# BA 932 Session 7 Modelling of Accounting Conservatism

Xu Jiang

April 4th, 2023

# What we have done so far regarding disclosure

- We discussed verifiable disclosure models and costly manipulation models
- Those settings belong to ex-post disclosure: managers choose what to disclose after observing the realization of signals
- Accounting rules are usually set up as ex-ante disclosure rules: e.g., expensing all research expenditures, lower of cost or market
- Therefore studying ex-ante disclosure issues may shed more light on accounting issues

# This class - accounting conservatism

- Conservatism is pervasive in accounting rules and there is a huge literature on it
- Empirically, we all know the famous Basu measure of accounting conservatism
- Conceptually, how do we think of conservatism and theoretically, how do we model conservatism?

# How to think about conservatism

- What is an example of a conservative earnings report?
- Suppose the earnings report always subtracts the true value by 5 cents, how would a rational decision maker respond when observing the earnings report?
- So, would subtracting true value by 5 cents a good way of modelling conservatism?

# How to think about conservatism, continued

- Think about conservatism as adopting a higher verification for good news versus bad news.
- What is the implication of higher verification for good news versus bad news?
- Implication is that bad earnings occurs more frequently but is less informative; good earnings occurs less frequently but is more informative. Overall the informativeness of earnings does not vary with bias, that is, the precision does not vary with bias.

# Modelling conservatism (Gigler et al. 2009, GKSV)

- In statistics, a measure of informativeness is the likelihood ratio.
- Denote $y$ as reported earnings, $x \in \{x_H, x_L\}$ as true earnings (e.g., cash flow in a static model) and $\delta$ as reporting bias with higher $\delta$ representing more conservative bias.
- Then $\frac{\varphi(y|x_H)}{\varphi(y|x_L)}$ is the likelihood ratio.

# Modelling conservatism (GKSV)

- Therefore more conservative bias (i.e., higher $\delta$) results in more informative good earnings and less informative bad earnings translates into higher $\frac{\varphi(y|x_H)}{\varphi(y|x_L)}$ (as for high $y$, $\frac{\varphi(y|x_H)}{\varphi(y|x_L)}$ moves away from 1 and for low $y$, $\frac{\varphi(y|x_H)}{\varphi(y|x_L)}$ moves towards 1).

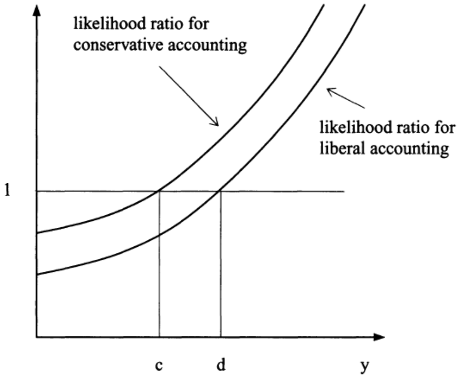



FIG. 1.—Informational implications of accounting conservatism.

# GKSV - Model Setup

- Setting: a firm needs investment $K$ in a project, which can generate a terminal cash flow $x$. The investment is financed by debt with a face value $D$ and a covenant based on accounting earnings $y$, denoted as $y^C$, where $y \in [0, \overline{y}]$ is a noisy signal about $\widetilde{x} \in \{x_H, x_L\}$.
- Continuation/liquidation decisions are chosen upon observing $y$, with liquidation generating a liquidation payoff of $M$. Assume that $E[\widetilde{x}] > K(1+R) > M$ so that the project is of positive NPV, where $R$ is the required rate of return for creditors.
- Timeline of the model
  - Date 0: debt contract $\{D, y^C\}$ is signed and creditors contribute the amount of investment $K$
  - Date 1: $y$ is observed, liquidation occurs if $y < y^C$ and continuation occurs otherwise.
  - Date 2: $\widetilde{x} \in \{x_H, x_L\}$ realized if the project is continued.

# GKSV - Results

- Debt contract is characterized by face value $D$ and covenant $y^C$.
- Socially efficient decision rule: liquidate if and only if $E[\widetilde{x}|y] \leq M$. Assume that higher $y$ is good news in the sense of Milgrom, then $E[\widetilde{x}|y]$ is strictly increasing in $y$ and $E[\widetilde{x}|y] \leq M$ if and only if $y \leq y^*$ where $E[\widetilde{x}|y^*] = M$.
- Lemma 1: $D > M$.
- Implication: debt is risky as in case of liquidation, creditors are not fully repaid.

# GKSV - Proof of Lemma 1

- Let $w \leq M$ be the amount paid to creditors when the project is terminated, when $y \leq y^0$ for some $y^0$.
- Then the lender's participation constraint is

$$\int_0^{y^0} wh(y)dy + \int_{y^0}^{\overline{y}} V(D,y)h(y)dy \geq K(1+R),$$

where

$$V(D,y) = \int_0^D xf(x|y)dx + \int_D^\infty Df(x|y)dx = D - \int_0^D F(x|y)dx,$$

and $h(y)$ is the (unconditional) distribution of $y$.

- Since $w \leq M$ and $V(D,y) < D$, $\int_0^{y^0} Mh(y)dy + \int_{y^0}^{\overline{y}} Dh(y)dy \geq K(1+R)$. Since $M < K(1+R)$, $D > K(1+R) > M$.

# GKSV - Results

- Proposition 1 characterizes the optimal covenant from the debtholder's perspective: for any $D$ there is a $\widehat{y}(D) > y^*$ such that liquidation occurs if and only if $y \leq \widehat{y}(D)$. $\widehat{y}(D)$ is strictly decreasing in $D$ and goes to $y^*$ when $D \to +\infty$.
- Implication: debtholders always want to liquidate more than the socially efficient level because their upside gains are capped at $D$.

# GKSV - Proof of Proposition 1

- Since $M < D$ from Lemma 1, debt holders get $M$ upon liquidation.
- Since $\frac{\partial V(D,y)}{\partial y} = f(x|y) > 0$, $V(D,y)$ increases in $y$. Therefore, there exists a unique $\widehat{y}(D)$ such that $V(D, \widehat{y}(D)) = M$.
- Since $V(D,y) < E[\widetilde{x}|y]$, $\widehat{y}(D) < y^*$.

# GKSV - Conflict between creditors and shareholders

- We know that $\widehat{y}(D) < y^*$, creditors like to liquidate a lot.
- Shareholders' payoff from liquidation is zero and their payoff from continuation is

$$U(D, y) = \int_D^\infty (x - D) f(x|y) dx > 0.$$

- Therefore they will always want the project to continue.
- Optimal debt contract balances the conflicting incentives of the two parties.

# GKSV - Optimal debt contract

- Given covenant $y^C$,

$$\widehat{V}(D, y^C) \equiv \int_0^{y^C} Mh(y)dy + \int_{y^C}^{\overline{y}} V(D, y)h(y)dy,$$

and

$$\widehat{U}(D, y^C) \equiv \int_{y^C}^{\overline{y}} U(D, y)h(y)dy.$$

- The optimal debt contract is characterized by $\{D, y^C\}$ to maximize $\widehat{U}(D, y^C)$ subject to $\widehat{V}(D, y^C) \geq K(1 + R)$.

# GKSV - Optimal debt contract

- Note that $U(D, y) = E[\widetilde{x}|y] - V(D, y) \ \forall y$.
- Therefore the constraint must binds, i.e., $\widehat{V}(D, y^C) = K(1 + R)$ and the maximization problem becomes

$$\begin{aligned} & \max_{D, y^C} \int_{y^C}^{\overline{y}} [E[\widetilde{x}|y] - V(D, y)] h(y) dy \\ = \quad & \max_{D, y^C} \int_{y^C}^{\overline{y}} E[\widetilde{x}|y] h(y) dy - K(1 + R) + \int_0^{y^C} Mh(y) dy. \end{aligned}$$

- First order condition with respect to $y^C$ results in

$$E[\widetilde{x}|y^C] h(y^C) - Mh(y^C) = 0 \Rightarrow E[\widetilde{x}|y^C] = M.$$

- In other words, $y^C = y^*$.

# GKSV - Optimal debt contract

- The optimal debt contract results in socially efficient liquidation/continuation decision.
- Intuition: creditors want excessive (relative to socially efficient) liquidation and shareholders want excessive continuation
- Optimal debt contract balances the two -> socially efficient decision maximizes the pie so that both parties can benefit
- Due to no frictions (no information asymmetry or moral hazard issue between creditors and shareholders)

# GKSV - Optimal debt contract

- Even though $y^C$ is socially efficient, since $y$ is a noisy signal of $x$, there are still two types of decision errors
  - False alarm errors (Type I errors) $L_I(y^C) = \int_0^{y^C} \int_M^{\infty} (x - M) f(x|y) dx h(y) dy$ -liquidated project that should be continued
  - Undue optimism errors (Type II errors) $L_{II}(y^C) = \int_{y^C}^{\overline{y}} \int_0^M (M - x) f(x|y) dx h(y) dy$ - continued project that should be liquidated
- $y^C = y^*$ minimizes the sum of the two errors (Proposition 3)

# GKSV - Proof of Proposition 3

$$
\begin{aligned}
& L_I(y^C) + L_{II}(y^C) \\
= \; & \int_0^{y^C} \int_M^{\infty} (x - M) f(x|y) dx h(y) dy \\
& + \int_{y^C}^{\overline{y}} \int_0^M (M - x) f(x|y) dx h(y) dy.
\end{aligned}
$$

- Take derivative w.r.t. $y^C$ results in

$$
\begin{aligned}
& \int_M^{\infty} (x - M) f(x|y^C) dx h(y^C) \\
= \; & \int_0^M (M - x) f(x|y^C) dx h(y^C) \\
\Rightarrow \; & E[\widetilde{x}|y^C] = M \Rightarrow y^C = y^*.
\end{aligned}
$$

In addition, the derivative is negative when $y^C < y^*$ and positive when $y^C > y^*$, implying a minimum.

# GKSV - how conservatism affects the summation of errors

- It would be more intuitive to discuss conservatism when $\widetilde{x}$ is binary, i.e., $\widetilde{x} \in \{x_H, x_L\}$.
- Conservatism parametrized by $\delta$, with higher $\delta$ indicating less conservative accounting as discussed before.
- Denote the distribution of $\widetilde{y}$ conditional on $\widetilde{x}$ by $\varphi(y|x, \delta)$.
- Assumptions on conservatism concerns how properties of $\varphi(y|x, \delta)$ varies with $\delta$.

# GKSV - how conservatism affects the summation of errors

- (A1) $\frac{\varphi(y|x_H,\delta)}{\varphi(y|x_L,\delta)} \uparrow$ in $y$ $\forall\delta$ ->MLRP in $y$ -> higher ys are better news in the sense of Milgrom (1981)
- (A2) $\int_a^y \varphi(y|x,\delta)dy \uparrow$ in $\delta$ $\forall x, a > 0$ -> more conservative accounting shifts the distribution of $y$ to the left in the sense of FOSD
- (A3) $\frac{\varphi(y|x_H,\delta)}{\varphi(y|x_L,\delta)} \downarrow$ in $\delta$ $\forall y$ -> lower $\delta$ (more conservative accounting) increases the information content of high $y$ but decreases the information content of low $y$
- Intuitive explanation using exam analogy:
  - (A1) -> higher grades more informative of better ability, for any fixed difficulty level of exams
  - (A2) -> easier exams results in grade inflation
  - (A3) -> high (low) grades in tougher (easier) exams are very informative but low (high) grades not very informative

# GKSV - how conservatism affects the summation of errors

- (A4) Unconditional conservatism:
  $\frac{\partial}{\partial\delta}(\int_a^y \varphi(y|x_H,\delta)dy) = \frac{\partial}{\partial\delta}(\int_a^y \varphi(y|x_L,\delta)dy) > 0$ ("$>0$" from A2)
- (A5) Conditional conservatism: $\exists\delta^0$ such that
  $\frac{\partial}{\partial\delta}(\int_a^y \varphi(y|x_H,\delta)dy) > \frac{\partial}{\partial\delta}(\int_a^y \varphi(y|x_L,\delta)dy) > 0 \ \forall\delta < \delta^0$ and
  $0 < \frac{\partial}{\partial\delta}(\int_a^y \varphi(y|x_H,\delta)dy) < \frac{\partial}{\partial\delta}(\int_a^y \varphi(y|x_L,\delta)dy) \ \forall\dot{\delta} > \delta^0$.
  -$>\delta < \delta^0 (\delta > \delta^0)$ conditionally conservative (aggressive) accounting
- Intuitive explanation using exam analogy:
  - (A4) -$>$ When exams get less difficult, inflation of grade distribution for students with high ability is the same as that for students with low ability
  - (A5) -$>$ When very difficult exams get a little easier ($\delta < \delta^0$), inflation of grade distribution for students with high ability will be larger than that for students with low ability; only when exams get significantly easier ($\dot{\delta} > \delta^0$) will students with low ability catch up

# GKSV - how conservatism affects the summation of errors

- Main result 1: Unconditionally conservative accounting decreases debt contracting efficiency (Proposition 5)
- Main results 2: Conditionally conservative accounting decreases debt contracting efficiency (Proposition 6)
- Intuition: more conservative accounting increases false alarm error but decreases undue optimism error. Since the project has positive NPV, false alarm error is more costly, resulting in more conservative accounting decreasing efficiency
- Put it in another way, since the project has positive NPV, termination is more costly. Therefore one want signals that call for liquidation (i.e., low signals) to be more informative, which calls for more aggressive accounting.

# GKSV - Proof of Propositions 5 and 6

$$
\begin{aligned}
& \frac{d}{d\delta}(L_I + L_{II}) \\
= \; & \frac{d}{d\delta}\left(\int_0^{y^*} p_H(x_H - M)\varphi(y|x_H,\delta)dy + \int_{y^*}^{\overline{y}} p_L(M - x_L)\varphi(y|x_L,\delta)dy\right) \\
= \; & p_H(x_H - M)\int_0^{y^*} \varphi_\delta(y|x_H,\delta)dy + p_L(M - x_L)\int_{y^*}^{\overline{y}} \varphi_\delta(y|x_L,\delta)dy \\
& + [p_H(x_H - M)\varphi(y^*|x_H,\delta) - p_L(M - x_L)\varphi(y^*|x_L,\delta)]\frac{dy^*}{d\delta} \\
= \; & -p_H(x_H - M)\int_{y^*}^{\overline{y}} \varphi_\delta(y|x_H,\delta)dy + p_L(M - x_L)\int_{y^*}^{\overline{y}} \varphi_\delta(y|x_L,\delta)dy.
\end{aligned}
$$

Since $p_H(x_H - M) > p_L(M - x_L)$, $\int_{y^*}^{\overline{y}} \varphi_\delta(y|x_H,\delta)dy \geq \int_{y^*}^{\overline{y}} \varphi_\delta(y|x_L,\delta)dy$ when $\delta < \delta^0$, $\frac{d}{d\delta}(L_I + L_{II}) < 0$.

# Summary of GKSV

- The first to carefully think about how to model accounting conservatism in influencing the information content of good versus bad news.
- Consistent with empirical findings in Dyreng et al. (2017).
- Inspired a lot of follow-up studies.
- However, conservatism is modelled in a reduced-form way: imply reasonable and intuitive assumptions on the distribution of earnings conditional on fundamentals

# Application of the GKSV way of modelling bias: Jiang (2016)

- Jiang (2016) introduces non-accounting information and explores the optimal bias of accounting information in the presence of non-accounting information, which may have its own bias. (WP version shows the result for a more general specification).
- Setup similar to GKSV with the addition of (potentially) biased non-accounting information.

# Jiang (2016), continued

- The general insight is that when non-accounting information has aggressive bias, the accounting information should also be aggressively biased, that is, the biases are complements.
- Intuition: aggressive bias means bad signals more informative and good news less informative. Therefore, non-accounting information system generates more decision errors when good news is present. This requires accounting information system to generate informative bad news, resulting in aggressive bias being optimal.
- This result is in contrast with conventional wisdom that accounting needs to be conservative to compensate for the optimal bias of other information.

# Going beyond modelling conservatism in a reduced form way

- Can conservative bias arise endogenously as part of a solution of the optimal accounting rules?
- This requires modelling accounting signals not simply as true cash flow plus normally distributed noise (for an exception, see Armstrong et al. 2016).
- If we assume the optimal accounting rules are designed ex-ante (i.e, a commitment to a set of rules), then we can apply the insights of Bayesian persuasion literature (Kamenica and Gentzkow 2011) into specific accounting settings. This literature does not impose any particular a priori structure on the specification of noise structure.
- There is a growing literature in accounting on this topic: Gox and Wagenhofer (2009), Bertomeu and Cheynel (2015), Huang (2016), Jiang and Yang (2017, 2021), Michaeli (2017), Friedman et al. (2020, 2021), Bertomeu, Cheynel, Cianciaruso (2021).

# Gox and Wagenhofer (2009) (GW)

- One of the first to study ex-ante optimal accounting rules (even before the Bayesian persuasion literature) in a debt contracting setting with agency problems.
- Show that ex-ante optimal accounting rules can be interpreted as consistent with lower of cost or market.

# GW - Model setup

- Thee dates $t = 0, 1, 2$. Everybody is risk-neutral. The firm (i.e., the borrower) has limited liability.
  - Date 0: The firm has some asset-in-place and an investment opportunity
  - Date 1: The firm may pledge some of the asset-in-place in return for debt financing of the investment opportunity
  - Date 2: All uncertainties are realized

# GW - Model details

- The investment opportunity requires an investment of $I > 0$ and generates a random cash flow $\widetilde{X} \in \{0, X\}$. $\Pr(\widetilde{X} = X) = p$ so the NPV of the project is $pX - I$.
- An agency problem between the firm and the lender: the manager can exert high effort to increase the success probability of the project from $p_L$ to $p_H$ with a personal cost to the manager of $v > 0$ (more on this type of problem next year)
- Assume that $p_H X - I > 0 > p_L X - I$, i.e., the project is positive NPV only when high effort is exerted.

# GW - Model details

- The debt contract contains a face value $d(X) \in \{d_S, d_F\}$, i.e., the payment to debt holders when the project succeeds and fails. WLOG set $d_F = 0$ and $d_S \leq X$ (limited liability).
- In addition to $d$, the creditor can seize the pledged assets $A$ in the event of bankruptcy and liquidate to get a proceed $V(A) = \gamma A$ with $\gamma \in [0, 1]$.

# GW - Solution in a benchmark without accounting info

- Optimal debt contract consists of $\{d_S, A\}$ such that high effort is induced, i.e.,

$$\max_{d_S, A} p_H(X - d_S) - (1 - p_H)A$$

s.t.

$$p_H d_S + (1 - p_H)\gamma A - I \geq 0 \text{ (IR)}$$

$$p_H(X - d_S) - (1 - p_H)A - v \geq p_L(X - d_S) - (1 - p_L)A \text{ (IC)}$$

- The IC constraint is equivalent to $d_S \leq X + A - \frac{v}{\Delta p}$ where $\Delta p \equiv p_H - p_L$.

# GW - Solution in a benchmark without accounting info

- Since the objective function is decreasing in $A$, setting $A = 0$ would be optimal if the constraints are all satisfied.
- If $A = 0$, IR becomes $p_H d_S \geq I$ and IC becomes $d_S \leq X - \frac{v}{\Delta p}$.
- Therefore, so long as $\frac{I}{p_H} \leq X - \frac{v}{\Delta p}$, then both IR and IC are satisfied and first-best investment policy is achieved.
- For the more interesting case of $\frac{I}{p_H} > X - \frac{v}{\Delta p}$, then $A^* \neq 0$.

# GW - Solution in a benchmark without accounting info

- In this case, we can show that both IR and IC bind in equilibrium.
- To see why IR must bind in equilibrium, suppose not, then $p_H d_S + (1 - p_H)\gamma A - I > 0$. Reduce $d_S$ by some small amount $\varepsilon > 0$, IC will still be satisfied but the objective function will increase so contradiction.
- To see why IC must bind in equilibrium, suppose not, then $d_S < X + A - \frac{v}{\Delta p}$ in equilibrium. We can decrease $A$ by some small amount $\varepsilon > 0$ and increase $d_S$ by $\frac{(1-p_H)\gamma}{p_H}\varepsilon$. Then IR will be satisfied, IC will be satisfied because of the slack, and the objective function will change by $(1 - p_H)\varepsilon - p_H \frac{(1-p_H)\gamma}{p_H}\varepsilon = (1 - p_H)(1 - \gamma)\varepsilon \geq 0$ so contradiction.

# GW - Solution in a benchmark without accounting info

- Since both IR and IC bind, we have two unknowns $(d_S, A)$ and two equations.
- Solving results in $A = \widehat{A} \equiv \frac{I - p_H(X - \frac{v}{\Delta p})}{p_H + (1 - p_H)\gamma}$ and $d_S = X - \frac{v}{\Delta p} + \widehat{A}$ (Proposition 1) -> severe agency problem results in the borrower having to pledge some assets to creditors in order for the creditors to provide financing

# GW - Adding accounting info

- Now consider incorporating an accounting system that provides info about $A$ -> this implies we need to make $A$ a random variable
- Assume $\widetilde{A} = \mu + \widetilde{\varepsilon} + \widetilde{\xi}$ where $\widetilde{\varepsilon}$ and $\widetilde{\xi}$ are two independent noise terms with zero expectation.
- Accounting system is modelled as a two-step process (See also Gao 2013 JAE)
    - Step 1: fundamental -> evidence $\widetilde{A} \rightarrow \widetilde{y} = \mu + \widetilde{\xi} = \widetilde{A} - \widetilde{\varepsilon}$
    - Step 2: evidence -> report $\widetilde{y} \rightarrow$ actual report
    - GW simplifies by restricting actual report to be either perfect reporting (report $\widetilde{y}$) or reporting nothing

# GW - Adding accounting info

- Assume that the support of $\widetilde{y}$ is $[\underline{y}, \overline{y}]$. Denote $D$ as the region where $y$ is reported and $N$ as the region where $y$ is not reported
- Since $\widetilde{A}$ is now random, replace $A$ in the deterministic case by $E[\widetilde{A}] = \mu$ and everything goes through.
- If $\mu \geq \widehat{A}$, then the analysis from the deterministic case implies that the project will always be financed.
- In this case, the firm prefers not to report anything (Proposition 2) since the default action in the absence of information is to finance the project.

# GW - optimal ex-ante reporting rule

- If $\mu < \widehat{A}$, then project will not be financed in the absence of disclosure, making disclosure potentially useful.
- Proposition 3 shows that when $\mu < \widehat{A}$, the optimal ex-ante reporting rule is
$$B = \begin{cases} y \text{ if } y < y^N \\ \varnothing \text{ if } y \geq y^N \end{cases},$$
where $y^N$ is defined as $E[\widetilde{A}|y \geq y^N] = \widehat{A}$.
- Essentially the insight from the Bayesian persuasion literature: to maximize the chance of providing financing, reduce the informativeness of signal that indicate providing finance such that the creditor is indifferent between providing financing or not.

# GW - Proof of Proposition 3

- Suppose that $B$ is not the optimal reporting rule, then there exists another reporting rule s.t. $N = N_1 \cup Y_1$ where $Y_1 \subset D$.
- Note that we need to have $E[\widetilde{A}|y \in N_1 \cup Y_1] \geq \widehat{A}$ as otherwise no financing is provided when there is no report.
- Assume that $N_1 = [y_3, \overline{y}]$ and $Y_1 = [y_1, y_2]$ where $y_3 > y^N$ s.t. $E[\widetilde{A}|y \in N_1 \cup Y_1] = \widehat{A}$.
- For this reporting rule to be better we need to show that $F(N) < F(Y_1) + F(N_1)$, where $F(z) = F(y \in z)$.

# GW - Proof of Proposition 3

- Denote $Y_3 = [y^N, y_3)$, then $N = Y_3 \cup N_1$.
- Thus $F(N) < F(Y_1) + F(N_1) \Leftrightarrow F(Y_3) < F(Y_1)$
- Note that $E[\widetilde{A}|y \in N_1 \cup Y_1] = E[\widetilde{A}|y \in N] = E[\widetilde{A}|y \in Y_3 \cup N_1] = \widehat{A}$.
- $E[\widetilde{A}|y \in N_1 \cup Y_1] = E[\widetilde{A}|y \in Y_3 \cup N_1]$ is equivalent to

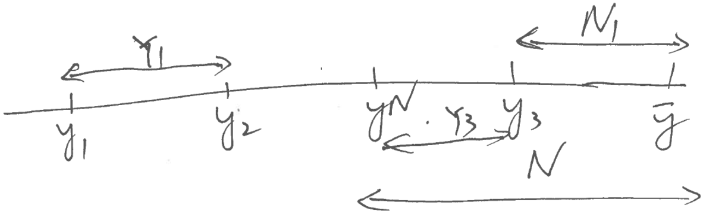
$$\frac{F(Y_1)E(Y_1) + F(N_1)E(N_1)}{F(Y_1) + F(N_1)} = \frac{F(Y_3)E(Y_3) + F(N_1)E(N_1)}{F(Y_3) + F(N_1)}.$$

# GW - Proof of Proposition 3

$$
\begin{aligned}
\Leftrightarrow \quad & F(Y_3)F(N_1)[E(N_1) - E(Y_3)] \\
= \quad & F(Y_1)F(Y_3)[E(Y_3) - E(Y_1)] + F(Y_1)F(N_1)[E(N_1) - E(Y_1)] \\
> \quad & F(Y_3)F(Y_3)[E(Y_3) - E(Y_1)] + F(Y_3)F(N_1)[E(N_1) - E(Y_1)] \\
\Leftrightarrow \quad & F(N_1)[E(Y_1) - E(Y_3)] > F(Y_3)[E(Y_3) - E(Y_1)] \\
\Leftrightarrow \quad & -F(N_1) > F(Y_3) \text{ and thus contradiction.}
\end{aligned}
$$

# GW - Proof of Proposition 3

- Graphical illustration:




- Intuition of the proof: Adding a smaller set (i.e., $Y_1$) into the no-reporting region requires an ever larger decrease (i.e., $Y_3$) out of the no-reporting region to keep the indifference condition, i.e., $E[\widetilde{A}|y \in N] = \widehat{A}$ to hold.

# GW - Optimal reporting rule

- Recall that the optimal reporting rule is

$$B = \begin{cases} y \text{ if } y < y^N \\ \varnothing \text{ if } y \geq y^N \end{cases}.$$

- GW interprets this as consistent with conservative accounting rules such as "lower of cost or market".
- Different from GKSV, here more conservative accounting is associated with good news being less informative and bad news being more informative.
- Consistent with GKSV, more informative good news and less informative bad news are always bad.

# GW - Compare and contrast with GKSV

- The mechanism that more informative good news and less informative bad news are always bad is different under the two papers
  - In GKSV, the project has positive NPV and will be continued in the absence of any information but continuation may not always be efficient -> Accounting provides intermediate bad news to tell creditors to liquidate -> The bad news therefore has to be sufficiently informative
  - In GW, the project cannot be financed without any info (due to agency problem) but financing the project is always (ex-ante) efficient -> Accounting provides ex-ante news (i.e., information about A but not whether the project succeeds or not) -> good news has to be uninformative to maximize the probability of receiving good news and thus financing

# Chen et al. 2007 (CHZ) - allowing earnings management

- CHZ is also an ex-ante information system design but allow for ex-ante earnings management in a moral hazard setting.
- Moral hazard setting: need to impose risk in agents' compensation (i.e., compensation varies with performance measure) to induce effort -> earnings management decreases informativeness of such performance measure and worsens agency problem
- They show that conservative accounting is beneficial as it curbs incentives for such ex-ante earnings management and alleviates agency problem
- Their definition of conservative accounting is based on the binary framework of Gigler and Hemmer (2001), which is later generalized by GKSV

# CHZ - Model set up

- A typical agency model with unobservable effort (more on this next year)
  - A risk-neutral current owner (shareholders) and a risk-averse and effort-averse agent (manager)
  - The agent can exert unobservable effort $a \in \{a_h, a_l\}$ that generates cash flow $x \in \{x_h, x_l \equiv 0\}$
  - the owner prefers $a_h$ and pays agent wage $s$ for the effort with $s$ depending on $a$
  - The agent's payoff is $U(s) - V(a)$, where $s$ is the wage provided by the owner. Normalize $V(a_l) = 0$ and $V(a_h) = D > 0$
  - Assume that $\Pr(x_l|a_l) = 1$ and $\Pr(x_h|a_h) \equiv p \geq \frac{1}{2}$.

# CHZ - a short introduction of moral hazard

- Risk-neutral principal, risk-averse agent (or risk-neutral with limited liability), agent exerts unobservable and costly effort
- Principal cannot contract on effort but has to contract on some observable outcome (e.g., earnings) that is stochastically affected by effort
- Since agent is risk-averse and the principal is risk-neutral, optimal risk-sharing indicates not letting agent bear any of the risk -> flat payment to the agent regardless of outcome -> not incentive compatible as the agent will not exert effort
- Optimal contract involves a tradeoff between risk-sharing and incentivizing the agent
- The more informative the signal, the less risk imposed on the agent, the lower the expected payment to the agent, and the less severe the agency problem

# CHZ - Model set up

- The current owner sells the firm before $x$ is realized and the price is based on accounting earnings
- Without no manipulation and no bias: $\Pr(e_h|x_h) = 1$ and $\Pr(e_l|x_l) = 1$
- Without manipulation and conservative accounting: $\Pr(e_h|x_h) = 1 - \Gamma$ and $\Pr(e_l|x_l) = 1$ where $\Gamma \in (0, 1)$ captures the degree of conservatism
- With (current owner) manipulation and conservative accounting: $\Pr(e_h|x_h) = 1 - \tau$ and $\Pr(e_l|x_l) = 1 - \Delta$ where $\Delta \in (0, \Delta^{max})$ for some $\Delta^{max} < 1$ is the amount of earnings manipulation; $\Delta$ observed by the current owner and the agent but not by future shareholders

# CHZ - graphical illustration of the information structure

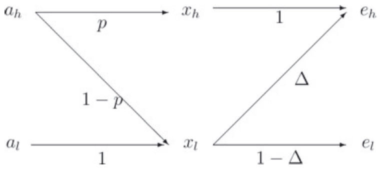
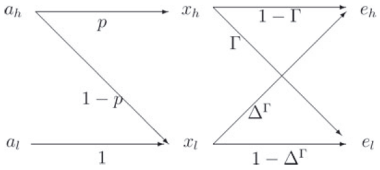



FIG. 1.—Information structure—unbiased accounting standard with $\Delta$ earnings manipulation.



# CHZ - Equilibrium definition

- Future shareholders will conjecture $\widehat{\Delta}$ when pricing the firm.
- Equilibrium thus characterized by $(\Delta, \widehat{\Delta}, P_{e_k}, s_k)$ for $k = h, l$ s.t.
  - $\widehat{\Delta} = \Delta$
  - $a_h$ is chosen given $s_k$ to maximize $E[U(s) - V(a)]$
  - Given $\widehat{\Delta}$, $\Delta$ and $s_k$ maximize $E[P_{e_k} - s]$
  - $P_{e_k} = E[\widetilde{x}|a_h, \widehat{\Delta}]$

# CHZ - unbiased accounting case

- $\widehat{P}_{eh} = \Pr(x_h|e_h, \widehat{\Delta})x_h + \Pr(x_l|e_h, \widehat{\Delta})x_l = \frac{p}{p+(1-p)\widehat{\Delta}}x_h$ and $\widehat{P}_{el} = 0$. $\widehat{P}_{eh} - \widehat{P}_{el} > 0$, generating an incentive for manipulation.
- The current owner therefore chooses $s_h, s_l$ and $\Delta$ to maximize

$$[p + (1-p)\Delta](\widehat{P}_{eh} - s_h) + (1-p)(1-\Delta)(\widehat{P}_{el} - s_l)$$

subject to

$$[p + (1-p)\Delta]U(s_h) + (1-p)(1-\Delta)U(s_l) - D \geq 0 \text{ (IR)}$$

and

$$\begin{aligned} & [p + (1-p)\Delta]U(s_h) + (1-p)(1-\Delta)U(s_l) - D \\ \geq \ & \Delta U(s_h) + (1-\Delta)U(s_l) \text{ (IC)} \end{aligned}$$

# CHZ - unbiased accounting case

- We can show that both IR and IC binds, resulting in $U(s_h^*) = \frac{D}{p}$ and $U(s_l^*) = \frac{D}{p}(1 - \frac{1}{1-\Delta})$.
- Note that $\frac{\partial[U(s_h^*)-U(s_l^*)]}{\partial(1-\Delta)} = -\frac{D}{p}\frac{1}{(1-\Delta)^2} < 0$. Therefore, as $\Delta \downarrow$, there is less earnings management so the accounting report becomes more informative and the spread between compensation payments decrease (agency cost decreases).
- $\frac{\partial E(s^*)}{\partial(1-\Delta)} = \frac{\partial[(p+(1-p)\Delta)s_h^*+(1-p)(1-\Delta)s_l^*]}{\partial(1-\Delta)} =$
  $(1-p)(s_l^* - s_h^*) + (1-p)(1-\Delta)\frac{\partial U^{-1}(U(s_l^*))}{\partial(1-\Delta)}$
  $= (1-p)(s_l^* - s_h^*) + (1-p)\frac{1}{U'(s_l^*)}\frac{D}{p(1-\Delta)} =$
  $-(1-p)[(s_h^* - s_l^*) - \frac{U(s_h^*)-U(s_l^*)}{U'(s_l^*)}] < 0$, as $U$ strictly concave -> earnings management increases the expected compensation to the agent (as earnings more likely to become high)

# CHZ - unbiased accounting case

- To solve for optimal $\Delta$, insert $s_h^*$ and $s_l^*$ into the current owner's problem results in

$$\max_{\Delta}[p+(1-p)\Delta](\widehat{P}_{eh}-s_h^*)+(1-p)(1-\Delta)(\widehat{P}_{el}-s_l^*).$$

- First order condition with respect to $1-\Delta$ results in

$$\begin{aligned} & -(1-p)(\widehat{P}_{eh}-\widehat{P}_{el})-\frac{\partial E(s^*)}{\partial(1-\Delta)} \\ = \quad & -\frac{(1-p)p}{p+(1-p)\widehat{\Delta}}x_h+(1-p)[(s_h^*-s_l^*)-\frac{U(s_h^*)-U(s_l^*)}{U'(s_l^*)}]=0. \end{aligned}$$

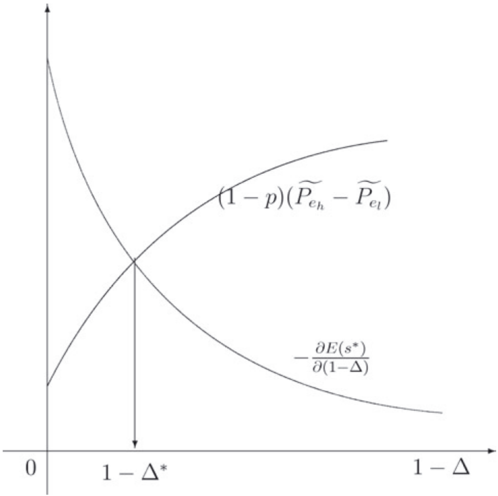
# CHZ - unbiased accounting case

- In equilibrium, $\widehat{\Delta} = \Delta^*$, resulting in

$$\frac{(1-p)p}{p+(1-p)\Delta^*}x_h = (1-p)[(s_h^* - s_l^*) - \frac{U(s_h^*) - U(s_l^*)}{U'(s_l^*)}]$$

- The left hand side is decreasing in $\Delta^*$ so increasing in $1-\Delta^*$ and the right hand side can be shown to be decreasing in $1-\Delta^*$ (see footnote 9) so there will be a unique interior solution $\Delta^* > 0$ if when $\Delta^* = 0$, the left hand side is larger than the right hand side.
- Under unbiased accounting, current owner cannot commit to not to involve in manipulation, which increases the compensation cost.

# CHZ - optimal solution in the unbiased case, graphical illustration



# CHZ - conservative accounting standard

- Let $\Delta^\Gamma$ denote the amount of earnings manipulation introduced when conservatism parameter is $\Gamma$ and denote $Z = 1 - \Delta^\Gamma - \Gamma$ as the informativeness of the accounting system that eventually generates the earnings number.
- Again let future owner's conjecture of $\Delta^\Gamma$ as $\widehat{\Delta^\Gamma}$. Then $\widehat{P}^\Gamma_{eh} = \Pr(x_h|e_h, \widehat{\Delta^\Gamma})x_h + \Pr(x_l|e_h, \widehat{\Delta^\Gamma})x_l = \frac{p(1-\Gamma)}{\widehat{\eta}}x_h$ and $\widehat{P}^\Gamma_{el} = \frac{p\Gamma}{1-\widehat{\eta}}x_h$,
  where
  $\widehat{\eta} \equiv \Pr(e_h|\widehat{\Delta^\Gamma}, \Gamma) = p(1-\Gamma) + (1-p)\widehat{\Delta^\Gamma} = 1 - \Gamma - (1-p)\widehat{Z}$.
  Again $\widehat{P}^\Gamma_{eh} - \widehat{P}^\Gamma_{el} = \frac{p(1-p)\widehat{Z}}{\widehat{\eta}(1-\widehat{\eta})}x_h$, generating an incentive for manipulation.

- The current owner therefore chooses $s_h^\Gamma, s_l^\Gamma$ and $\Delta^\Gamma$ to maximize

$$\eta(\widehat{P}_{eh}^\Gamma - s_h^\Gamma) + (1-\eta)(\widehat{P}_{el}^\Gamma - s_l^\Gamma)$$

subject to

$$\eta U(s_h^\Gamma) + (1-\eta)U(s_l^\Gamma) - D \geq 0 \text{ (IR)}$$

and

$$\begin{aligned} & \eta U(s_h^\Gamma) + (1-\eta)U(s_l^\Gamma) - D \\ \geq \;& \Delta^\Gamma U(s_h) + (1-\Delta^\Gamma)U(s_l) \text{ (IC)} \end{aligned}$$

# CHZ - conservative accounting standard

- Again both IR and IC binds, resulting in $U(s_h^{\Gamma*}) = (1 + \frac{\Gamma}{Z})\frac{D}{p}$ and $U(s_l^{\Gamma*}) = (1 + \frac{\Gamma}{Z} - \frac{1}{Z})\frac{D}{p}$.
- It can be shown that $\frac{\partial E[s^{\Gamma*}]}{\partial Z} < 0$ (Lemma) -> Intuitively, a higher informativeness earnings reduces the agency cost.
- Again substituting $s_h^{\Gamma*}$ and $s_l^{\Gamma*}$ into the current owner's problem results in

$$\max_{\Delta^\Gamma} \eta(\widehat{P}_{eh}^{\Gamma} - s_h^{\Gamma*}) + (1-\eta)(\widehat{P}_{el}^{\Gamma} - s_l^{\Gamma*}).$$

- Since $\Delta^\Gamma$ and $Z$ are one-to-one, optimizing over $\Delta^\Gamma$ is equivalent to optimizing over $Z$.

# CHZ - Proof of Lemma

$$
\begin{aligned}
\frac{\partial E[s^{\Gamma *}]}{\partial Z} &= -(1-p)(s_h^{\Gamma *} - s_l^{\Gamma *}) + \eta \frac{\partial s_h^{\Gamma *}}{\partial Z} + (1-\eta)\frac{\partial s_l^{\Gamma *}}{\partial Z} \\
&= -(1-p)(s_h^{\Gamma *} - s_l^{\Gamma *}) + \eta \frac{\partial}{\partial Z}[U^{-1}((1+\frac{\Gamma}{Z})\frac{D}{p})] \\
&\quad +(1-\eta)\frac{\partial}{\partial Z}[U^{-1}((1+\frac{\Gamma}{Z}-\frac{1}{Z})\frac{D}{p})] \\
&= -(1-p)(s_h^{\Gamma *} - s_l^{\Gamma *}) + \frac{\eta D}{p}\frac{1}{U'(s_h^{\Gamma *})}\frac{-\Gamma}{Z^2} \\
&\quad +\frac{(1-\eta)D}{p}\frac{1}{U'(s_l^{\Gamma *})}\frac{1-\Gamma}{Z^2} \\
&\le -(1-p)(s_h^{\Gamma *} - s_l^{\Gamma *}) + \frac{\eta D}{p}\frac{1}{U'(s_l^{\Gamma *})}\frac{-\Gamma}{Z^2} \\
&\quad +\frac{(1-\eta)D}{p}\frac{1}{U'(s_l^{\Gamma *})}\frac{1-\Gamma}{Z^2}.
\end{aligned}
$$

# CHZ - Proof of Lemma, continued

We then have

$$
\begin{aligned}
\frac{\partial E[s^{\Gamma *}]}{\partial Z} &\leq -(1-p)(s_h^{\Gamma *} - s_l^{\Gamma *}) + \frac{1}{U'(s_l^{\Gamma *})}\frac{D}{p}\frac{1}{Z^2}[(1-\eta)(1-\Gamma) - \eta\Gamma] \\
&= -(1-p)(s_h^{\Gamma *} - s_l^{\Gamma *}) + \frac{1}{U'(s_l^{\Gamma *})}\frac{D}{p}\frac{1}{Z^2}(1-\eta-\Gamma) \\
&= -(1-p)(s_h^{\Gamma *} - s_l^{\Gamma *}) + \frac{1}{U'(s_l^{\Gamma *})}\frac{D}{p}\frac{1}{Z^2}(1-p)Z \\
&= -(1-p)(s_h^{\Gamma *} - s_l^{\Gamma *}) + \frac{1}{U'(s_l^{\Gamma *})}\frac{D(1-p)}{pZ} \\
&= -(1-p)[\frac{U(s_h^{\Gamma *}) - U(s_l^{\Gamma *})}{U'(s_l^{\Gamma *})} - (s_h^{\Gamma *} - s_l^{\Gamma *})] < 0.
\end{aligned}
$$

# CHZ - conservative accounting standard

- Again substituting $s_h^{\Gamma*}$ and $s_l^{\Gamma*}$ into the current owner's problem results in
$$\max_{\Delta^\Gamma} \eta(\widehat{P}_{eh}^{\Gamma} - s_h^{\Gamma*}) + (1-\eta)(\widehat{P}_{el}^{\Gamma} - s_l^{\Gamma*}).$$
- Since $\Delta^\Gamma$ and $Z$ are one-to-one, optimizing over $\Delta^\Gamma$ is equivalent to optimizing over $Z$.

# CHZ - conservative accounting standard

- First order condition with respect to $Z$ results in

$$(1-p)(\widehat{P}^{\Gamma}_{eh}-\widehat{P}^{\Gamma}_{el})=-\frac{\partial E[s^{\Gamma *}]}{\partial Z}.$$

- Letting $\widehat{Z}=Z$ results in

$$(1-p)px_h[\frac{(1-\Gamma)}{\eta^*}-\frac{\Gamma}{1-\eta^*}]=-\frac{\partial E[s^{\Gamma *}]}{\partial Z}|_{Z=Z^*}.$$

# CHZ - when would conservative accounting reduce earnings management

- Proposition 2 provides a sufficient condition: $-\frac{U''(s_l^{\Gamma*})}{U'(s_l^{\Gamma*})}\frac{U'(s_h^{\Gamma*})}{U'(s_l^{\Gamma*})}\frac{U(s_h^{\Gamma*})-U(s_l^{\Gamma*})}{U'(s_l^{\Gamma*})-U'(s_h^{\Gamma*})} \leq \frac{1}{1-p}$ when $\Gamma$ is relatively small
- Intuition: conservative bias makes good news less good and bad news less bad, therefore reducing the benefit of managing earnings to get a good report; also conservative bias makes it more costly to induce the agent to work, therefore increasing the cost of earnings management

# CHZ - Proof of Proposition 2, sketch

- Recall that $Z^*$ is the solution of $FOC = -\frac{\partial E[s^{\Gamma *}]}{\partial Z}|_{Z=Z^*} - (1-p)(P_{eh}^{\Gamma *} - P_{el}^{\Gamma *}) = 0.$
- Implicit function theorem then implies that

$$\frac{dZ^*}{d\Gamma} = -\frac{\frac{\partial FOC}{\partial \Gamma}}{\frac{\partial FOC}{\partial Z}}|_{Z=Z^*} = \frac{\frac{\partial FOC}{\partial \Gamma}}{\frac{\partial^2 E[s^{\Gamma *}]}{\partial Z^2} + (1-p)\frac{\partial (P_{eh}^{\Gamma *} - P_{el}^{\Gamma *})}{\partial Z}}|_{Z=Z^*}.$$

- Assuming an interior solution exists, then SOC must be negative, implying that $\frac{\partial^2 E[s^{\Gamma *}]}{\partial Z^2} > 0$. In addition, $\frac{\partial (P_{eh}^{\Gamma *} - P_{el}^{\Gamma *})}{\partial Z}|_{\Gamma=0} = \frac{p(1-p)x_h}{\eta^{*2}} > 0$ so the denominator of $\frac{dZ^*}{d\Gamma}$ is positive. Need to find sufficient conditions for $\frac{\partial FOC}{\partial \Gamma} > 0$.

# CHZ - Proof of Proposition 2, sketch

- We can show that

$$\frac{\partial FOC}{\partial \Gamma}|_{Z=Z^*} = -\frac{\partial^2 E[s^{\Gamma *}]}{\partial Z \partial \Gamma}|_{Z=Z^*} + \frac{p(1-p)^2 Z^* x_h}{[\eta^*(1-\eta^*]^2}(2\eta^* - 1).$$

- The second term is non-negative as $\eta^* = p + (1-p)\Delta^{\Gamma^*} \geq p \geq \frac{1}{2}$. So need to find sufficient conditions for $\frac{\partial^2 E[s^{\Gamma *}]}{\partial Z \partial \Gamma}|_{Z=Z^*} < 0$.
- Since $E[s^{\Gamma *}] = \eta^* s_h^{\Gamma *} + (1-\eta^*) s_l^{\Gamma *}$ where $\eta^* = 1 - \Gamma - (1-p)Z^*$,

$$\begin{aligned} \frac{\partial E[s^{\Gamma *}]}{\partial \Gamma} &= \eta^* \frac{\partial s_h^{\Gamma *}}{\partial \Gamma} + (1-\eta^*) \frac{\partial s_l^{\Gamma *}}{\partial \Gamma} + \frac{\partial \eta^*}{\partial \Gamma}(s_h^{\Gamma *} - s_l^{\Gamma *}) \\ &= .\eta^* \frac{\partial s_h^{\Gamma *}}{\partial \Gamma} + (1-\eta^*) \frac{\partial s_l^{\Gamma *}}{\partial \Gamma} - (s_h^{\Gamma *} - s_l^{\Gamma *}). \end{aligned}$$

- Therefore

$$
\begin{aligned}
\frac{\partial^2 E[s^{\Gamma *}]}{\partial Z \partial \Gamma}|_{Z=Z^*} &= \frac{\partial \eta^*}{\partial Z^*}\frac{\partial s_h^{\Gamma *}}{\partial \Gamma} + \eta^* \frac{\partial^2 s_h^{\Gamma *}}{\partial \Gamma \partial Z^*} - \frac{\partial \eta^*}{\partial Z^*}\frac{\partial s_l^{\Gamma *}}{\partial \Gamma} \\
&\quad + (1-\eta^*)\frac{\partial^2 s_l^{\Gamma *}}{\partial \Gamma \partial Z^*} - \frac{\partial (s_h^{\Gamma *} - s_l^{\Gamma *})}{\partial Z^*} \\
&= -(1-p)\frac{\partial (s_h^{\Gamma *} - s_l^{\Gamma *})}{\partial \Gamma} - \frac{\partial (s_h^{\Gamma *} - s_l^{\Gamma *})}{\partial Z^*} \\
&\quad + \eta^* \frac{\partial^2 s_h^{\Gamma *}}{\partial \Gamma \partial Z^*} + (1-\eta^*)\frac{\partial^2 s_l^{\Gamma *}}{\partial \Gamma \partial Z^*}.
\end{aligned}
$$

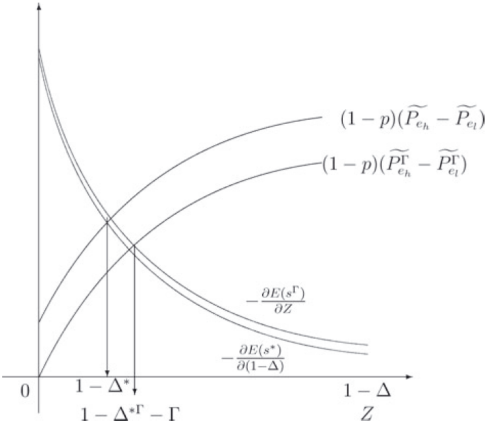
# CHZ - Proof of Proposition 2, sketch

- Evaluate all those derivatives at $\Gamma = 0$, insert into the above expressions and collecting terms results in

$$
\begin{aligned}
& \frac{\partial^2 E[s^{\Gamma *}]}{\partial Z \partial \Gamma}|_{Z=Z^*,\Gamma=0} \\
= \quad & \frac{D}{pZ^*}\left(\frac{1}{U'(s_h^{\Gamma *})} - \frac{1}{U'(s_l^{\Gamma *})}\right) \times \\
& [-\frac{1}{Z^*} + (1-p)\frac{-U''(s_l^{\Gamma *})}{U'(s_l^{\Gamma *})}\frac{D}{pZ^*}\left(\frac{1}{U'(s_l^{\Gamma *})}\right)^2 \frac{1}{\frac{1}{U'(s_h^{\Gamma *})} - \frac{1}{U'(s_l^{\Gamma *})}}].
\end{aligned}
$$

- A sufficient condition is thus $(1-p)\frac{-U''(s_l^{\Gamma *})}{U'(s_l^{\Gamma *})}\frac{D}{pZ^*}\left(\frac{1}{U'(s_l^{\Gamma *})}\right)^2 \frac{1}{\frac{1}{U'(s_h^{\Gamma *})} - \frac{1}{U'(s_l^{\Gamma *})}} < 0$, which is equivalent to the condition in Proposition 2.

# CHZ - Graphical illustration of Proposition 2



# CHZ - when would conservative accounting alleviate agency problem

- Need to look at $\frac{dE[s^{\Gamma*}]}{d\Gamma} = \frac{\partial E[s^{\Gamma*}]}{\partial Z^*}\frac{dZ^*}{d\Gamma} + \frac{\partial E[s^{\Gamma*}]}{\partial \Gamma}$.
- The first term is always negative under the conditions of Proposition 2, as more conservative accounting, through reducing earnings management, increases $Z$ and alleviates agency problem.
- The second term is the direct effect of conservative accounting on expected payment.
- $\frac{\partial E[s^{\Gamma*}]}{\partial \Gamma} = -(s_h^{\Gamma*} - s_l^{\Gamma*}) + [\eta^* \frac{\partial s_h^{\Gamma*}}{\partial \Gamma} + (1-\eta^*)\frac{\partial s_l^{\Gamma*}}{\partial \Gamma}]$ -> First term is negative as more conservative accounting makes the agent more likely to be paid lower wages with low earnings; the second term implies that conservative accounting, by introducing noise, exacerbates agency problem
- Proposition 3 shows a sufficient condition is that $\frac{1}{U'(s_h^{\Gamma*})}\eta^* + \frac{1}{U'(s_l^{\Gamma*})}(1-\eta^*) \leq \frac{s_h^{\Gamma*} - s_l^{\Gamma*}}{U(s_h^{\Gamma*}) - U(s_l^{\Gamma*})}$ when $\Gamma$ is relatively small

$$\frac{dE[s^{\Gamma*}]}{d\Gamma} = \frac{\partial E[s^{\Gamma*}]}{\partial Z^*}\frac{dZ^*}{d\Gamma} + \frac{\partial E[s^{\Gamma*}]}{\partial \Gamma}.$$

We can show that

$$\frac{\partial E[s^{\Gamma*}]}{\partial Z^*}|_{\Gamma=0} = -(1-p)(s_h^{\Gamma*} - s_l^{\Gamma*}) + (1-\eta^*)\frac{D}{p(Z^*)^2}\frac{1}{U'(s_l^{\Gamma*})},$$

and

$$\frac{\partial E[s^{\Gamma*}]}{\partial \Gamma}|_{\Gamma=0} = -(s_h^{\Gamma*} - s_l^{\Gamma*}) + \eta^*\frac{D}{pZ^*}\frac{1}{U'(s_h^{\Gamma*})} + (1-\eta^*)\frac{D}{pZ^*}\frac{1}{U'(s_l^{\Gamma*})}.$$

In addition, $\frac{dZ^*}{d\Gamma} = K > 0$ if the conditions in Proposition 2 are satisfied.

# CHZ - Proof of Proposition 3, sketch

We therefore have

$$
\begin{aligned}
& \frac{dE[s^{\Gamma *}]}{d\Gamma}|_{\Gamma=0} \\
= \; & -[(1-p)K+1](s_h^{\Gamma *} - s_l^{\Gamma *}) \\
& \underbrace{+(U(s_h^{\Gamma *}) - U(s_l^{\Gamma *}))[\frac{\eta^*}{U'(s_h^{\Gamma *})} + \frac{1-\eta^*}{U'(s_l^{\Gamma *})} + \frac{(1-p)K}{U'(s_l^{\Gamma *})}]}_{A}.
\end{aligned}
$$

# CHZ - Proof of Proposition 3, sketch

So $\frac{dE[s^{\Gamma*}]}{d\Gamma}|_{\Gamma=0} < 0$ if and only if

$$\frac{A}{s_h^{\Gamma*} - s_l^{\Gamma*}} < (1-p)K + 1,$$

which can be shown to be equivalent to

$$\frac{\eta^*}{U'(s_h^{\Gamma*})} + \frac{1-\eta^*}{U'(s_l^{\Gamma*})} < \frac{s_h^{\Gamma*} - s_l^{\Gamma*}}{U(s_h^{\Gamma*}) - U(s_l^{\Gamma*})} + [\frac{s_h^{\Gamma*} - s_l^{\Gamma*}}{U(s_h^{\Gamma*}) - U(s_l^{\Gamma*})} - \frac{1}{U'(s_l^{\Gamma*})}](1-p)K$$

# CHZ - Proof of Proposition 3, sketch

For the condition

$$\frac{\eta^*}{U'(s_h^{\Gamma*})} + \frac{1-\eta^*}{U'(s_l^{\Gamma*})} < \frac{s_h^{\Gamma*} - s_l^{\Gamma*}}{U(s_h^{\Gamma*}) - U(s_l^{\Gamma*})} + [\frac{s_h^{\Gamma*} - s_l^{\Gamma*}}{U(s_h^{\Gamma*}) - U(s_l^{\Gamma*})} - \frac{1}{U'(s_l^{\Gamma*})}](1-p)K,$$

since $\frac{dZ^*}{d\Gamma} = K > 0$ and $\frac{s_h^{\Gamma*} - s_l^{\Gamma*}}{U(s_h^{\Gamma*}) - U(s_l^{\Gamma*})} - \frac{1}{U'(s_l^{\Gamma*})} > 0$, a sufficient condition for the condition to hold is that $\frac{\eta^*}{U'(s_h^{\Gamma*})} + \frac{1-\eta^*}{U'(s_l^{\Gamma*})} < \frac{s_h^{\Gamma*} - s_l^{\Gamma*}}{U(s_h^{\Gamma*}) - U(s_l^{\Gamma*})}$, which is the condition in Proposition 3.

# Summary of the conservatism literature

- Two ways of modelling conservatism:
  - Think of reasonable assumptions of how conservatism affects the mapping from firm fundamentals to earnings (more conservative accounting not only reduces the frequency of good signals but increases the informativeness of good news)
  - Solve for optimal ex-ante reporting rule in the presence of various agency problems and find if the results are consistent with conservative accounting rules used in practice
- Conservatism has benefits and costs and whether accounting should be more conservative or not depends on settings and the underlying frictions.

# Next class

- Real effects of accounting disclosure
  - How accounting disclosure affects firms' real investment decisions if firms care about capital market prices rather than terminal cash flows
  - Different from the feedback effect: here the firm is informed and the uninformed market tries to make inferences from firms' disclosures
  - Because the market has less information than the firm, the law of iterated expectations may not apply
  - Such failure generates investment distortions (manager chooses investment decisions to maximize perceptions rather than firm value) lying at the core of real effects studies