BA 932 Session 6 Costly Misreporting and Earnings Management

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What we have done last class

- Managers have information that investors do not know and can choose to strategically disclose to the investors
- Managers can choose to strategically withhold information but cannot lie
- With no frictions full disclosure will ensure; with frictions (either disclosure cost or the manager may not necessarily have information) good news are disclosed and bad news are withheld
- How to precisely define investor skepticism

This class - relaxing the assumption that managers cannot lie

- We assume that the manager always have private information but they can choose not to disclose truthfully
- Lying is costly and the cost is usually exogenous (endogenizing lying cost would be an important contribution)
- As we will see, allowing for lying does not necessarily mean that investors are always fooled

A simple model of earnings management

- Assume that the manager observes privately the true value of firm value, v.
- The manager can choose to misreport $r = v + b$, with a quadratic lying cost of $\frac{1}{2}cb^2$.
- The manager chooses b to maximize $E[P|r] - \frac{1}{2}cb^2$ and investors set $P = E[v|r]$.

Investors completely back out the misreporting bias in equilibrium

- The investor will conjecture that $E[v|r] = r - \widehat{b}$ for some $\widehat{b} > 0$.
- The manager therefore chooses b to maximize $E[P|r] - \frac{1}{2}cb^2 = r - \widehat{b} - \frac{1}{2}cb^2 = v + b - \widehat{b} - \frac{1}{2}cb^2$.
- First order condition results in $b^* = \frac{1}{c}$ and in equilibrium, the conjecture has to be consistent so $\widehat{b} = b^* = \frac{1}{c}$.

Implications of the simple model

- In equilibrium there is no loss in information content of the report as investors can perfectly infer v from r.
- Nevertheless, earnings management generates deadweight cost of $\frac{1}{2}cb^{*2} = \frac{1}{2c}$.
- Managers are forced to manage earnings as investors will always subtract b^* from their reports so if they do not manipulate, they would be considered to have lower v so they are ex-ante worse off from the ability to manage earnings.
- This does not seem to be consistent with empirical findings.

Fisher and Verrecchia (2000) (FV) - simple way that investors cannot completely back out

- How to make sure that investors cannot completely back out? Add some other stuff that the investors also do not know.
- FV introduces uncertainty about the managers' incentives, i.e., instead of managers maximizing $E[P|r] - \frac{1}{2}cb^2$, managers will be maximizing $\widetilde{x}E[P|r] - \frac{1}{2}cb^2$ with $\widetilde{x}$ only known to the manager.
- In this way, investors cannot completely backout the reporting bias (although they back out on average).
- Justification for such assumption: investors are not completely sure to what extent managers want to maximize short-term prices (manager may want a lower short-term price, the weight of short-term price depends on managers' compensation contracts that may not be perfectly known to investors)

FV - Model Setup (simplified version)

- Firm value $\widetilde{v}$ has a prior normal distribution with mean 0 and variance σ_v^2
- The manager observes $\widetilde{v}$ and $\widetilde{x}$ and chooses the report r to maximize $\widetilde{x}E[P|r] - \frac{1}{2}c(r-v)^2$
- Investors only know that $\widetilde{x}$ is distributed normally with mean μ_x and variance σ_x^2; $\widetilde{x}$ is independent of $\widetilde{v}$.
- Investors price the firm at $E[P|r]$.

FV - Equilibrium definition

- The manager has to infer investors' pricing strategy and the investors have to infer the manager's reporting strategy
- The manager conjectures the investors's pricing strategy, $\widehat{P}(r)$, and chooses $b(v,x)$ such that $r = v + b$ to maximize $\widetilde{x}\widehat{P}(r) - \frac{1}{2}c(r-v)^2$.
- The investors conjecture the manager's reporting strategy, $\widehat{b}(v,x)$, and sets $P(r) = E[\widetilde{v}|r, \widehat{b}(v,x)]$.
- In equilibrium conjectures are consistent with actual, i.e., $\widehat{P}(r) = P(r)$, and $\widehat{b}(v,x) = b(v,x)$.

FV - Solving for an equilibrium

- Focus on linear equilibrium, i.e., $b(v,x) = \lambda_v v + \lambda_x x + \delta$ and $P(r) = \beta r + \alpha$; the conjecture is therefore $\widehat{b}(v,x) = \widehat{\lambda}_v v + \widehat{\lambda}_x x + \widehat{\delta}$ and $\widehat{P}(r) = \widehat{\beta} r + \widehat{\alpha}$.
- Given $\widehat{P}(r)$, the manager chooses $b(v,x)$ to maximize $x[\widehat{\beta}(v+b) + \widehat{\alpha}] - \frac{1}{2}cb^2$.
- First order condition therefore results in $b(v,x) = \frac{\widehat{\beta}x}{c}$.
- Matching coefficient results in $\lambda_v = 0$, $\lambda_x = \frac{\widehat{\beta}}{c}$, and $\delta = 0$, resulting in $\widehat{\lambda}_v = \widehat{\delta} = 0$.

FV - Solving for an equilibrium

- Given the conjecture that $\widehat{b}(v,x) = \widehat{\lambda}_x x$, investors will set

$$\begin{aligned} P(r) &= E[v|r, \widehat{\lambda}_x x] \\ &= E[v] + \frac{cov(v,r)}{var(r)}(r - E[r]) \\ &= \frac{\sigma_v^2}{\sigma_v^2 + \widehat{\lambda}_x^2 \sigma_x^2}(r - \widehat{\lambda}_x \mu_x). \end{aligned}$$

- Matching coefficients results in

$$\beta = \frac{\sigma_v^2}{\sigma_v^2 + \widehat{\lambda}_x^2 \sigma_x^2},$$

and

$$\alpha = -\frac{\sigma_v^2}{\sigma_v^2 + \widehat{\lambda}_x^2 \sigma_x^2}\widehat{\lambda}_x \mu_x.$$

FV - Solving for an equilibrium

- In equilibrium conjecture has to be the same as the actual, resulting in

$$\lambda_x = \frac{\beta}{c},$$

and

$$\beta = \frac{\sigma_v^2}{\sigma_v^2 + \lambda_x^2 \sigma_x^2},$$

or, equivalently,

$$\beta = \frac{\sigma_v^2}{\sigma_v^2 + (\frac{\beta}{c})^2 \sigma_x^2},$$

FV - equilibrium characterization

- Rearranging terms result in

$$\beta^3\sigma_x^2 + (\beta - 1)\sigma_v^2 c^2 = 0.$$

- The left hand side is increasing in β, is negative when $\beta \longrightarrow 0$, and positive when $\beta \longrightarrow 1$. Therefore there is a unique solution $\beta^* \in (0, 1)$. (Proposition 1) ->β can be interpreted as ERC.
- Note that when $\sigma_x^2 = 0$ and $\mu_x = 1$, $\beta = 1$, and $b = \frac{1}{c}$ and we are back to the fully-revealing benchmark.
- FV assumes that instead of the manager observing $\widetilde{v}$, the manager observes a noisy signal of $\widetilde{v}$: $\widetilde{e} = \widetilde{v} + \widetilde{n}$ -> results do not change qualitatively but one more comparative statics with respect to the variance of $\widetilde{n}$

FV - comparative statics

- Denote $L = \beta^3\sigma_x^2 + (\beta - 1)\sigma_v^2 c^2$ so the equation of β can be written as $L = 0$.
- Then $\frac{\partial\beta}{\partial c} = -\frac{\frac{\partial L}{\partial c}}{\frac{\partial L}{\partial\beta}} = -\frac{2(\beta-1)\sigma_v^2 c}{3\beta^2\sigma_x^2+\sigma_v^2 c^2} > 0$ as $\beta < 1$. Therefore, higher marginal cost of earnings management increases ERC.
- Intuition: higher marginal cost of earnings management reduces earnings management, letting investors put more weight on earnings signal and increases ERC
- When $c \to 0$, $\beta \to 0$; When $c \to \infty$, $\beta \to 1$.

FV - comparative statics

- $\frac{\partial \beta}{\partial \sigma_x^2} = -\frac{\frac{\partial L}{\partial \sigma_x^2}}{\frac{\partial L}{\partial \beta}} = -\frac{3\beta^2 \sigma_v^2}{3\beta^2 \sigma_x^2 + \sigma_v^2 c^2} < 0$. Therefore, more uncertainty about managers' short-term price incentives decreases ERC.
- Intuition: more uncertainty about managers' short-term price incentives reduces the informativeness of earnings as investors are not sure whether high earnings is because of high reporting bias because of short-term price incentives or because fundamentals are high, therefore reducing ERC.
- When $\sigma_x^2 \to 0$, $\beta \to 1$ and when $\sigma_x^2 \to \infty$, $\beta \to 0$.
- Ferri et al. (2019) empirical confirms this comparative statics using CD&A disclosure.

FV - comparative statics

- $\frac{\partial \beta}{\partial \sigma_v^2} = -\frac{\frac{\partial L}{\partial \sigma_v^2}}{\frac{\partial L}{\partial \beta}} = -\frac{(\beta-1)c^2}{3\beta^2\sigma_x^2+\sigma_v^2 c^2} > 0$ as $\beta < 1$. Therefore, more prior uncertainty about firm value increases ERC.
- Intuition: more prior uncertainty about firm value results in investors putting more weight on earnings report, further increasing ERC.

FV - comparative statics

- The intercept of the price function $\alpha = -\beta\lambda_x\mu_x = -\frac{\beta^2}{c}\mu_x$ so $\alpha < 0$ when $\mu_x > 0$ -> if the manager wants short-term stock price to be high (on average), the manager will bias earnings upwards (on average) and the investors adjust for that by subtracting a positive intercept. Focus on the case when $\mu_x > 0$.
- $\frac{\partial \alpha}{\partial c} = (\frac{\beta^2}{c^2} - \frac{2\beta}{c}\frac{\partial \beta}{\partial c})\mu_x \propto (\frac{\beta}{c} - 2\frac{\partial \beta}{\partial c}) \propto [3\beta^3\sigma_x^2 + \beta\sigma_v^2 c^2 - 4c^2(1 - \beta)\sigma_v^2] = \beta\sigma_v^2 c^2 - c^2(1-\beta)\sigma_v^2 = \sigma_v^2 c^2(2\beta - 1) > 0$ if and only if $\beta > \frac{1}{2}$.
- Since β increases with c, α will decrease with c first and then increase with c
- Intuition: increase of c has two effects on reporting bias: first, it reduces earnings management and thus reporting bias; second, it increases the weight investors put on earnings and thus increases reporting bias; the first effect dominate if and only if c is sufficiently large.

FV - comparative statics

- $\frac{\partial \alpha}{\partial \sigma_x^2} = -\frac{2\beta}{c^2}\mu_x \frac{\partial \beta}{\partial \sigma_x^2} > 0$; intuition: higher uncertainty about managerial incentives reduces earnings management and thus reduces the discount on the report;
- $\frac{\partial \alpha}{\partial \sigma_v^2} = -\frac{2\beta}{c^2}\mu_x \frac{\partial \beta}{\partial \sigma_v^2} < 0$; intuition: higher prior uncertainty about firm value increases earnings management and thus increases the discount on the report;
- $\frac{\partial \alpha}{\partial \mu_x} = -\frac{\beta^2}{c} < 0$; intuition: higher (expected) weight on short-term price increases earnings management and thus increases the discount on the report.

FV - comparative statics

- Expected reporting bias $E[b] = \frac{\beta}{c} E[\widetilde{x}] = \frac{\beta}{c}\mu_x$ and we focus on the case when $\mu_x > 0$.
- $\frac{\partial E[b]}{\partial c} = \mu_x(-\frac{\beta}{c^2} + \frac{1}{c}\frac{\partial \beta}{\partial c}) \propto -\frac{\beta}{c} + \frac{\partial \beta}{\partial c} = -\frac{\beta}{c} + \frac{2(1-\beta)\sigma_v^2 c}{3\beta^2\sigma_x^2 + \sigma_v^2 c^2} \propto -\beta\sigma_v^2 c^2 - (1-\beta)\sigma_v^2 c^2 < 0.$
- Intuition: higher c has the direct effect of reducing reporting bias and the indirect effect of increasing reporting bias as investors put more weight on reported earnings (i.e., β becomes larger). The direct effect always dominates.

FV - comparative statics

- $\frac{\partial E[b]}{\partial \sigma_x^2} = \frac{\mu_x}{c} \frac{\partial \beta}{\partial \sigma_x^2} < 0$; intuition: more reporting uncertainty reduces expected earnings management and thus expected bias
- $\frac{\partial E[b]}{\partial \sigma_v^2} = \frac{\mu_x}{c} \frac{\partial \beta}{\partial \sigma_v^2} > 0$; intuition: more prior uncertainty increases expected earnings management and thus expected bias
- $\frac{\partial E[b]}{\partial \mu_x} = \frac{\beta}{c} > 0$; intuition: more (expected) short-term price incentives increase expected earnings management and thus expected bias

FV - does the manager benefit from reporting bias

- Recall in the simple model without uncertainty about reporting incentives that the manager ex-ante is worse off from the opportunity to misreport earnings
- In FV, note that $\widetilde{b} = \frac{\beta \widetilde{x}}{c}$, $\alpha = -\frac{\beta^2}{c}\mu_x$, and the manager's ex-ante payoff is

$$
\begin{aligned}
& E[\widetilde{x}(\beta(\widetilde{v}+\widetilde{b})+\alpha)-\frac{c\widetilde{b}^2}{2}] \\
= \; & \beta E[\widetilde{x}\widetilde{v}] + \frac{\beta^2}{c}E[\widetilde{x}^2] + \alpha E[\widetilde{x}] - \frac{\beta^2}{2c}E[\widetilde{x}^2] \\
= \; & \frac{\beta^2}{2c}(\mu_x^2+\sigma_x^2) - \frac{\beta^2}{c}\mu_x^2 = \frac{\beta^2}{2c}(\sigma_x^2-\mu_x^2).
\end{aligned}
$$

- When the manager cannot misreport earnings (i.e., when $c \rightarrow +\infty$), the manager's ex-ante payoff is zero.

FV - does the manager benefit from reporting bias

- Therefore, the manager can ex-ante be better off from managing earnings if and only if $\sigma_x^2 > \mu_x^2$.
- Intuition: the manager benefits from information advantage over investor (regarding his reporting incentives) when this information advantage is sufficiently high.

FV - does the manager benefit from reporting bias

- Now look at whether the manager benefits from reporting bias ex-post, i.e., after the manager observes $\widetilde{x}$ but before observing $\widetilde{v}$.
- The manager's ex-post payoff is

$$
\begin{aligned}
& E[\widetilde{x}(\beta(\widetilde{v}+\widetilde{b})+\alpha)-\frac{c\widetilde{b}^2}{2}|\widetilde{x}=x] \\
= \quad & \beta x E[\widetilde{v}]+\frac{\beta^2 x^2}{c}+\alpha x-\frac{\beta^2 x^2}{2c} \\
= \quad & \frac{\beta^2 x^2}{2c}-\frac{\beta^2}{c} x \mu_x=\frac{\beta^2}{2c}[(x-\mu_x)^2-\mu_x^2].
\end{aligned}
$$

- When the manager cannot misreport earnings (i.e., when $c \longrightarrow +\infty$), the manager's ex-post payoff is zero for any realization of $\widetilde{x}$.

FV - does the manager benefit from reporting bias

- Therefore, ex-post the manager may benefit from reporting bias as well, when $x < 0$ or $x > 2\mu_x$.
- Intuition: when x is sufficiently away from the mean, the manager has much larger benefit from managing earnings than investors expect and therefore the manager will be better off.

Summary of FV

- A parsimonious way of not allowing investors to fully back out the reporting bias in a costly misreporting setting
- Provide empirical implications regarding ERC and expected reporting bias that can be tested
- Provide a justification for why and when managers would like to have discretion in misreporting earnings

Dye and Sridhar (2004) -DS

- Another way of introducing uncertainty of the manager's reporting incentives so investors cannot fully back out the reporting bias.
- Also introduce real effects - how accounting reports affect real investments
- Can then be used to study accounting issues: reliability versus relevance and aggregation versus disaggregation (impossible if the reports can be fully backed out by investors)

DS - Model Setup

- Firm endowed with the following technology: investment of I generates $\widetilde{\omega} \sim N(\frac{I^\alpha}{\alpha}, \sigma_\omega^2)$ where $0 < \alpha < 1$ so the net cash flow is $\widetilde{\omega} - I$.
- $\widetilde{\omega}$ is soft information that can only be observed by the manager (therefore introducing an opportunity for earnings management)
- The manager can choose to report $\widehat{\omega}$ with cost $\frac{c}{2}(\widehat{\omega} - \omega - \widetilde{\varepsilon}_\omega)^2$ where $\widetilde{\varepsilon}_\omega \sim N(0, \sigma_{\varepsilon_\omega}^2)$ to maximize selling price conditional on disclosed accounting report.
- The accountant observes $\widetilde{x} = \widetilde{\omega} + \widetilde{\delta}$ where $\widetilde{\delta} \sim N(0, \sigma_\delta^2)$.

DS - Model Setup

- The accountant reports $r = \lambda\widehat{\omega} + (1-\lambda)x$ where $\widehat{\omega}$ is the soft info and x is the hard info
- Example: think of r as provision for bad debt, $\widehat{\omega}$ as estimates from the manager based on proprietary algorithms and x as estimates based on, e.g., macroeconomic trends and industry trends.
- λ represents how much weight to put on the manager's proprietary estimates.
- It also introduces a relevance-reliability tradeoff: $\widetilde{\omega}$ is more relevant than $\widetilde{x}$ so if there is no manipulation one should set $\lambda = 1$. However, manipulation results in $\widehat{\omega}$ less reliable.

DS - equilibrium definition

- Given λ, an equilibrium consists of $P(\bullet)$, $\widehat{\omega}(\bullet)$, and I^* such that
  - Given $P(\bullet)$, x, ω, and ε_ω, $\widehat{\omega}(\omega, \varepsilon_\omega)$ maximizes $P(\lambda\widehat{\omega} + (1-\lambda)x) - \frac{c}{2}(\widehat{\omega} - \omega - \varepsilon_\omega)^2$
  - Given $P(\bullet)$ and $\widehat{\omega}(\bullet)$, I^* maximizes $E[P(\lambda\widehat{\omega} + (1-\lambda)x) - \frac{c}{2}(\widehat{\omega} - \omega - \widetilde{\varepsilon}_\omega)^2 | I] - I$.
  - Given I^* and $\widehat{\omega}(\bullet)$, $P(r) = E[\widetilde{\omega} | r, I^*]\ \forall r$.

DS - solving for equilibrium

- Again focus on linear equilibrium, where $P(r) = a + br$.
- Given $P(r)$, the owner chooses $\widehat{\omega}$ to maximize

$$a + b[\lambda\widehat{\omega} + (1 - \lambda)x] - \frac{c}{2}(\widehat{\omega} - \omega - \varepsilon_\omega)^2.$$

- First order condition therefore results in

$$\widehat{\omega}(\omega, \varepsilon_\omega) = \frac{b\lambda}{c} + \omega + \varepsilon_\omega.$$

DS - solving for equilibrium

- The aggregated report r can then be written as

$$\begin{aligned} r &= \lambda\widehat{\omega}(\omega, \varepsilon_\omega) + (1-\lambda)x \\ &= \omega + \lambda(\frac{b\lambda}{c} + \varepsilon_\omega) + (1-\lambda)\delta. \end{aligned}$$

- Standard Bayesian updating then results in

$$\begin{aligned} & E[\widetilde{\omega}|r, I^*] \\ = \ & E[\widetilde{\omega}|I^*] + \frac{cov(\widetilde{\omega}, r)}{var(r)}(r - E[r|I^*]) \\ = \ & E[\widetilde{\omega}|I^*] + \frac{\sigma_\omega^2}{\sigma_\omega^2 + \lambda^2\sigma_{\varepsilon_\omega}^2 + (1-\lambda)^2\sigma_\delta^2}(r - E[\widetilde{\omega}|I^*] - \frac{b\lambda^2}{c}). \end{aligned}$$

DS - solving for equilibrium

- Matching coefficient results in

$$b = \frac{\sigma_\omega^2}{\sigma_\omega^2 + \lambda^2 \sigma_{\varepsilon_\omega}^2 + (1-\lambda)^2 \sigma_\delta^2},$$

  and

$$a = E[\widetilde{\omega}|I^*] - b(E[\widetilde{\omega}|I^*] + \frac{b\lambda^2}{c}).$$

- Still need to figure out I^*, which requires solving for the manager's optimization problem.

DS - solving for equilibrium

- The manager chooses I to maximize

$$
\begin{aligned}
& a + bE[\widetilde{r}|I] - I - \frac{c}{2}E[(\widehat{\omega} - \omega - \varepsilon_\omega)^2|I] \\
= \quad & a + bE[\widetilde{r}|I] - I - \frac{c}{2}(\frac{b\lambda}{c})^2 \\
= \quad & a + b(E[\widetilde{\omega}|I + \frac{b\lambda^2}{c}) - I - \frac{c}{2}(\frac{b\lambda}{c})^2.
\end{aligned}
$$

- First order condition results in

$$
bI^{\alpha-1} - 1 = 0 \Rightarrow I^* = b^{\frac{1}{1-\alpha}}.
$$

DS - solving for equilibrium

- We therefore have $E[\widetilde{\omega}|I^*] = \frac{b^{\frac{\alpha}{1-\alpha}}}{\alpha}$ and thus $a = \frac{b^{\frac{\alpha}{1-\alpha}}}{\alpha}(1-b) - \frac{b^2\lambda^2}{c^2}$.
- This completes the characterization of equilibrium (Theorem 1).
- The parameter b is similar to β in FV: it measures ERC, or sensitivity of the price to report; higher b is, the higher the incentive for the manager to misreport.
- When real effect is introduced, I^* is also increasing in b so more efficient investment if ERC is higher.

DS - comparative statics of b

- Recall that $b = \frac{\sigma_\omega^2}{\sigma_\omega^2 + \lambda^2 \sigma_{\varepsilon_\omega}^2 + (1-\lambda)^2 \sigma_\delta^2}$.
- So it is straightforward to show that b is
  - increasing in σ_ω^2 (higher prior uncertainty increases investors' weight on reports)
  - decreasing in $\sigma_{\varepsilon_\omega}^2$ and σ_δ^2 (higher noise in either the hard or soft information decreases investors' weight on reports)
  - Optimal amount of λ to maximize b is $\lambda^* = \frac{\sigma_\delta^2}{\sigma_{\varepsilon_\omega}^2 + \sigma_\delta^2}$ (more weight on the information that is less noisy)

DS - comparative statics of expected reporting bias

- The expected amount of reporting bias is $\frac{b\lambda}{c} = \frac{1}{c}\frac{\lambda\sigma_\omega^2}{\sigma_\omega^2+\lambda^2\sigma_{\varepsilon_\omega}^2+(1-\lambda)^2\sigma_\delta^2}$.
- Straightforward algebra shows that $\frac{d(\frac{b\lambda}{c})}{d\lambda} \propto (1-\lambda)^2\sigma_\delta^2 + \sigma_\omega^2 - \lambda^2\sigma_{\varepsilon_\omega}^2$ so
  - increases in λ if $\sigma_\omega^2 \geq \sigma_{\varepsilon_\omega}^2$
  - may decrease in λ if $\sigma_\omega^2 < \sigma_{\varepsilon_\omega}^2$
  - increases in σ_ω^2 and decreases in $\sigma_{\varepsilon_\omega}^2$ and σ_δ^2 both when λ is fixed or when $\lambda^* = \frac{\sigma_\delta^2}{\sigma_{\varepsilon_\omega}^2+\sigma_\delta^2}$.

DS - intuition for the comparative statics of expected reporting bias

- Increasing λ has two effects on incentives to misreport:
  - direct effect of increasing the weight on the report -$>$ increases incentives to misreport
  - indirect effect that investors will put less weight on the report when pricing -$>$ decreases incentives to misreport
  - Direct effect dominates when prior uncertainty is sufficiently high so investors have to put a high weight on report
- Increases in σ^2_ω and decreases in $\sigma^2_{\varepsilon_\omega}$ and σ^2_δ results in investors putting more weight on reports and thus increases the managers' incentives to misreport.

DS - The disaggregation regime

- In the disaggregated regime, $\widehat{\omega}$ and x are separately reported
- An equilibrium consists of $P(\bullet)$, $\widehat{\omega}(\bullet)$, and I^* such that
  - Given $P(\bullet)$, x, ω, and ε_ω, $\widehat{\omega}(\omega, \varepsilon_\omega)$ maximizes $P(\widehat{\omega}, x) - \frac{c}{2}(\widehat{\omega} - \omega - \varepsilon_\omega)^2$
  - Given $P(\bullet)$ and $\widehat{\omega}(\bullet)$, I^* maximizes $E[P(\widehat{\omega}, x) - \frac{c}{2}(\widehat{\omega} - \omega - \widetilde{\varepsilon}_\omega)^2 | I] - I$.
  - Given I^* and $\widehat{\omega}(\bullet)$, $P(r) = E[\widetilde{\omega} | \widehat{\omega}, x, I^*]\ \forall \widehat{\omega}, x$.

DS - Solving for the equilibrium in the disaggregation regime

- Again focus on linear equilibrium $P(\widehat{\omega}, x) = a + b_\omega \widehat{\omega} + b_x x$.
- Given $P(\widehat{\omega}, x)$, the firm chooses $\widehat{\omega}$ to maximize $b_\omega \widehat{\omega} - \frac{c}{2}(\widehat{\omega} - \omega - \varepsilon_\omega)^2$ (as a and x is not affected by $\widehat{\omega}$)
- First order condition results in $\widehat{\omega}(\omega, \varepsilon_\omega) = \frac{b_\omega}{c} + \omega + \varepsilon_\omega$.

DS - Solving for the equilibrium in the disaggregation regime

- Using standard Bayesian updating (see the mathematica file),

$$\begin{aligned} P(\widehat{\omega}, x) &= E]\widetilde{\omega}|\widehat{\omega}, x, I^*] \\ &= E]\widetilde{\omega}|I^*] + \frac{1}{\sigma_\omega^2\sigma_{\varepsilon_\omega}^2 + \sigma_\omega^2\sigma_\delta^2 + \sigma_{\varepsilon_\omega}^2\sigma_\delta^2} \\ &\quad \times[\sigma_\omega^2\sigma_\delta^2(\widehat{\omega} - E[\widetilde{\omega}|I^*] - \frac{b_\omega}{c}) + \sigma_\omega^2\sigma_{\varepsilon_\omega}^2(x - E[\widetilde{\omega}|I^*])]. \end{aligned}$$

DS - Solving for the equilibrium in the disaggregation regime

- Matching coefficients therefore results in

$$b_\omega = \frac{\sigma_\omega^2 \sigma_\delta^2}{\sigma_\omega^2 \sigma_{\varepsilon_\omega}^2 + \sigma_\omega^2 \sigma_\delta^2 + \sigma_{\varepsilon_\omega}^2 \sigma_\delta^2},$$

$$b_x = \frac{\sigma_\omega^2 \sigma_{\varepsilon_\omega}^2}{\sigma_\omega^2 \sigma_{\varepsilon_\omega}^2 + \sigma_\omega^2 \sigma_\delta^2 + \sigma_{\varepsilon_\omega}^2 \sigma_\delta^2},$$

and

$$a = E[\widetilde{\omega}|I^*](1 - b_\omega - b_x) - \frac{b_\omega^2}{c}.$$

- Again need to solve for the firm's optimization problem to get I^*.

DS - Solving for the equilibrium in the disaggregation regime

- The firm chooses I to maximize

$$\begin{aligned} & E[P(\widehat{\omega}, x) - \frac{c}{2}(\widehat{\omega} - \omega - \widetilde{\varepsilon}_{\omega})^2 | I] - I \\ = \quad & a + b_{\omega}(E[\widetilde{\omega}|I] + \frac{b_{\omega}}{c}) + b_x E[\widetilde{\omega}|I] - \frac{c}{2}(\frac{b_{\omega}}{c})^2 - I. \end{aligned}$$

- First order condition therefore results in

$$(b_{\omega} + b_x) I^{\alpha - 1} - 1 = 0 \Rightarrow I^* = (b_{\omega} + b_x)^{\frac{1}{1-\alpha}}.$$

DS - Solving for the equilibrium in the disaggregation regime

- We therefore have $E[\widetilde{\omega}|I^*] = \frac{(b_\omega+b_x)^{\frac{\alpha}{1-\alpha}}}{\alpha}$, and $a = \frac{(b_\omega+b_x)^{\frac{\alpha}{1-\alpha}}}{\alpha}(1 - b_\omega - b_x) - \frac{b_\omega^2}{c}$.
- This completes the characterization of the equilibrium in the disaggregation regime (Theorem 2).

DS - comparative statics in the disaggregation regime

- Note that $b_\omega = \frac{\sigma_\omega^2 \sigma_\delta^2}{\sigma_\omega^2 \sigma_{\varepsilon_\omega}^2 + \sigma_\omega^2 \sigma_\delta^2 + \sigma_{\varepsilon_\omega}^2 \sigma_\delta^2}$ and $b_x = \frac{\sigma_\omega^2 \sigma_{\varepsilon_\omega}^2}{\sigma_\omega^2 \sigma_{\varepsilon_\omega}^2 + \sigma_\omega^2 \sigma_\delta^2 + \sigma_{\varepsilon_\omega}^2 \sigma_\delta^2}$.
- Straightforward algebra (see Mathematica file) shows that
  - b_ω increases in σ_ω^2 and σ_δ^2 and decreases in $\sigma_{\varepsilon_\omega}^2$
  - b_x increases in σ_ω^2 and $\sigma_{\varepsilon_\omega}^2$ and decreases in σ_δ^2
  - $b_\omega + b_x$ (and therefore I^*) increases in σ_ω^2 and decreases in $\sigma_{\varepsilon_\omega}^2$ and σ_δ^2
  - All consistent with intuition (more weight on reports with less noise)

DS - comparison of the two regimes

- First compare disaggregated regime and aggregated regime when $\lambda = \lambda^* = \frac{\sigma_\delta^2}{\sigma_{\varepsilon_\omega}^2 + \sigma_\delta^2}$.
- The expected manipulation cost in the aggregated regime is thus $E[\frac{c}{2}(\widehat{\omega} - \omega - \varepsilon_\omega)^2] = \frac{c}{2}(\frac{b\lambda^*}{c})^2 = \frac{(b\lambda^*)^2}{2c}$.
- Similarly, the expected manipulation cost in the disaggregated regime is $\frac{b_\omega^2}{2c}$.
- Note that $b\lambda^* = \frac{\lambda^* \sigma_\omega^2}{\sigma_\omega^2 + \lambda^{*2}\sigma_{\varepsilon_\omega}^2 + (1-\lambda^*)^2 \sigma_\delta^2} = \frac{\sigma_\delta^2 \sigma_\omega^2}{\sigma_\omega^2 \sigma_{\varepsilon_\omega}^2 + \sigma_\delta^2 \sigma_{\varepsilon_\omega}^2 + \sigma_\delta^2 \sigma_\omega^2} = b_\omega$. Therefore the expected manipulation cost is the same.

DS - comparison of the two regimes

- In addition, the equilibrium investment level is $b^{\frac{1}{1-\alpha}}$ in the aggregation regime and $(b_\omega + b_x)^{\frac{1}{1-\alpha}}$ in the disaggregation regime.
- When $\lambda = \lambda^*$, $b = \frac{(\sigma_\delta^2 + \sigma_{\varepsilon_\omega}^2)\sigma_\omega^2}{\sigma_\omega^2 \sigma_{\varepsilon_\omega}^2 + \sigma_\delta^2 \sigma_{\varepsilon_\omega}^2 + \sigma_\delta^2 \sigma_\omega^2} = b_\omega + b_x$ so investment efficiency is the same in the two regimes; so those two regimes have the same expected payoffs
- Intuition: when setting λ at the appropriate level, aggregation regime provides the same incentive for earnings management and thus the information content as the disaggregation regime.

DS - comparison of the two regimes

- Let $\Pi_g^A(\lambda) = E[\widetilde{\omega}|I^{A*}]$, i.e., the gross return of investment in the aggregation regime, and $C^A(\lambda) = \frac{c}{2}(\frac{b\lambda}{c})^2$. Similarly, let $\Pi_g^D = E[\widetilde{\omega}|I^{D*}]$ and $C^D = \frac{c}{2}(\frac{b_\omega}{c})^2$. We just showed that $\Pi_g^A(\lambda^*) - C^A(\lambda^*) = \Pi_g^D - C^D$.
- Now $\frac{\partial}{\partial\lambda}[\Pi_g^A(\lambda) - C^A(\lambda)]|_{\lambda=\lambda^*} = -\frac{\partial C^A(\lambda)}{\partial\lambda}|_{\lambda=\lambda^*} = -\frac{1}{2c}\frac{\partial(b^2\lambda^2)}{\partial\lambda}|_{\lambda=\lambda^*} = -\frac{b^2\lambda^*}{c} < 0$.
- Therefore by continuity when we decrease λ from λ^* by a little, $\Pi_g^A(\lambda) - C^A(\lambda) > \Pi_g^D - C^D$, i.e., aggregation regime will result in the manager strictly better off.
- Intuition: by reducing λ the manager's misreporting incentives are dampened, resulting in a lower manipulation cost. The reduction in earnings manipulation cost dominates the reduction in investment efficiency (as by envelope theorem the latter effect is second order).

DS - summary and follow up

- DS provides a parsimonious way of thinking about how to address accounting issues (in this case, aggregation and relevance vs. reliability) in a costly misreporting framework.
- The general theme: if you want less earnings manipulation, put less weight on soft signals -> aggregating with hard signals is a way to achieve this
- However, in DS aggregation never improves investment efficiency due to loss of information
- Gao and Jiang (2020) show that suppressing information through a special form of aggregation (discrete recognition) can improve investment efficiency, precisely by introducing less manipulation (they look at ex-ante manipulation which significantly simplifies the algebra)

Summary of the earnings management literature

- In a pure costly misreporting framework with only information asymmetry about the manager's privately observed signals, the report is fully revealing as investors can fully back out the misreporting (kind of similar to Grossman and Milgrom result in verifiable disclosure).
- Adding information asymmetry about other dimensions (e.g., uncertainty about the manager's incentives) can not only result in the report not fully revealing but also generate novel empirical implications
- The report not fully revealing also allows us to study issues related to accounting: suppressing information may be optimal if such information is subject to managerial manipulation

Next class

- Accounting conservatism: how to model it and what are the implications