambda = 2\lambda - 2\lambda\beta(\mu) - \alpha\beta^2(\mu).$$

- Note that the right hand side is strictly decreasing in β , strictly positive at $\beta = 0$ and strictly negative at $\beta = 1$. Thus, there is a unique solution $\beta_F \in (0, 1)$ such that $2\lambda - 2\lambda\beta_F - \alpha\beta_F^2 = 0$. Note that this implies $2\beta'(\mu)k(\mu)\sigma\alpha\lambda = 0$ thus $\beta'(\mu) = 0$. This means that $\beta(\mu_L) = 0$ will then increase as μ increases until it reaches $\beta_F < 1$. Then β will stay at β_F even as μ increases further. Intuition: recall that too large β is inefficient risk-sharing. When β becomes sufficiently large, this effect outweighs the disciplinary effect of more precise financial reports

Summary of KL

- Illustration of the real effects of financial disclosure: discipline firm into more efficient investment behavior
- Why does the discipline work? Firms choose investment to maximize prices and prices respond to earnings reports and thus the disciplinary behavior of earnings
- Firms with low μ want to choose high k to mimic firms with high μ , but they are not able to because firms with high μ can also choose high precision of financial reports that is too costly for firms with low μ
- Higher financial reporting quality thus results in lower overinvestment (Biddle, Hilary and Verdi 2009)
- Sapra (2002) applies this insight in studying the real effect of hedging disclosures

Kanodia, Singh and Spero (2005) (KSS) overview

- Reporting noise usually considered to be not desirable (as you have seen in agency models last class)
- In a model with real effects and multiple frictions, KSS show that this is not necessarily the case.
- Reason: removing the friction of measurement noise may exacerbate other frictions, making the economy worse off

- Manager privately observes θ and chooses k to maximize the selling price of the firm, $\theta k - c(k) + v(k, \theta)$. k may be measured with noise
- $v_k(k, \theta) > 0$, $v_{kk}(k, \theta) \leq 0$, $v_\theta(k, \theta) > 0$, $v_{k\theta}(k, \theta) > 0$, $\forall k, \theta$.
- Again: information asymmetry between the manager and the firm and that the manager only cares about short-term prices
- Two frictions: information asymmetry about θ and measurement noise about k

KSS - First-best Benchmark

- First best refers to the no friction case. In this model, the friction is the unobservability of θ to the buyer of the firm and the measurement noise of k . So, first best is equivalent to θ and k being observed by everybody.
- First-order condition: $\theta - c_k + v_k(k, \theta) = 0$. Optimal k increases with θ .

KSS - Second-best Benchmark 1

- θ is known but k is measured and reported imprecisely. Denote accounting report on k as s with $f(s|k)$ as the distribution satisfying MLRP (i.e., higher k is good news in the sense of Milgrom and shifts $f(s|k)$ to the right in the sense of FOSD) and having fixed support $[\underline{s}, \bar{s}]$.
- Capital market price is $\varphi(s, \theta)$.
- Definition of equilibrium: $\{k(\theta), \varphi(s, \theta)\}$ s.t.
 - 1 $k(\theta) \in \arg \max_k \theta k - c(k) + \int_{\underline{s}}^{\bar{s}} \varphi(s, \theta) f(s|k) ds$
 - 2 $\varphi(s, \theta) = E[v(k(\theta), \theta) | s]$

KSS - Second-best Benchmark 1

- Result: $\frac{\partial \varphi}{\partial k} = 0$ and FOC becomes $\theta = c'(k)$, i.e., optimal k only maximizes short-term price. We have myopic investment and severe underinvestment.
- Intuition: When θ is known, investor can exactly pin down $k(\theta)$ and attribute any difference between s and $k(\theta)$ to noise.

KSS - Second-best Benchmark 2

- θ is not known but k is measured perfectly. We now have a signalling equilibrium where manager uses the investment k to signal private information θ .
- Investors conjecture manager's investment schedule $k(\theta)$, which is confirmed in equilibrium.

- Definition of equilibrium: $\{k(\theta), \varphi(k), I(k)\}$ s.t.

- ① $k(\theta) \in \arg \max_k \theta k - c(k) + \varphi(k)$
- ② $\varphi(k) = v(k, I(k))$
- ③ $I(k(\theta)) = \theta, \forall \theta.$

- Note that 3 implies that $k(\theta)$ has to be incentive compatible, which requires that

$$\begin{aligned} \Omega(\theta) &= \theta k(\theta) - c(k(\theta)) + v(k(\theta), \theta) \geq \\ \theta k(\theta') - c(k(\theta')) + v(k(\theta'), \theta') &= \Omega(\theta') - k(\theta')(\theta' - \theta) \end{aligned}$$

KSS - Second-best Benchmark 2

- Using similar argument from KL, $k(\theta)$ is IC if and only if (i) $\Omega'(\theta) = k(\theta)$ and $k(\theta)$ increases in θ .
- $\Omega'(\theta) = k(\theta)$ implies that $k'(\theta)[- \theta + c'(k(\theta)) - v_k] = v_\theta$. Note that FB implies that $- \theta + c'(k(\theta)) - v_k = 0$.
- Since $v_\theta > 0$ and $k'(\theta) > 0$, we have $- \theta + c'(k(\theta)) - v_k > 0$, i.e., overinvestment.
- Intuition: as in the typical signalling model, overinvestment to signal high θ .

KSS - A parametric example

- $c(k) = \frac{1}{2}ck^2$ and $v(k, \theta) = \gamma k\theta$ with $c > 0, \gamma > 0$
- We then have $k_{FB} = \frac{1+\gamma}{c}\theta$; $k_{SB1} = \frac{1}{c}\theta$ and $k_{sb2} = \frac{1+2\gamma}{c}\theta$
- So underinvestment in the first benchmark and overinvestment in the second benchmark.

KSS - With both information asymmetry and measurement noise

- A “noisy signalling equilibrium”.
- Investor conjecture manager's investment schedule $k(\theta)$, but can only observe k with noise.
- Inferences will take the form of a Bayesian posterior density of θ conditional on s with $g(\theta|s) = \frac{f(s|k(\theta))h(\theta)}{\int_{\Theta} f(s|k(t))h(t)dt}$

KSS - With both information asymmetry and measurement noise

- Definition of equilibrium: $\{k(\theta), \varphi(s), g(\theta|s)\}$ s.t.

- 1 $k(\theta) \in \arg \max_k \theta k - c(k) + \int_{\underline{s}}^{\bar{s}} \varphi(s) f(s|k) ds$

- 2 $\varphi(s) = E[v(k(\theta), \theta)|s] = \int_{\Theta} v(k(\theta), \theta) g(\theta|s) d\theta$

- 3 $g(\theta|s) = \frac{f(s|k(\theta))h(\theta)}{\int_{\Theta} f(s|k(t))h(t)dt}$

- FOC from 1 results in $\theta - c'(k(\theta)) + \int_{\underline{s}}^{\bar{s}} \varphi(s) f_k(s|k(\theta)) ds = 0$ and $k(\theta)$ increasing in θ

- Insert $\varphi(s) = \int_{\Theta} v(k(\theta), \theta) \frac{f(s|k(\theta))h(\theta)}{\int_{\Theta} f(s|k(t))h(t)dt} d\theta$ into FOC gets

$$\int_{\underline{s}}^{\bar{s}} \int_{\Theta} v(k(\theta), \theta) \frac{f(s|k(\theta))h(\theta)}{\int_{\Theta} f(s|k(t))h(t)dt} d\theta f_k(s|k(\theta)) ds = c'(k(\theta)) - \theta \text{ and } k(\theta)$$

increasing in θ (Proposition 3).

KSS - With both information asymmetry and measurement noise

- Hard to make any general statements but corollary to Proposition 3 says if $f(s|k)$ satisfies MLRP, then firm's investment is greater than the second-best benchmark 1 for any $\theta > 0$.
- Proof: Let $n(s|\theta) \equiv f(s|k(\theta))$. Then

$$\frac{n_\theta(s|\theta)}{n(s|\theta)} = \frac{f_k(s|k(\theta))k'(\theta)}{f(s|k(\theta))}.$$

- Thus, $f(s|k)$ satisfies MLRP $\rightarrow \frac{f_k(s|k)}{f(s|k)}$ strictly increases in $s \rightarrow \frac{n_\theta(s|\theta)}{n(s|\theta)}$ strictly increases in s as $k'(\theta) > 0 \rightarrow$ higher s moves $g(\theta|s)$ to the right in the sense of FOSD $\rightarrow \varphi'(s) > 0 \rightarrow$

$$\int_{\underline{s}}^{\bar{s}} \varphi(s) f_k(s|k(\theta)) ds = \varphi(s) F_k(s|k(\theta)) \Big|_{\underline{s}}^{\bar{s}} - \int_{\underline{s}}^{\bar{s}} \varphi'(s) F_k(s|k(\theta)) ds =$$
$$- \int_{\underline{s}}^{\bar{s}} \varphi'(s) F_k(s|k(\theta)) ds > 0 \text{ as } F_k(s|k(\theta)) < 0.$$

KSS - Closed-form solution with parametric structure

- Suppose $s = k + \varepsilon$ with $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ and prior of $\theta \sim N(\mu, \sigma_\theta^2)$ with θ and ε independent. Still assume $c(k) = \frac{1}{2}ck^2$ and $v(k, \theta) = \gamma k\theta$.
- Guess (and later verify) that $k(\theta) = a + b\theta$.
- $\varphi(s) = E[v(k(\theta), \theta)|s] = E[\gamma(a + b\theta)\theta|s] = \gamma a E[\theta|s] + \gamma b E[\theta^2|s] = \alpha_0 + \alpha_1 s + \alpha_2 s^2$.
- Note that Standard Bayesian updating results in $E[\theta|s] = (1 - \beta)\mu + \beta \frac{s - a}{b}$ and $\text{var}[\theta|s] = (1 - \beta)\sigma_\theta^2$, where $\beta = \frac{b^2 \sigma_\theta^2}{b^2 \sigma_\theta^2 + \sigma_\varepsilon^2}$.
- This results in $\alpha_0 = \alpha\gamma[(1 - \beta)\mu - \frac{\beta a}{b}] + b\gamma\{(1 - \beta)\sigma_\theta^2 + [(1 - \beta)\mu - \frac{\beta a}{b}]^2\}$, $\alpha_1 = \frac{\beta}{b}\{\alpha\gamma + 2b\gamma[(1 - \beta)\mu - \frac{\beta a}{b}]\}$, and $\alpha_2 = \frac{\beta^2 \gamma}{b}$.

KSS- Closed-form solution

- We can then calculate

$$\begin{aligned}\int_{-\infty}^{\infty} \varphi(s) f_k(s|k) ds &= \int_{-\infty}^{\infty} (\alpha_0 + \alpha_1 s + \alpha_2 s^2) f(s-k) \frac{s-k}{\sigma_\varepsilon^2} ds \\ &= \frac{\alpha_1}{\sigma_\varepsilon^2} (E[s^2|k] - kE[s|k]) \\ &\quad + \frac{\alpha_2}{\sigma_\varepsilon^2} (E[s^3|k] - kE[s^2|k]) \\ &= \frac{\alpha_1}{\sigma_\varepsilon^2} (k^2 + \sigma_\varepsilon^2 - k^2) + \frac{\alpha_2}{\sigma_\varepsilon^2} (k^3 + 3k\sigma_\varepsilon^2 - k^3 - k\sigma_\varepsilon^2) \\ &= \alpha_1 + 2\alpha_2 k.\end{aligned}$$

- FOC now becomes $\alpha_1 + 2\alpha_2 k = ck - \theta$. This results in $k(\theta) = \frac{\alpha_1 + \theta}{c - 2\alpha_2}$ which confirms the guess.

- This is equivalent to $a = \frac{\alpha_1}{c-2\alpha_2} = \alpha_1 b$ and $b = \frac{1}{c-2\alpha_2} = \frac{1}{c-2\frac{\beta^2\gamma}{b}}$.

When $\beta \rightarrow 0$, $b \rightarrow \frac{1}{c}$ and when $\beta \rightarrow 1$, $b \rightarrow \frac{1+2\gamma}{c}$. Therefore $b \in (\frac{1}{c}, \frac{1+2\gamma}{c})$, i.e., lie between the value for the two second-best benchmarks.

- Use the expression for α_1 and α_2 we have $a = \frac{2b\gamma\beta(1-\beta)\mu}{1-\beta\gamma(1-2\beta)}$ where $\beta = \frac{b^2\sigma_\theta^2}{b^2\sigma_\theta^2 + \sigma_\varepsilon^2}$ and $\beta^2 = \frac{b(bc-1)}{2\gamma b}$ (equation (37)) (simplified version of Lemma 1)

KSS - When can first-best be achieved

- First-best can be achieved if $a = 0$ and $b = \frac{1+\gamma}{c}$. Therefore, we need $\mu = 0$ and $\frac{1}{c-2\frac{\beta^2\gamma}{\frac{1+\gamma}{c}}} = \frac{1+\gamma}{c}$. Solving for σ_ε^2 results in $\sigma_\varepsilon^2 = (\frac{1+\gamma}{c})^2 \sigma_\theta^2 (\sqrt{2} - 1) > 0$, the optimal level of reporting noise (simplified version of Proposition 4)
- The higher σ_θ^2 is, the higher σ_ε^2 should be (Corollary to Proposition 4); the higher γ is, the higher σ_ε^2 should be; the higher c is, the lower σ_ε^2 should be
- Intuition: information asymmetry results in overinvestment (signalling) and measurement noise results in underinvestment that dampens overinvestment; the higher signalling incentive for overinvestment (higher σ_θ^2 , higher γ and lower c), the higher incentive for measurement noise to dampen

- Should tolerate more investment imprecision (or perhaps not measuring investment) when
 - The information asymmetry between the market and the firm is higher
 - Growth option is more important to the firm (Gao and Liang 2013 has a similar prediction based on feedback effect)
 - Investment is less costly
 - In general, when the signalling incentive and overinvestment incentives are larger

Summary of the real effects literature

- When manager is more informed than the market, law of iterated expectations fails -> Manager's expectation of short-term market prices are not the same as manager's expectation of terminal firm value
- When manager cares about short-term market prices, such differences in expectations generate distortions of the investment
- The type and magnitude of distortion depends on what the market knows -> disclosure then plays a role
- Different disclosure rules result in different information set of the market and different distortions to firms' real investment, which has real economic consequences -> More information is not necessarily better