

# BA 932 Session 5 Verifiable Disclosure with Strategic Withholding

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# What we have done so far

- Price formation in markets with informed traders: Grossman-Stiglitz and Hellwig (perfect competition) vs. Kyle and Glosten and Milgrom (imperfect competition)
- Prices aggregate information from informed traders (with noise)
- Such prices are important for managers in their decision making (feedback effect)

# This class - introduction to managerial disclosure

- Market does not have information and have to rely on manager as the informed party to provide information
- Whether and how manager provides information to the market is the core of studies on information disclosure
- Presumption: manager and the market has different objective functions (otherwise managers always disclose the whole truth)
- This and next class focuses on ex-post disclosure (i.e., manager chooses what to disclose after observing the realization of some value-relevant information)

# Assumptions on managerial disclosure

- Three types of models depending on assumptions on the truthfulness of managerial disclosure (see Stocken 2013)
  - Verifiable disclosure: if the manager disclose, any disclosure has to be truthful (persuasion game)
  - Cheap talk: the manager can disclose anything with zero cost
  - Costly misreporting: the manager can disclose differently from what s/he observes, with some cost
- This class focuses on verifiable disclosure and next focuses on costly misreporting

- When there is information asymmetry between the manager and the market, no disclosure results in market breakdown (Akerlof 1970)
- Verifiable disclosure completely resolves this issue, if the following additional crucial assumptions are satisfied:
  - The manager always have information and the market knows that the manager always have information
  - Disclosure incurs no cost
- This result is independently shown by Grossman (1981) and Milgrom (1981). We briefly discuss Grossman (1981) and discuss Milgrom (1981) in more detail as it also contains materials relevant for future classes.

# Grossman (1981) - illustrative example

- Suppose a manager wants to sell the firm to the market to maximize the selling price. The firm's value  $\tilde{v}$ , is uniformly distributed on  $[0,1]$ .
- Suppose first there is no disclosure. The knee-jerk reaction is that the price of the firm should be the average value of  $\tilde{v}$ , i.e.,  $\frac{1}{2}$ .
- Then what do firms with values above  $\frac{1}{2}$  do? Disclose their value (as disclosure is verifiable and therefore truthful).
- Knowing this, then the market would price silence as firms with value uniformly distributed on  $[0, \frac{1}{2}]$  so the price upon no disclosure would be  $\frac{1}{4}$ .

# Grossman (1981) - illustrative example continued

- Then what do firms with value between  $\frac{1}{4}$  and  $\frac{1}{2}$  do? Disclose their value.
- Knowing this, then the market would price silence as firms with value uniformly distributed on  $[0, \frac{1}{4}]$  so the price upon no disclosure would be  $\frac{1}{8}$ .
- The firms with value between  $\frac{1}{8}$  and  $\frac{1}{4}$  will now have incentives to disclose.
- The logic goes on till eventually all firms disclose - we have full disclosure and information asymmetry is not an issue.

# Milgrom (1981) - basics

- Two numbers:  $x$  and  $y$ , the definition of  $x > y$  is quite straightforward.
- What if  $\tilde{x}$  and  $\tilde{y}$  are random variables? What is the notion of  $\tilde{x} > \tilde{y}$ ?
- Solution: comparing the distribution of  $\tilde{x}$ , denoted as  $F_X$  and that of  $\tilde{y}$ , denoted as  $F_Y$ .
- $\tilde{x} > \tilde{y}$  if  $F_X$  first order stochastically dominates  $F_Y$



# Milgrom (1981) - basics on “better news”

- Extend to the case when  $\tilde{x}$  and  $\tilde{y}$  are two observable signals that are informative of some unobservable  $\tilde{\theta}$  (e.g.,  $\tilde{\theta}$  as fundamental value of the firm and  $\tilde{x}$  and  $\tilde{y}$  as news about the firm)
- Then knowing  $\tilde{x}$  ( $\tilde{y}$ ) will generate a posterior distribution of  $\theta$ , denoted as  $G(\theta|x)$  ( $G(\theta|y)$ ).
- $\tilde{x} > \tilde{y}$  if  $G(\theta|x)$  first order stochastically dominates  $G(\theta|y)$  for any non-degenerate prior  $G$  of  $\theta$  -> Milgrom calls this as “ $\tilde{x}$  is better news (or more favorable) than  $\tilde{y}$ ”

# Milgrom (1981) - basics on “better news”

- Proposition 1 in Milgrom (1981) shows that  $\tilde{x}$  is better news (or more favorable) than  $\tilde{y}$  if and only if  $\forall \theta' > \theta$ ,  $\frac{f(x|\theta')}{f(x|\theta)} > \frac{f(y|\theta')}{f(y|\theta)}$ .
- Proof of “only if”: pick the prior of  $G$  such that the support of  $G$  is  $\theta'$  and  $\theta$  with  $g(\theta') = g(\theta) = \frac{1}{2}$ . Then  $G(\theta|x)$  first order stochastically dominates  $G(\theta|y)$  implies that  $g(\theta|x) < g(\theta|y)$  and  $g(\theta'|x) > g(\theta'|y)$ , resulting in  $\frac{g(\theta'|x)}{g(\theta|x)} > \frac{g(\theta'|y)}{g(\theta|y)}$ . Bayes' Rule implies that this is equivalent to

$$\frac{f(x|\theta')g(\theta')}{f(x|\theta)g(\theta)} > \frac{f(y|\theta')g(\theta')}{f(y|\theta)g(\theta)},$$

which is equivalent to

$$\frac{f(x|\theta')}{f(x|\theta)} > \frac{f(y|\theta')}{f(y|\theta)}.$$

# Milgrom (1981) - basics on “better news”

- Proof of “if”: For any non-degenerate posterior  $G$  with support  $[\underline{\theta}, \bar{\theta}]$ , there must exist a  $\theta^*$  such that  $G(\theta^*) \in (0, 1)$ . Then for  $\theta \leq \theta^*$  and  $\theta' > \theta^*$ , we have

$$\frac{f(x|\theta')}{f(x|\theta)} > \frac{f(y|\theta')}{f(y|\theta)}.$$

Integrate the numerator from  $\theta^*$  to  $\bar{\theta}$  results in

$$\frac{\int_{\theta^*}^{\bar{\theta}} f(x|\theta') dG(\theta')}{f(x|\theta)} > \frac{\int_{\theta^*}^{\bar{\theta}} f(y|\theta') dG(\theta')}{f(y|\theta)},$$

which is equivalent to

$$\frac{f(x|\theta)}{\int_{\theta^*}^{\bar{\theta}} f(x|\theta') dG(\theta')} < \frac{f(y|\theta)}{\int_{\theta^*}^{\bar{\theta}} f(y|\theta') dG(\theta')}.$$

# Milgrom (1981) - basics on “better news”

- Integrate the numerator from  $\underline{\theta}$  to  $\theta^*$  results in

$$\frac{\int_{\underline{\theta}}^{\theta^*} f(x|\theta) dG(\theta) / f(x)}{\int_{\theta^*}^{\bar{\theta}} f(x|\theta') dG(\theta') / f(x)} < \frac{\int_{\underline{\theta}}^{\theta^*} f(y|\theta) dG(\theta) / f(y)}{\int_{\theta^*}^{\bar{\theta}} f(y|\theta') dG(\theta') / f(y)},$$

which from Bayes' Rule is equivalent to

$$\frac{G(\theta^*|x)}{1 - G(\theta^*|x)} < \frac{G(\theta^*|y)}{1 - G(\theta^*|y)},$$

i.e.,

$$G(\theta^*|x) < G(\theta^*|y).$$

# Milgrom (1981) - The monotone likelihood ratio property (MLRP)

- Any distribution  $f(\bullet|\theta)$  satisfies MLRP if for any  $\theta' > \theta$  and  $x > y$ ,

$$\frac{f(x|\theta')}{f(x|\theta)} > \frac{f(y|\theta')}{f(y|\theta)}.$$

- Sometimes it is convenient to write the inequality as

$$\frac{f(x|\theta')}{f(y|\theta')} > \frac{f(x|\theta)}{f(y|\theta)}$$

and express MLRP as  $\frac{f(x|\theta)}{f(y|\theta)}$  is increasing in  $\theta$ .

- When  $f(\bullet|\theta)$  satisfies MLRP, higher information is “good news” in the sense of Milgrom (1981).
- Example: if  $\theta$  has a prior normal distribution, then  $x = \theta + a + \tilde{\varepsilon}_1$  and  $y = \theta + \tilde{\varepsilon}_2$  with  $a > 0$  and  $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2 \sim N(0, 1)$  and independent of each other, then  $\frac{f(x|\theta)}{f(y|\theta)}$  is increasing in  $\theta$ .

# Applying MLRP to verifiable disclosure (simplified setting)

- The seller sells the firm of unknown quality  $\tilde{\theta}$  to a buyer: buyer's payoff is  $\tilde{\theta} - p$  and the seller's payoff is  $p$ .
- The seller has  $N$  pieces of information about the product, denoted by  $\tilde{x} = \{\tilde{x}_1, \dots, \tilde{x}_N\}$ . The seller issues a report  $r(x)$ . The report needs to be truthful, i.e.,  $x \in r(x)$  but it does not need to contain the whole truth, e.g.,  $r(x) = \{x_1\}$  is allowed.

# Applying MLRP to verifiable disclosure

- Result: Full disclosure is optimal.
- Proof: Suppose that one of the information, i.e.,  $x_j$  is not disclosed. Then the only rational inference from the buyer upon no disclosure is that  $x_j$  achieves the worst realization, denoted as  $\underline{x}$  (suppose not, that the inference is that conditional upon no disclosure,  $E[x_j|ND] = x_0$  for some  $x_0 > \underline{x}$ , then firms observing values of  $x_j > x_0$  will all disclose, but then  $E[x_j|ND] = E[x_j|x_j \leq x_0] < x_0$ , resulting in a contradiction). We then have full disclosure.

# Applying MLRP to verifiable disclosure - modified version

- Suppose the seller is only allowed to disclose  $k < N$  signals; assume that  $\tilde{x}_1, \dots, \tilde{x}_N$  are independent conditional upon  $\theta$  and the family of distributions  $F(\bullet|\theta)$  satisfies MLRP, then the best  $k$  pieces of news will be disclosed.
- Proof: by the property of MLRP we know that  $E[\theta|x_j]$  is increasing in  $x_j$ . Therefore, to maximize the expected selling price, which is  $E[\theta|x_j]$ , the best  $k$  pieces of news will be disclosed.



# Adding friction to verifiable disclosure - disclosure cost

- The manager sells the firm to the market and chooses disclosure strategy to maximize the selling price.
- Everybody is risk-neutral so price is equal to expected firm value conditional on disclosure.
- Firm value  $\tilde{u}$  has a prior normal distribution with mean  $u_0$  and precision  $h_0$ .
- The manager privately observes  $\tilde{u}$ .
- The manager can choose to remain silent; however, any disclosure has to be truthful and incurs a cost of  $c > 0$  (proprietary cost).

# Verifiable disclosure with disclosure cost - Verrechia (1983)

- Equilibrium does not feature full disclosure but a threshold-based disclosure: the firm will only disclose if and only if  $u > x$  (i.e., only good news is disclosed) where  $x$  is the unique solution of

$$x - c = E[\tilde{u}|ND] = E[\tilde{u}|\tilde{u} \leq x],$$

or, equivalently,

$$x - c = u_0 - \frac{1}{\sqrt{h_0}} \frac{\phi(\sqrt{h_0}(x - u_0))}{\Phi(\sqrt{h_0}(x - u_0))},$$

where  $\phi$  and  $\Phi$  are pdf and cdf of standard normal distribution.

- Proof: For firm observing  $u$ , disclosing results in a price of  $u - c$ , which is increasing in  $u$ . For firm not disclosing, the price is  $E[\tilde{u}|ND]$ , which is a constant of  $u$ . When  $u \rightarrow -\infty$ ,  $u - c < E[\tilde{u}|ND]$  and when  $u \rightarrow +\infty$ ,  $u - c > E[\tilde{u}|ND]$ . Therefore, there is a unique threshold  $x$  such that  $x - c = E[\tilde{u}|ND]$  and the firm will disclose if and only if  $u > x$ .

- One can rewrite the equation for  $x$  as

$$\sqrt{h_0}(x - u_0) + \frac{\phi(\sqrt{h_0}(x - u_0))}{\Phi(\sqrt{h_0}(x - u_0))} = \sqrt{h_0}c,$$

or

$$t + \frac{\phi(t)}{\Phi(t)} = \sqrt{h_0}c,$$

where

$$t = \sqrt{h_0}(x - u_0).$$

# Verifiable disclosure with disclosure cost - V83

- One can show that  $t$  increases with respect to  $c$ , so  $x$  increases with respect to  $c \rightarrow$  Higher proprietary cost results in less disclosure (recall disclose only when  $u > x$ ).
- Underlines the motivation of empirical studies relating competition to voluntary disclosure  $\rightarrow$  more competition results in less disclosure
- Intuition: higher proprietary cost makes disclosure more costly, need better news to disclose

# Verifiable disclosure with disclosure cost - V83

- Proof: sufficient to show that  $t + \frac{\phi(t)}{\Phi(t)}$  is increasing in  $t$ , i.e.,

$$1 + \frac{d}{dt} \left( \frac{\phi(t)}{\Phi(t)} \right) > 0.$$

- $1 + \frac{d}{dt} \left( \frac{\phi(t)}{\Phi(t)} \right) = 1 + \frac{\phi'(t)\Phi(t) - \phi^2(t)}{\Phi^2(t)} = 1 + \frac{-t\phi(t)\Phi(t) - \phi^2(t)}{\Phi^2(t)} = \frac{\Phi^2(t) - t\phi(t)\Phi(t) - \phi^2(t)}{\Phi^2(t)}.$

- Note that

$$\begin{aligned} & \frac{d}{dt} [\Phi^2(t) - t\phi(t)\Phi(t) - \phi^2(t)] \\ &= 2\Phi(t)\phi(t) - \phi(t)\Phi(t) - t\phi^2(t) - t\Phi(t)\phi'(t) - 2\phi(t)\phi'(t) \\ &= \Phi(t)\phi(t) - t\phi^2(t) + t^2\Phi(t)\phi(t) + 2t\phi^2(t) \\ &= \Phi(t)\phi(t) + t\phi^2(t) + t^2\Phi(t)\phi(t) > 0, \end{aligned}$$

and  $\lim_{t \rightarrow -\infty} \Phi^2(t) - t\phi(t)\Phi(t) - \phi^2(t) = 0$ . Therefore

$\Phi^2(t) - t\phi(t)\Phi(t) - \phi^2(t) > 0$  and  $t + \frac{\phi(t)}{\Phi(t)}$  is increasing in  $t$  is proved.

# Verifiable disclosure with uncertain information endowment

## - DJK

- The manager sells the firm to the market and chooses disclosure strategy to maximize the selling price.
- Firm value  $\tilde{x}$  has a prior density distribution of  $f$  on support  $[\underline{x}, \bar{x}]$  and mean  $\mu$ .
- The manager privately observes  $\tilde{x}$  with probability  $1 - p$  and observes nothing with probability  $p$ .
- The manager can choose to remain silent upon observing  $\tilde{x}$ ; however, any disclosure has to be truthful.
- The manager has to be silent if observing nothing.

# Verifiable disclosure with uncertain information endowment -DJK

- Equilibrium does not feature full disclosure but a threshold-based disclosure: the firm will only disclose if and only if  $x > y$  (i.e., only good news is disclosed) where  $y \in (\underline{x}, \mu)$  is the unique solution of

$$p(\mu - y) = (1 - p) \int_{\underline{x}}^y F(x) dx.$$

- Proof: For the firm observing  $x$ , disclose results in  $x$ , non-disclosure results in  $P(ND)$ , which is independent of  $x$ . Therefore, disclose if and only if  $x > y$ , where

$$y = P(ND) = E[x|ND]$$



# Verifiable disclosure with uncertain information endowment -DJK

- Proof continued: how do we calculate  $E[x|ND]$ ?
- Use Bayes' Rule:  $ND = A \cup B$ , where  $A = \{\text{Firm uninformed}\}$ , and  $B = \{\text{Firm informed and not disclosing}\}$ .
- Therefore

$$\begin{aligned} E[x|ND] &= E[x|A] \Pr(A|ND) + E[x|B] \Pr(B|ND) \\ &= E[x|A] \frac{\Pr(A)}{\Pr(ND)} + E[x|B] \frac{\Pr(B)}{\Pr(ND)} \\ &= \mu \frac{p}{p + (1-p)F(y)} + \frac{\int_{\underline{x}}^y x dF(x)}{\int_{\underline{x}}^y dF(x)} \frac{(1-p)F(y)}{p + (1-p)F(y)} \\ &= \mu \frac{p}{p + (1-p)F(y)} + \frac{(1-p) \int_{\underline{x}}^y x dF(x)}{p + (1-p)F(y)}. \end{aligned}$$

# Verifiable disclosure with uncertain information endowment -DJK

- Proof continued: The indifference condition then implies that

$$y[p + (1 - p)F(y)] = \mu p + (1 - p) \int_{\underline{x}}^y x dF(x).$$

- Using integration by parts,

$$\begin{aligned} \int_{\underline{x}}^y x dF(x) &= xF(x)|_{\underline{x}}^y - \int_{\underline{x}}^y F(x) dx \\ &= yF(y) - \int_{\underline{x}}^y F(x) dx. \end{aligned}$$

- The final equation is therefore

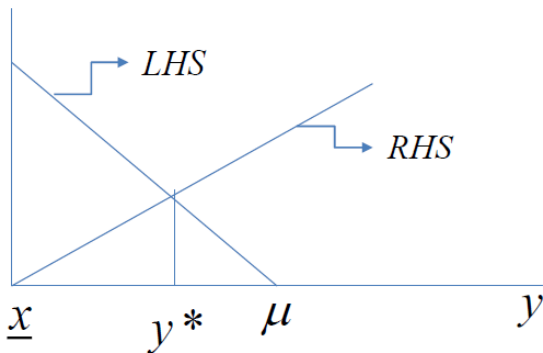
$$y[p + (1 - p)F(y)] = \mu p + (1 - p)[yF(y) - \int_{\underline{x}}^y F(x) dx],$$

resulting in

$$p(\mu - y) = (1 - p) \int_{\underline{x}}^y F(x) dx.$$

# Verifiable disclosure with uncertain information endowment -DJK

- Graphical illustration: there is a unique solution of  $y$ , as the left hand side is decreasing in  $y$ , the right hand side is increasing in  $y$ , and left hand side  $< (>)$  right hand side when  $y = \underline{x}$  ( $y = \mu$ ).



# Verifiable disclosure with uncertain information endowment -DJK

- Intuition: since the firm may be uninformed, investors give some benefit of doubt to non-disclosure.
- This allows managers with bad news to exploit such benefit of doubt to choose not to disclose.
- Firms with sufficiently good news would prefer disclosing to get a higher price.
- Disclosure threshold  $y < \mu$ : investors are skeptical (but not extremely skeptical) about non-disclosure.

# Verifiable disclosure with uncertain information endowment -DJK

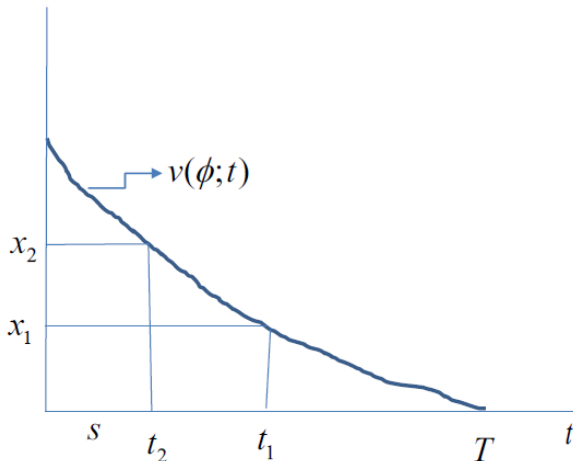
- Recall that the threshold  $y$  is defined by

$$p(\mu - y) = (1 - p) \int_{\underline{x}}^y F(x) dx.$$

- Comparative statics 1: decreasing  $p$  (increasing  $1 - p$ ) decreases  $y \rightarrow$  intuition: manager more likely to be informed, less benefit of doubt and more investor skepticism so lower no disclosure price and firms disclose more
- Proof: differentiate the condition by  $p$  results in  $\mu - y - p \frac{\partial y}{\partial p} = - \int_{\underline{x}}^y F(x) dx + (1 - p) F(y) \frac{\partial y}{\partial p}$  so  $\frac{\partial y}{\partial p} = \frac{\mu - y + \int_{\underline{x}}^y F(x) dx}{p + (1 - p) F(y)} > 0$ .

# Verifiable disclosure with uncertain information endowment -DJK

- Graphical illustration: good news disclosed earlier than bad news



# Verifiable disclosure with uncertain information endowment -DJK

- Comparative statics 2: two distributions of  $x$ :  $f$  and  $g$ , if  $f$  dominates  $g$  in terms of FOSD or SOSD then  $y_g \leq y_f$  so less disclosure under distribution  $f \rightarrow$  Intuition: FOSD or SOSD dominance means better prior distribution so more benefit of doubt and higher no disclosure price so firms disclose less
- Proof: Suppose  $y_g > y_f$ . Then  $F$  dominating  $G$  implies  $\mu_f \geq \mu_g$  and

$$\begin{aligned} p(\mu_f - y_f) &= (1 - p) \int_{\underline{x}}^{y_f} F(x) dx \leq (1 - p) \int_{\underline{x}}^{y_f} G(x) dx \\ &< (1 - p) \int_{\underline{x}}^{y_g} G(x) dx = p(\mu_g - y_g), \end{aligned}$$

implying that

$$y_f > y_g,$$

which is a contradiction.

# Empirical implications of DJK model

- Firms voluntarily disclose good news and hide bad news
- Closer to the earnings announcement date (when  $1 - p$  becomes lower), the firm is more likely to disclose bad news
- Bad news from other sources (e.g., information intermediaries, industry peers, etc.) is more likely to trigger firms disclosing bad news (see Archaya et al. 2011 for a N-firm dynamic disclosure model to fully explain firms clustering disclosure of bad news)
- However, since the firm either knows value perfectly or not knowing anything, disclosure is perfectly informative so investor skepticism applies only to non-disclosure but not to disclosure
- If want to study investor skepticism to disclosure, need to relax the assumption that firms know the value perfectly - the Shin model.



- The manager may not know the value perfectly so investors will apply skepticism even to disclosure
- Shin introduces a smart and tractable way of modelling how the manager may not know the value perfectly and how to precisely define investor skepticism
- In equilibrium, manager discloses all good news and suppress bad news and investors are skeptical (than naive Bayesian updating) about such disclosure

# Shin (1994) - model setup

- Manager chooses voluntary disclosure strategy  $D$  to maximize price upon disclosure,  $E[\tilde{x}|D]$  and no lying is allowed.
- $\tilde{x} \in X = \{x_1, x_2, \dots, x_N\}$  with prior probabilities  $p_1, p_2, \dots, p_N$  is the value of the firm.
- $N$  possible signals that may or may not be observed by the manager:  $\sigma_1, \sigma_2, \dots, \sigma_N \in \{0, 1\}$
- Realization of  $\sigma_j$  depends on the realization of  $\tilde{x}$ :  $\sigma_j(x_i) = \begin{cases} 1 & \text{if } i \geq j \\ 0 & \text{if } i < j \end{cases}$   
->  $\sigma_1 = 1$  and the rest  $\sigma_j$  can be either 0 or 1, depending in  $x_i$ .
- Manager observes  $\sigma_j$  with probability  $\theta_j \in (0, 1)$ .

# Shin (1994) - Manager's information structure

- This is an elegant way of modelling that the manager in general learns an interval  $I$  in which the true value of the firm lies.
- An illustrative example:  $\tilde{x} \in X = \{x_1, x_2, \dots, x_6\}$  with prior probabilities  $p_1, p_2, \dots, p_6$ .
  - Six possible signals that may or may not be observed by the manager:  $\sigma_1, \sigma_2, \dots, \sigma_6 \in \{0, 1\}$
  - Suppose that the manager observes  $\sigma_3 = 1$  and  $\sigma_5 = 0$ . Then the manager knows that  $\tilde{x} \in \{x_3, x_4\} \equiv I$ .
  - The manager cannot lie but can withhold information so  $D \supset I$ , e.g.,  $D = \{x_3, x_4, x_5, x_6\}$  is allowed but  $D = \{x_4, x_5, x_6\}$  not allowed.

# Shin (1994) - Model Timeline

- Three dates:
  - Date 0:  $x_i$  and  $\sigma_j(x_i)$  generated by nature
  - Date 1: manager observes  $I$  and discloses  $D$  subject to the constraint that  $D \supset I$  to maximize  $E[\tilde{x}|D]$ .
  - All uncertainties realized

# Shin (1994) - Truthful disclosure not an equilibrium

- Lemma 1: Truthful disclosing (i.e.,  $D = I$ ) is not an equilibrium.
- Proof: suppose not, then in equilibrium the market will conjecture that the manager is disclosing the whole truth. Suppose for some state  $x_i$ , the manager observes that  $\sigma_j(x_i) = 0$ , then given the market conjecture, the manager has an incentive to not disclose  $\sigma_j(x_i)$  to achieve a higher stock price.

# Shin (1994) - Truthful disclosure not an equilibrium

- To be more accurate, if  $\sigma_j(x_i) = 0$ , then  $\sigma_k(x_i) = 0$  for all  $k > j$ .  
Thus, if the manager discloses  $\sigma_j(x_i) = 0$ , then  $P(D) = \sum_{i=1}^{j-1} \Pi_i x_i$  for some  $\Pi_i$ .
- However, if the manager chooses an alternative  $D'$  that does not disclose  $\sigma_j(x_i) = 0$ , then  $P(D') = \sum_{i=1}^N \Pi'_i x_i$  for some  $\Pi'_i$ . Note that  $\Pi'_i$  FOSD  $\Pi_i$  so  $P(D') > P(D)$  and full disclosure cannot be an equilibrium.

# Shin (1994) - equilibrium disclosure strategy

- Shin shows that a plausible and intuitive equilibrium disclosure strategy is the “sanitization strategy”, disclose all good news (i.e., those with  $\sigma_j = 1$ ) and suppress all bad news (i.e., those with  $\sigma_j = 0$ ), i.e., disclosure is of the form  $D_i = \{x \geq x_i\}$ .
- What is the market response to the “sanitization strategy”? Use an example to illustrate.

# Shin (1994) - equilibrium disclosure strategy

- Think of  $N = 6$  and  $D_3 = \{x \geq x_3\}$ . Then from Bayes' Rule

$$\Pr(x = x_3 | D_3) = \frac{\Pr(D_3 | x = x_3) p_3}{\sum_{i=3}^6 \Pr(D_3 | x = x_i) p_i}.$$

- Clearly  $\Pr(D_3 | x = x_3) = \theta_3$  as one needs to know  $\sigma_3 = 1$  to disclose  $D_3$ ;
  - $\Pr(D_3 | x = x_4) = \theta_3(1 - \theta_4)$  as the manager must observe  $\sigma_3 = 1$  and not observe  $\sigma_4 = 1$  (as otherwise the manager will disclose  $x \geq x_4$ )
  - Similarly,  $\Pr(D_3 | x = x_5) = \theta_3(1 - \theta_4)(1 - \theta_5)$  and  $\Pr(D_3 | x = x_6) = \theta_3(1 - \theta_4)(1 - \theta_5)(1 - \theta_6)$ .
- Therefore  $\Pr(x = x_3 | D_3) =$ 
$$\frac{\theta_3 p_3}{\theta_3 p_3 + \theta_3(1 - \theta_4)p_4 + \theta_3(1 - \theta_4)(1 - \theta_5)p_5 + \theta_3(1 - \theta_4)(1 - \theta_5)(1 - \theta_6)p_6} =$$
$$\frac{p_3}{p_3 + (1 - \theta_4)p_4 + (1 - \theta_4)(1 - \theta_5)p_5 + (1 - \theta_4)(1 - \theta_5)(1 - \theta_6)p_6}.$$



# Shin (1994) - equilibrium disclosure strategy

- Suppose the market is naive, i.e., the market takes disclosure at face value, in other words, the manager discloses everything he or she knows.
- Then  $\Pr(D_3|x = x_3) = \theta_3(1 - \theta_4)(1 - \theta_5)(1 - \theta_6)$ , i.e., the manager observes  $\sigma_3 = 1$  but fails to observe  $\sigma_4 = \sigma_5 = \sigma_6 = 0$ . Similarly, we can calculate  $\Pr(D_3|x = x_4) = \Pr(D_3|x = x_5) = \Pr(D_3|x = x_6) = \theta_3(1 - \theta_4)(1 - \theta_5)(1 - \theta_6)$ .
- Bayes' Rule then implies that  $\Pr(x = x_3|D_3)$  given the naive belief is  $\frac{p_3}{p_3+p_4+p_5+p_6} < \Pr(x = x_3|D_3)$  given the sanitization strategy, in other words, the market belief given the sanitization strategy is more skeptical than the naive belief.
- In general, we can write, for  $k \geq i + 1$ , given sanitization strategy,

$$\Pr(x = x_k|D = D_i) = \frac{p_k \prod_{l=i}^k (1 - \theta_l)}{p_i + \sum_{s=i+1}^k \prod_{l=i+1}^s (1 - \theta_l) p_s}.$$

# Shin (1994) - sanitization strategy is an equilibrium strategy

- Theorem 1: Sanitization strategy is an equilibrium strategy.
- Proof: Already show that the price response is the equilibrium response to the sanitization strategy. Now need to show that sanitization strategy is optimal given the beliefs.
- Let  $A$  be the disclosure corresponding to sanitization strategy. Consider another feasible disclosure strategy  $B \supset I$  but  $B \neq A$ . If we show that  $E[\tilde{x}|A] > E[\tilde{x}|B]$  given the market's belief consistent with the sanitization strategy then we are done. Since  $A \supset I$  and  $B \supset I$ ,  $A \cap B \supset I$ . We can partition  $A \cup B$  into  $\{B|A(\equiv X), A \cap B(\equiv Y), A|B(\equiv Z)\}$ .

# Shin (1994) - sanitization strategy is an equilibrium strategy

- From sanitization strategy, conditional on  $D_i = \{x \geq x_i\}$ ,  $A = \{s_i, s_{i+1}, \dots, s_N\}$ . Any  $B \neq A$  can only exclude higher states as  $B \supset I$  and  $A$  is the most comprehensive set including higher states when  $\sigma_i = 1$ . Therefore  $E[\tilde{x}|X] < E[\tilde{x}|Y] < E[\tilde{x}|Z]$ . Since  $E[\tilde{x}|A]$  is a convex combination of  $Y$  and  $Z$ , and  $E[\tilde{x}|B]$  is a convex combination of  $X$  and  $Y$ ,  $E[\tilde{x}|A] > E[\tilde{x}|B]$  and the proof is complete.

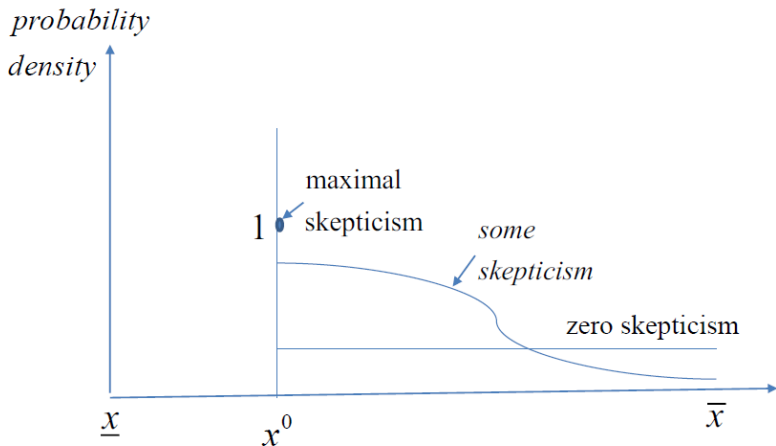
# Shin (1994) - precise meaning of more or less skepticism

- Benefit of doubt comes from the possibility that the manager may be uninformed, captured by the parameter  $\theta_i$  (or, more precisely,  $1 - \theta_i$ ). Thus less benefit of doubt, or more skepticism, if  $\theta_i$  is higher  $\forall i$ .
- $\theta_i = 0 \rightarrow$  manager always uninformed  $\rightarrow \Pr(x = x_k | D_i) \rightarrow \frac{p_k}{\sum_{j \geq i} p_j}$ ,  
i.e., the naive belief  $\rightarrow$  least level of skepticism and highest benefit of doubt.
- $\theta_i = 1 \rightarrow$  manager always informed  $\rightarrow \Pr(x = x_i | D_i) \rightarrow 1$  and  $\Pr(x = x_k | D_i) \rightarrow 0$  for  $k > i$ , i.e., the Grossman-Milgrom benchmark, assuming the worst when no full disclosure  $\rightarrow$  highest level of skepticism and least benefit of doubt.
- When  $\theta_i \in (0, 1)$ , level of skepticism is in the middle.

# Shin (1994) - comparative statics of more or less skepticism

- Theorem 2: if  $\theta_i$  is higher  $\forall i$ , the level of skepticism increases, and  $E[\tilde{x}|D]$  decreases  $\forall$  equilibrium disclosure strategy  $D$ .
- Intuition of the proof:  $\theta_i$  is higher  $\rightarrow$  distribution of  $\tilde{x}|x \geq x_i$  shifts to the left in the sense of FOSD so it is “bad news” in the sense of Milgrom (1981)  $\rightarrow E[\tilde{x}|D]$  decreases.

# Shin (1994) - graphical illustration of more or less skepticism



# Shin (1994) - empirical prediction

- higher  $\theta_i$  causes positive skewness in the distribution of earnings reports and also causes lower  $\frac{P}{E}$  ratio because of more skepticism.
- Therefore Shin (1994) predicts a positive association between the skewness in the distribution of earnings reports and a lower  $\frac{P}{E}$  ratio.
- Krishnan et al. (1996) confirms this prediction empirically.

# Shin (1994) - further implications

- Shin (1994) provides a precise measure of the degree of skepticism to manager's disclosures, which can potentially be used in various contexts:
  - Time-series variation of degree of skepticism (e.g., does SOX increase market confidence by reducing the degree of market skepticism?)
  - Cross-sectional variation of degree of skepticism (e.g., does market apply more skepticism to earnings that are perceived of lower quality?)
  - Event test of degree of skepticism: does market become more skeptical to disclosure after certain events (e.g., missing earnings targets, cutting of dividend, etc.?)



# Summary of the verifiable disclosure literature

- Without any frictions, information asymmetry alone cannot result in market malfunctioning - full disclosure will always arise if 1) the market knows that the manager has information; 2) disclosure incurs no cost; and 3) the manager cannot lie if chooses to disclose
- Relaxing assumptions 1 and 2 will result in no full disclosure -> good news will be disclosed and bad news will be withheld
- Further relaxing assumption 1 (that even when the manager is informed the manager may not know the whole truth) leads to richer predictions, in particular, a precise measure of the degree of skepticism.

- Relaxing assumption 3, i.e., the manager can lie, albeit at some cost (earnings management)
- It is shown that relaxing assumption 3 may not necessarily result in disclosure not being completely informative
- There are some subtleties in relaxing assumption 3, which generates some novel empirical implications