BA 932 Session 4 Feedback Effect With Disclosure

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March 7th, 2023

What we have done last class

- Feedback effect: manager/firm infers from information from prices for real decisions
- Price anticipates such possible managerial learning behavior (or not learning) -> price efficiency is not equivalent to real efficiency
- Introducing feedback effect may generate some counterintuitive result: it directly affects traders' trading behavior as good news versus bad news revelation impact firm value and thus affect their trading profits differentially

This class- feedback effect with disclosure

- Feedback effect presumes that the market has something that the manager, or more general, a decision maker does not know about.
- For tractability reasons early feedback effect paper assumes that the manager does not have any information.
- This also results in such papers unable to address disclosure issues in the presence of feedback effect.

This class- feedback effect with disclosure

- When modelling disclosure, need to think about what information the manager possesses and how is managerial information related to traders' information.
- If traders are required to acquire information, such disclosure will affect the feedback effect through traders' information acquisition activities. Disclosure -> Traders' private information acquisition -> Information incorporated into prices -> Strength of the feedback effect

This class- feedback effect with disclosure

- The two papers we discuss today exploit such interactions
  - Gao and Liang (2013) (GL): manager's information is the same as the trader's information (so disclosure substitutes traders' information acquisition)
  - Goldstein and Yang (2019) (GY): manager has overlapping information as the trader's but the information is multi-dimensional so disclose which dimension makes a difference (again through affecting traders' information acquisition) -> in their model disclosure comes from a third party in terms of a public signal (and the manager can learn from it) so cannot be interpreted as accounting disclosure (think of disclosure by regulators or credit rating agencies) -> they do have a robustness check (Section 5.3) when the disclosure comes from the manager but no closed-form solutions

GL - overview

- The information that manager knows (with some probability) is identical to the information that informed investor may know.
- Therefore more public disclosure crowds out information acquisition of informed investors, weakening feedback effect and reducing firm value - cost of more public disclosure
- Need to introduce a benefit of more public disclosure - reduces information asymmetry and improves liquidity in the secondary market (recall that Kyle's lambda and bid-ask spread increases with information asymmetry) $->$ reduces the discount in the primary market as primary market participants become noisy traders in the secondary market later
- Therefore there is an optimal interior degree of disclosure quality.
- We discuss a simplified model of GL.

GL - model timeline

Date 1	2	3	4
Firm chooses a disclosure level β; then primary market for firm shares opens.	Speculator acquires a signal y; then the firm makes disclosure x and liquidity shock n is realized. Firm shares traded in secondary market at price P.	Firm observes price P and chooses investment K.	Cash flow is realized.

FIG. 1.—The time-line.

GL - timeline explanation

- Firm chooses an ex-ante disclosure policy, which is the probability of disclosure β and sell one share in the primary market (think about IPO) at price V.
- Then firm shares get traded in the secondary market, primary market investors are subject to liquidity shocks and become noisy traders. They trade with a speculator in a discrete Kyle setting and the speculator can (and will) choose to acquire information y.
- Firm learns from secondary market prices and chooses investment decisions correspondingly (the feedback effect).

GL - firm's production technology

- The relevant information denoted by $\mu \in \{H, L\}$ with $H \equiv \mu_0 + \sigma_\mu$ and $L \equiv \mu_0 - \sigma_\mu$ and $\Pr(\mu = H) = \frac{1}{2}$
- μ determines the profitability of the firm's growth option.
- Terminal cash flow from the growth option is $G = \mu\sqrt{2gK} - K$.

GL - speculator's information acquisition technology

- The speculator can acquire information y about μ with some cost.
- The information acquisition technology follows Verrecchia 1982, can acquire more precise information with higher cost.
- Can acquire $y \in \{h, l\}$ with $\Pr(y = h|\mu = H) = \Pr(y = l|\mu = L) = \frac{\gamma+1}{2}$ with a cost of $C(\gamma) = \frac{c}{2}\gamma^2$.
- This also implies that the posterior of μ conditional on y, following Bayes rule, would be

$$\Pr(\mu = H|y = h) = \Pr(\mu = L|y = l) = \frac{\gamma + 1}{2}.$$

- $\gamma = 0$ -> completely uninformative signal; $\gamma = 1$ -> perfect signal

GL - firm's information endowment and disclosure strategy

- The firm observes μ with probability f and observes nothing with probability $1-f$ (need firm to be not perfectly informed as otherwise there is nothing to learn from the market). Denote the firm's information as $z \in \{\mu, \phi\}$.
- Conditional on observing μ, the firm then chooses the disclosure quality β.
- Therefore, the firm's disclosed signal

$$x = \begin{cases} \mu \text{ with probability } \beta f \\ \varnothing \text{ with probability } 1-\beta f \end{cases}.$$

- The firm also incurs a direct cost of disclose $W(\beta)$ that is strictly increasing and convex in β with $W(0) = W_\beta(0) = 0$ and $W_\beta(1) = \infty$ (unsatisfying as this implies the original trade-off is not enough)
- The firm chooses β to maximize $V(\beta) \equiv E[G] - \Pi(\beta) - W(\beta)$, where $\Pi(\beta)$ is the liquidity discount as the expected loss to the speculator at date 2.

GL - trading in the secondary market

- Standard discrete Kyle setting: the primary market investor is subject to a liquidity shock (serves as noise traders) and has to trade $n \in \{-\sigma_n, \sigma_n\}$ with equal probability.
- The speculator chooses $d(x, y) \in \{-\sigma_n, \sigma_n\}$.
- The market maker observes the total order flow $Q = n + d \in \{-2\sigma_n, 0, 2\sigma_n\}$ and sets the price

$$P = E[G - W|x, Q]$$

GL - solving for the model

- Ex-ante disclosure on top of a standard feedback model
- Solve by backward induction -> solve the feedback model first taking the disclosure quality as given and then solve for the optimal disclosure quality
  - solve investment decision
  - solve the trading game next by conjecturing and then confirming the speculator's trading decision
  - solve for the speculator's information acquisition decision
  - solve disclosure quality last

GL - solving for investment decisions

- The firm observes z, P and chooses the investment K to maximize

$$E[G|z,K] = E[\mu\sqrt{2gK} - K|z,P].$$

- Note that W is not included as W only depends on the disclosure policy and not affected by investment.
- First order condition results in

$$K^* = \frac{g}{2}(E[\mu|z,P])^2.$$

- We therefore have

$$E[G|z,K] = \frac{g}{2}(E[\mu|z,P])^2.$$

GL - solving for the trading game

- The price will depends on the firm's disclosure when the firm discloses and the order flow when the firm does not disclose.
- Details in the following table:

TABLE 1
The Information Structure

Case	Probability	Firm Information z	Firm Disclosure x	Speculator Information y	Price P
1	$f\beta$	μ	μ	y	$P(\mu)$
2	$f(1-\beta)$	μ	$\varnothing$	y	$P(y)$
3	$1-f$	$\varnothing$	$\varnothing$	y	$P(y)$

GL - solving for the trading game

- Clearly, when the firm discloses, $z = \mu$ so $E[G|z, K] = \frac{g\mu^2}{2}$. In this case the speculator will not trade as their information is inferior relative to the firm's disclosure.
- When the firm does not disclose, apply the usual intuitive conjecture that the speculator chooses $d(h) = \sigma_n$ and $d(l) = -\sigma_n$ (i.e., buy when observing good news and sell when observing bad news)
- The market maker infers $E[\mu|\phi, 2\sigma_n] = E[\mu|h]$, $E[\mu|\phi, -2\sigma_n] = E[\mu|l]$, and $E[\mu|\phi, 0] = E[\mu] = \mu_0$.
- Verification similar to what we did last class (and in the discrete Kyle setting) so left for exercise.

GL - solving for the speculator's information acquisition decision

- From table 1, the speculator will only make profit under cases 2 and 3, with happens with probability $1 - f\beta$.
- Prices in case 2 and 3:
  - When $Q = 2\sigma_n$, $P(2\sigma_n) = \frac{g(E[\mu|h])^2}{2} = \frac{g(\frac{\gamma+1}{2}H + \frac{1-\gamma}{2}L)^2}{2} = \frac{g}{2}(\mu_0 + \gamma\sigma_\mu)^2$ and
  - When $Q = -2\sigma_n$, $P(-2\sigma_n) = \frac{g(E[\mu|l])^2}{2} = \frac{g(\frac{1-\gamma}{2}H + \frac{\gamma+1}{2}L)^2}{2} = \frac{g}{2}(\mu_0 - \gamma\sigma_\mu)^2$ and
  - When $Q = 0$, $P(0) = \frac{g(E[\mu])^2}{2} = \frac{g(\frac{1}{2}H + \frac{1}{2}L)^2}{2} = \frac{g}{2}(\mu_0)^2$ and

GL - solving for the speculator's information acquisition decision

- Speculator's expected trading profit (gross of information acquisition cost):
  - buying when $y = h$ results in an expected trading profit of

$$\begin{aligned} & \frac{1}{2}\sigma_n[\frac{g}{2}(\mu_0+\gamma\sigma_\mu)^2 - P(2\sigma_n)] + \frac{1}{2}\sigma_n[\frac{g}{2}(\mu_0+\gamma\sigma_\mu)^2 - P(0)] \\ = \; & \frac{g(2\mu_0+\gamma\sigma_\mu)\gamma\sigma_n\sigma_\mu}{4}. \end{aligned}$$

  - selling when $y = l$ results in an expected trading profit of

$$\begin{aligned} & \frac{1}{2}\sigma_n[P(-2\sigma_n) - \frac{g}{2}(\mu_0-\gamma\sigma_\mu)^2] + \frac{1}{2}\sigma_n[P(0) - \frac{g}{2}(\mu_0-\gamma\sigma_\mu)^2] \\ = \; & \frac{g(2\mu_0-\gamma\sigma_\mu)\gamma\sigma_n\sigma_\mu}{4}. \end{aligned}$$

  - The total expected trading profit is therefore $\frac{1}{2}(1-f\beta)[\frac{g(2\mu_0+\gamma\sigma_\mu)\gamma\sigma_n\sigma_\mu}{4} + \frac{g(2\mu_0+\gamma\sigma_\mu)\gamma\sigma_n\sigma_\mu}{4}] = \frac{(1-f\beta)g\mu_0\gamma\sigma_n\sigma_\mu}{2}$.

GL - solving for the speculator's information acquisition decision

- The speculator therefore chooses γ to maximize
$$\frac{(1-f\beta)g\mu_0\gamma\sigma_n\sigma_\mu}{2} - \frac{c\gamma^2}{2}.$$
- First order condition results in
$$\gamma^* = \frac{(1-f\beta)}{2c} g\mu_0\sigma_n\sigma_\mu.$$
- Therefore in equilibrium the speculator's expected trading profit is
$$\begin{aligned} & \frac{(1-f\beta)g\mu_0\gamma^*\sigma_n\sigma_\mu}{2} - \frac{c\gamma^{*2}}{2} \\ = \; & \frac{[(1-f\beta)g\mu_0\sigma_n\sigma_\mu]^2}{4c}. \end{aligned}$$
- This is also the liquidity discount in the primary market, i.e.,
$$\Pi(\beta) = \frac{[(1-f\beta)g\mu_0\sigma_n\sigma_\mu]^2}{4c} = c\gamma^{*2},$$

Solving for the model - solve for the optimal disclosure quality

- What is left is calculation of $E[G] = E[E[G|z, K]] = \frac{g}{2}E[(E[\mu|z, P])^2]$.
  - In case 1, $E[\mu|z, P] = \mu$ so $E[G|z, K] = \frac{g\mu^2}{2}$.
  - In case 2, $E[\mu|z, P] = \mu$ so $E[G|z, K] = \frac{g\mu^2}{2}$.
  - In case 3, with probability $\frac{1}{4}$, $Q = 2\sigma_n$, and $E[\mu|z, P] = \mu_0 + \gamma^*\sigma_\mu$; with probability $\frac{1}{4}$, $Q = -2\sigma_n$, and $E[\mu|z, P] = \mu_0 - \gamma^*\sigma_\mu$; with probability $\frac{1}{2}$, $Q = 0$, and $E[\mu|z, P] = \mu_0$.

Solving for the model - solve for the optimal disclosure quality

- Therefore

$$
\begin{aligned}
E[G] &= fE[\frac{g\mu^2}{2}] + (1-f)[\frac{1}{4}\frac{g}{2}(\mu_0+\gamma^*\sigma_\mu)^2 + \frac{1}{4}\frac{g}{2}(\mu_0-\gamma^*\sigma_\mu)^2 \\
&\quad +\frac{1}{2}\frac{g}{2}(\mu_0)^2] \\
&= f\frac{g}{2}[\frac{1}{2}(\mu_0+\sigma_\mu)^2 + \frac{1}{2}(\mu_0-\sigma_\mu)^2] + (1-f)[\frac{g}{2}(\mu_0)^2 \\
&\quad +\frac{g}{4}\gamma^{*2}\sigma_\mu^2] \\
&= f\frac{g}{2}(\mu_0^2+\sigma_\mu^2) + (1-f)[\frac{g}{2}(\mu_0)^2 + \frac{g}{4}\gamma^{*2}\sigma_\mu^2] \\
&= \frac{g}{2}[\mu_0^2 + f\sigma_\mu^2 + (1-f)\frac{\gamma^{*2}\sigma_\mu^2}{2}].
\end{aligned}
$$

Solving for the model - solve for the optimal disclosure quality

- Note that $E[G]$ depends on β implicitly via γ^*. Since γ^* decreases with β, $E[G]$ decreases with β -> higher disclosure quality crowds out private information acquisition, resulting in lower learning from price and thus the expected firm value.

Solving for the model - solve for the optimal disclosure quality

- You now see why we need the disclosure cost term $W(\beta)$.
- Suppose we do not have the $W(\beta)$ term, then the firm chooses β to maximize

$$
\begin{aligned}
& \frac{g}{2}[\mu_0^2 + f\sigma_\mu^2 + (1-f)\frac{\gamma^{*2}(\beta)\sigma_\mu^2}{2}] - c\gamma^{*2}(\beta) \\
= \; & \frac{g}{2}[\mu_0^2 + f\sigma_\mu^2] + [\frac{g(1-f)\sigma_\mu^2}{4} - c]\gamma^{*2}(\beta).
\end{aligned}
$$

- Therefore the solution is always a corner solution (no disclosure or full disclosure), either the benefit of more disclosure (decrease in Π) dominates the cost (decrease in $E[G]$) or vice versa.

$$
V_\beta = 0.
$$

Solving for the model - solve for the optimal disclosure quality

- When introducing $W(\beta)$ and assume that $\frac{g(1-f)\sigma_\mu^2}{4} - c < 0$ (means full disclosure without $W(\beta)$), we have the first order condition for β^* as

$$[c - \frac{g(1-f)\sigma_\mu^2}{4}]\frac{f(1-f\beta^*)}{2c^2}(g\mu_0\sigma_n\sigma_\mu)^2 - W'(\beta^*) = 0,$$

equivalently,

$$V_\beta = 0.$$

Properties of the equilibrium and comparative statics

- By envelope theorem,

$$
\begin{aligned}
\frac{dV}{dc} &= \frac{\partial V}{\partial c} \\
&= -\gamma^{*2}(\beta) + [\frac{g(1-f)\sigma_\mu^2}{4} - c]2\gamma^*(\beta)\frac{\partial \gamma^*(\beta)}{\partial c} \\
&\propto -\gamma^*(\beta) + 2[\frac{g(1-f)\sigma_\mu^2}{4} - c]\frac{\partial \gamma^*(\beta)}{\partial c} \\
&= -\gamma^*(\beta) - 2[\frac{g(1-f)\sigma_\mu^2}{4} - c]\frac{\gamma^*(\beta)}{c} \\
&\propto 1 - \frac{g(1-f)\sigma_\mu^2}{2c}.
\end{aligned}
$$

- Therefore, $\frac{dV}{dc} < 0$ if and only if $g > \frac{2c}{(1-f)\sigma_\mu^2}$. Intuition: sufficiently large growth option -> feedback more important -> reduction in information acquisition more detrimental to firm value.

Properties of the equilibrium and comparative statics

- How does β^* vary with σ_μ^2? From the implicit function theorem

$$sgn(\frac{\partial \beta^*}{\partial \sigma_\mu^2}) = sgn(-\frac{V_{\beta\sigma_\mu^2}}{V_{\beta\beta}}) = sgn(V_{\beta\sigma_\mu^2}),$$

and

$$\begin{aligned} V_{\beta\sigma_\mu^2} &= -\frac{(1-f)g}{4}\frac{f(1-f\beta^*)}{2c^2}(g\mu_0\sigma_n\sigma_\mu)^2 \\ &\quad +[c-\frac{g(1-f)\sigma_\mu^2}{4}]\frac{f(1-f\beta^*)}{2c^2}(g\mu_0\sigma_n)^2 \\ &= \frac{fg(1-f\beta^*)(\mu_0\sigma_n\sigma_\mu)^2}{2c^2}[c-\frac{g(1-f)\sigma_n^2}{2}] < 0 \end{aligned}$$

if and only if $g > \frac{2c}{(1-f)\sigma_n^2}$. Intuition: when growth option is sufficiently important, firms with higher growth option disclose less to induce feedback from the market.

Properties of the equilibrium and comparative statics

- How does β^* vary with g? From the implicit function theorem

$$sgn(\frac{\partial \beta^*}{\partial g}) = sgn(-\frac{V_{\beta\sigma_\mu^2}}{V_{\beta\beta}}) = sgn(V_{\beta g}),$$

and

$$\begin{aligned} V_{\beta g} &= -\frac{(1-f)\sigma_\mu^2}{4}\frac{f(1-f\beta^*)}{2c^2}(g\mu_0\sigma_n\sigma_\mu)^2 \\ &\quad +[c-\frac{g(1-f)\sigma_\mu^2}{4}]\frac{f(1-f\beta^*)}{2c^2}2g(\mu_0\sigma_n\sigma_\mu)^2 \\ &= \frac{fg(1-f\beta^*)(\mu_0\sigma_n\sigma_\mu)^2}{2c^2}[c-\frac{3g(1-f)\sigma_n^2}{4}] < 0 \end{aligned}$$

if and only if $g > \frac{4c}{3(1-f)\sigma_n^2}$. Intuition: when growth option is sufficiently important, firms with higher growth option disclose less to induce feedback from the market.

Properties of the equilibrium and comparative statics

- How does $f\beta^*$ (total amount of disclosure vary with f? We have

$$\begin{aligned}
\frac{d(f\beta^*)}{df} &= \beta^* + f\frac{\partial\beta^*}{\partial f} \\
&= \frac{\beta^*(G_{\beta\beta} - \Pi_{\beta\beta} - W_{\beta\beta})}{V_{\beta\beta}} - f\frac{G_{\beta f} - \Pi_{\beta f}}{V_{\beta\beta}} \\
&= \frac{\beta^*(G_{\beta\beta} - \Pi_{\beta\beta}) - f(G_{\beta f} - \Pi_{\beta f})}{V_{\beta\beta}} - \frac{\beta^* W_{\beta\beta}}{V_{\beta\beta}}.
\end{aligned}$$

- The last term is positive as $W_{\beta\beta} > 0$ and $V_{\beta\beta} < 0$.

Properties of the equilibrium and comparative statics

- The first term

$$\begin{aligned}
& \beta^*(G_{\beta\beta} - \Pi_{\beta\beta}) - f(G_{\beta f} - \Pi_{\beta f}) \\
\propto \quad & \beta^*[c - \frac{g(1-f)\sigma_\mu^2}{4}]\frac{-f^2}{2c^2} \\
& -f\frac{g\sigma_\mu^2}{4}\frac{f(1-f\beta^*)}{2c^2} - f[c - \frac{g(1-f)\sigma_\mu^2}{4}]\frac{1-2f\beta^*}{2c^2} \\
= \quad & -f\frac{g\sigma_\mu^2}{4}\frac{f(1-f\beta^*)}{2c^2} - f[c - \frac{g(1-f)\sigma_\mu^2}{4}]\frac{f(1-f\beta^*)}{2c^2} < 0.
\end{aligned}$$

- Therefore the first term is also positive as $V_{\beta\beta} < 0$ so total disclosure quality increases with the probability of the firm getting informed -> there is less to learn from the market so levelling the playing field becomes more important.

Implications of GL

- For growth firms, more information acquisition from private speculators increases firm value.
- Growth firms also choose more opaque disclosure to facilitate private information acquisition.
- So various liquidity measures (e.g., bid-ask spread, PIN) should show more illiquidity for growth firms, in particular when the feedback effect is important.
- Mandatory disclosure rules will hurt growth firms more, in particular when the feedback effect is important (Jayaranan and Wu 2020).

More studies on how feedback effect shapes properties of disclosure

- GL focuses on disclosure precision -> good news and bad news are equally informative
- Accounting disclosures are known to exhibit more timely recognition of bad news versus good news
- Can this property naturally arise due to the feedback effect?
- Chen et al. (2021) shows that this is indeed the case, building on the framework of Edmans et al. (2015).
- Key tradeoff: Edmans et al. (2015) shows that traders are less likely to trade on bad news versus good news -> firms should then disclose more bad news and less good news to induce traders to trade. (you will work on a simplified version in the homework)

GY - overview

- Instead of assuming one-dimensional information, assuming information relevant for the manager is multidimensional but the decision maker knows one dimension better than the other one.
- Disclosure is about which dimension of information to disclose.
- Disclosing the dimension that the manager knows more is better (not necessarily better) -> induces more (less) informed trading on the dimension that the decision maker knows less -> positive (negative) real effects
- The manager can be interpreted to include decision makers in general (e.g., bank regulators or credit agencies)

GY - model setup

- Three dates:
  - Date 0, trading occurs in a Grossman-Stiglitz like setting with both private and public info (a continuum of investors so each investor act as price takes but does not learn from price)
  - Date 1, a decision maker makes decisions based on prices
  - Date 2, all uncertainties are realized

GY - the real decision side of the model

- The firm, when investing K, generates cash flow $Q(K) = \widetilde{A}\widetilde{F}K$.
- $\widetilde{A} \geq 0$ and $\widetilde{F} \geq 0$ are two production factors and where the two factors represent the two-dimensional uncertainty of relevant information.
- Assume that $\widetilde{A}$ and $\widetilde{F}$ are log-normally distributed, i.e., $\widetilde{a} \equiv \log \widetilde{A}$ and $\widetilde{f} \equiv \log \widetilde{F}$ are normally distributed, independent of each other, and $\widetilde{a} \sim N(0, \tau_a^{-1})$ and $\widetilde{f} \sim N(0, \tau_f^{-1})$.
- $\widetilde{a}$ and $\widetilde{f}$ can be interpreted as two different factors that affect cash flows: e.g., macroeconomic factor $(\widetilde{f})$ and firm-specific factor $(\widetilde{a})$.
- Assume that the manager knows perfectly about $\widetilde{a}$ but nothing about $\widetilde{f}$ (assuming knowing $\widetilde{a}$ more than $\widetilde{f}$ is sufficient).
- The manager chooses K to maximize $\beta Q(K) - \frac{1}{2}cK^2$ for some $\beta \in (0, 1)$: β can capture the equity incentives offered to the manager and $\frac{1}{2}cK^2$ can be interpreted as a private cost.

GY - Information structure

- Each speculator $i \in [0,1]$ observes two private noisy signals about $\widetilde{a}$ and $\widetilde{f}$:

$$\widetilde{x}_i = \widetilde{a} + \widetilde{\varepsilon}_{x,i};\ \widetilde{y}_i = \widetilde{f} + \widetilde{\varepsilon}_{y,i},$$

  where

$$\widetilde{\varepsilon}_{x,i} \sim N(0, \tau_x^{-1}) \text{ and } \widetilde{\varepsilon}_{y,i} \sim N(0, \tau_y^{-1}).$$

- Everybody also observe two public noisy signals about $\widetilde{a}$ and $\widetilde{f}$:

$$\widetilde{\omega} = \widetilde{a} + \widetilde{\varepsilon}_{\omega};\ \widetilde{\eta} = \widetilde{f} + \widetilde{\varepsilon}_{\eta},$$

  where $\widetilde{\varepsilon}_{\omega} \sim N(0, \tau_{\omega}^{-1})$ and $\widetilde{\varepsilon}_{\eta} \sim N(0, \tau_{\eta}^{-1})$. Focus will be on how τ_{ω} and τ_{η} affects investment efficiency and thus firm value through affecting the feedback effect.
- All noises are independent of each other and independent of $\widetilde{a}$ and $\widetilde{f}$.

GY- trading

- Each speculator i can submit a demand $d(i) \in [-1, 1]$ for a security that pays off $\widetilde{V} = (1-\beta)\widetilde{Q} = (1-\beta)\widetilde{A}\widetilde{F}K^*$ (for tractability reason, has to assume the manager bears the investment cost).
- The speculator therefore chooses $d(i)$ to maximize $d(i)E[\widetilde{V} - \widetilde{P}|I_i]$, where $I_i = \{\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}\}$.
- Given that each speculator is risk-neutral, $d(i) = \pm 1$ almost surely. Aggregate demand $D \equiv \int_0^1 d(i)di$.
- For tractability, assume the noisy supply is given by

$$L(\widetilde{\xi}, \widetilde{P}) = 1 - 2\Phi(\widetilde{\xi} - \lambda \log \widetilde{P}),$$

where $\widetilde{\xi} \sim N(0, \tau_{\xi}^{-1})$ is the noise independent of all other random variables, and $\widetilde{P}$ is the price. L is increasing in $\widetilde{P}$ and $\lambda > 0$ is the supply elasticity.
- Market clears (i.e., $\widetilde{P}$ is determined in equilibrium) by setting $D = L(\widetilde{\xi}, \widetilde{P})$.

Solving for the equilibrium

- Manager's decision making is straightforward from the first-order condition:

$$K^* = \frac{\beta}{c} E[\widetilde{A}\widetilde{F} | \widetilde{a}, \widetilde{P}, \widetilde{\omega}, \widetilde{\eta}] = \frac{\beta}{c} \widetilde{A} E[\widetilde{F} | \widetilde{a}, \widetilde{P}, \widetilde{\omega}, \widetilde{\eta}].$$

- For trading, follow the similar logic to that of Grossman and Stiglitz, focusing on linear monotone equilibrium where

$$d(\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}) = 1 \text{ if and only if } \widetilde{x}_i + \phi_y \widetilde{y}_i + \phi_\omega \widetilde{\omega} + \phi_\eta \widetilde{\eta} > g$$

for some endogenous constants $\phi_y, \phi_\omega, \phi_\eta$ and g.

Solving for the equilibrium - information revealed by the price

- $\widetilde{x}_i + \phi_y \widetilde{y}_i + \phi_\omega \widetilde{\omega} + \phi_\eta \widetilde{\eta} > g$ is equivalent to

$$\frac{\widetilde{\varepsilon}_{x,i} + \phi_y \widetilde{\varepsilon}_{y,i}}{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}} > \frac{g - (\widetilde{a} + \phi_y \widetilde{f}) - \phi_\omega \widetilde{\omega} - \phi_\eta \widetilde{\eta}}{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}.$$

- Note that $\frac{\widetilde{\varepsilon}_{x,i} + \phi_y \widetilde{\varepsilon}_{y,i}}{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}$ follows a standard normal distribution. By the law of large numbers, there will be $1 - \Phi(\frac{g - (\widetilde{a} + \phi_y \widetilde{f}) - \phi_\omega \widetilde{\omega} - \phi_\eta \widetilde{\eta}}{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}})$ of speculators buying and $\Phi(\frac{g - (\widetilde{a} + \phi_y \widetilde{f}) - \phi_\omega \widetilde{\omega} - \phi_\eta \widetilde{\eta}}{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}})$ of speculators selling so aggregate demand

$$D(\widetilde{a}, \widetilde{f}, \widetilde{\omega}, \widetilde{\eta}) = 1 - 2\Phi(\frac{g - (\widetilde{a} + \phi_y \widetilde{f}) - \phi_\omega \widetilde{\omega} - \phi_\eta \widetilde{\eta}}{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}).$$

Solving for the equilibrium - information revealed by the price

- Market clearing determines the price, i.e., (now you see why such a supply function is assumed)

$$1-2\Phi\left(\frac{g-(\widetilde{a}+\phi_y\widetilde{f})-\phi_\omega\widetilde{\omega}-\phi_\eta\widetilde{\eta}}{\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\right)=1-2\Phi(\widetilde{\xi}-\lambda\log\widetilde{P}),$$

resulting in equilibrium price

$$\widetilde{P}=\exp\left(\frac{\widetilde{a}+\phi_y\widetilde{f}+\phi_\omega\widetilde{\omega}+\phi_\eta\widetilde{\eta}-g}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}+\frac{\widetilde{\xi}}{\lambda}\right).$$

Solving for the equilibrium - information revealed by the price

- Since the manager knows $\widetilde{a}, \widetilde{\omega}$ and $\widetilde{\eta}$, $\widetilde{P}$ is another (noisy) informative signal about $\widetilde{f}$ (the Grossman-Stiglitz logic) so $\widetilde{P}$ is informationally equivalent to

$$\widetilde{s}_P \equiv \frac{\lambda\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}} \log \widetilde{P} - \widetilde{a} - \phi_\omega \widetilde{\omega} - \phi_\eta \widetilde{\eta} + g}{\phi_y} = f + \widetilde{\varepsilon}_P,$$

where $\widetilde{\varepsilon}_P = \frac{\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}{\phi_y} \widetilde{\zeta}$. So price informativeness is the precision of $\widetilde{\varepsilon}_P$, denoted as $\tau_P \equiv \frac{1}{var(\widetilde{\varepsilon}_P)} = \frac{\phi_y^2 \tau_x \tau_y \tau_\zeta}{\tau_y + \phi_y^2 \tau_x}$. Note that τ_P increases in ϕ_y: the more aggressively speculators trade on their private signal about $\widetilde{f}$, the more informative price is about f.

Solving for the equilibrium - learning from prices in investing

- Recall that $K^* = \frac{\beta}{c}\widetilde{A}E[\widetilde{F}|\widetilde{a}, \widetilde{P}, \widetilde{\omega}, \widetilde{\eta}]$. The signals $\widetilde{P}$ and $\widetilde{\eta}$ are informative about $\widetilde{f}$ and thus $\widetilde{F}$.
- Note that conditional on $\widetilde{P}$ and $\widetilde{\eta}$, standard Bayesian updating results in $\widetilde{f}|\widetilde{P}, \widetilde{\eta}$ being normally distributed with mean $\frac{\tau_p \widetilde{s}_P + \tau_\eta \widetilde{\eta}}{\tau_p + \tau_\eta + \tau_f}$ and variance $\frac{1}{\tau_p + \tau_\eta + \tau_f}$.
- Therefore $E[\widetilde{F}|\widetilde{a}, \widetilde{P}, \widetilde{\omega}, \widetilde{\eta}] = E[e^{\widetilde{f}}|\widetilde{P}, \widetilde{\eta}] = e^{\frac{\tau_p \widetilde{s}_P + \tau_\eta \widetilde{\eta}}{\tau_p + \tau_\eta + \tau_f} + \frac{1}{2}\frac{1}{\tau_p + \tau_\eta + \tau_f}}$.
- So

$$K^* = \frac{\beta}{c} e^{\widetilde{a}} e^{\frac{\tau_p \widetilde{s}_P + \tau_\eta \widetilde{\eta}}{\tau_p + \tau_\eta + \tau_f} + \frac{1}{2}\frac{1}{\tau_p + \tau_\eta + \tau_f}}.$$

Solving for the equilibrium - optimal trading strategy

- Note that speculator i will choose $d_i = 1$ if and only if $E[\widetilde{V}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}] > E[\widetilde{P}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}]$. so need expression of $E[\widetilde{V}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}]$ and $E[\widetilde{P}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}]$.
- Recall that $\widetilde{P} = \exp(\frac{\widetilde{a}+\phi_y\widetilde{f}+\phi_\omega\widetilde{\omega}+\phi_\eta\widetilde{\eta}-g}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}} + \frac{\widetilde{\xi}}{\lambda})$.

$$
\begin{aligned}
E[\widetilde{a}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}] &= E[\widetilde{a}|\widetilde{x}_i, \widetilde{\omega}] = \frac{\tau_x\widetilde{x}_i + \tau_\omega\widetilde{\omega}}{\tau_x + \tau_\omega + \tau_a}, \\
var[\widetilde{a}|\widetilde{x}_i, \widetilde{\omega}] &= \frac{1}{\tau_x + \tau_\omega + \tau_a},
\end{aligned}
$$

$$
\begin{aligned}
E[\widetilde{f}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}] &= E[\widetilde{f}|\widetilde{y}_i, \widetilde{\eta}] = \frac{\tau_y\widetilde{y}_i + \tau_\eta\widetilde{\eta}}{\tau_y + \tau_\eta + \tau_f}, \\
var[\widetilde{a}|\widetilde{y}_i, \widetilde{\eta}] &= \frac{1}{\tau_y + \tau_\eta + \tau_f}, \\
E[\widetilde{\xi}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}] &= E[\widetilde{\xi}] = 0,\ var[\widetilde{\xi}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}] = var[\widetilde{\xi}] = \frac{1}{\tau_\xi}.
\end{aligned}
$$

$$
\begin{aligned}
& E[\widetilde{P}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}] \\
= \; & E[\exp(\frac{\widetilde{a} + \phi_y \widetilde{f} + \phi_\omega \widetilde{\omega} + \phi_\eta \widetilde{\eta} - g}{\lambda\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}} + \frac{\widetilde{\xi}}{\lambda})|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}] \\
= \; & \exp(\frac{\frac{\tau_x \widetilde{x}_i + \tau_\omega \widetilde{\omega}}{\tau_x + \tau_\omega + \tau_a}}{\lambda\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}} + \frac{1}{2}\frac{\frac{1}{\tau_x + \tau_\omega + \tau_a}}{\lambda^2(\tau_x^{-1} + \phi_y^2 \tau_y^{-1})} \\
& + \frac{\phi_y}{\lambda\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}}\frac{\tau_y \widetilde{y}_i + \tau_\eta \widetilde{\eta}}{\tau_y + \tau_\eta + \tau_f} + \frac{1}{2}\frac{\phi_y^2 \frac{1}{\tau_y + \tau_\eta + \tau_f}}{\lambda^2(\tau_x^{-1} + \phi_y^2 \tau_y^{-1})} \\
& + \frac{\phi_\omega \widetilde{\omega}}{\lambda\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}} + \frac{\phi_\eta \widetilde{\eta}}{\lambda\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}} - \frac{g}{\lambda\sqrt{\tau_x^{-1} + \phi_y^2 \tau_y^{-1}}} \\
& + \frac{1}{2}\frac{1}{\lambda^2 \tau_\xi}).
\end{aligned}
$$

Solving for the equilibrium - optimal trading strategy

- Can reduce $E[\widetilde{P}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}]$ to $\exp(b_0^P + b_x^P \widetilde{x}_i + b_y^P \widetilde{y}_i + b_\omega^P \widetilde{\omega} + b_\eta^P \widetilde{\eta})$, where

Solving for the equilibrium - optimal trading strategy

$$
\begin{aligned}
b_0^P &= -\frac{g}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}} + \frac{1}{2\lambda^2\tau_\xi} \\
&\quad + \frac{1}{2\lambda^2(\tau_x^{-1}+\phi_y^2\tau_y^{-1})} \times \left(\frac{1}{\tau_x+\tau_\omega+\tau_a} + \frac{\phi_y^2}{\tau_y+\tau_\eta+\tau_f}\right), \\
b_x^P &= \frac{1}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\frac{\tau_x}{\tau_x+\tau_\omega+\tau_a}, \\
b_y^P &= \frac{\phi_y}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\frac{\tau_y}{\tau_y+\tau_\eta+\tau_f}, \\
b_\omega^P &= \frac{1}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\left(\phi_\omega + \frac{\tau_\omega}{\tau_x+\tau_\omega+\tau_a}\right), \\
b_\eta^P &= \frac{1}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\left(\phi_\eta + \frac{\tau_\eta}{\tau_y+\tau_\eta+\tau_f}\right).
\end{aligned}
$$

- Recall that

$$
\begin{aligned}
& E[\widetilde{V}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}] \\
= \ & \frac{\beta(1-\beta)}{c} E[\widetilde{A}\widetilde{F}K^*|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}] \\
= \ & \frac{\beta(1-\beta)}{c} E[e^{\widetilde{a}} e^{\frac{\tau_p \widetilde{s}_P + \tau_\eta \widetilde{\eta}}{\tau_p+\tau_\eta+\tau_f} + \frac{1}{2}\frac{1}{\tau_p+\tau_\eta+\tau_f}} |\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}] \\
= \ & \frac{\beta(1-\beta)}{c} e^{\frac{1}{2}\frac{1}{\tau_p+\tau_\eta+\tau_f}} E[e^{2\widetilde{a}} e^{\widetilde{f}} e^{\frac{\tau_p \widetilde{s}_P + \tau_\eta \widetilde{\eta}}{\tau_p+\tau_\eta+\tau_f}} |\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}] \\
= \ & \frac{\beta(1-\beta)}{c} e^{\frac{1}{2}\frac{1}{\tau_p+\tau_\eta+\tau_f}} E[e^{2\widetilde{a}} e^{\widetilde{f}} e^{\frac{\tau_p (f+\widetilde{\varepsilon}_P) + \tau_\eta \widetilde{\eta}}{\tau_p+\tau_\eta+\tau_f}} |\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}] \\
= \ & \frac{\beta(1-\beta)}{c} e^{\frac{1}{2}\frac{1}{\tau_p+\tau_\eta+\tau_f}} E[e^{2\widetilde{a}} e^{\widetilde{f}} e^{\frac{(2\tau_p+\tau_\eta+\tau_f) f + \tau_P \widetilde{\varepsilon}_P + \tau_\eta \widetilde{\eta}}{\tau_p+\tau_\eta+\tau_f}} |\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}].
\end{aligned}
$$

Solving for the equilibrium - optimal trading strategy

- Note that

$$
\begin{aligned}
E[\widetilde{a}|\widetilde{x}_i,\widetilde{y}_i,\widetilde{\omega},\widetilde{\eta}] &= E[\widetilde{a}|\widetilde{x}_i,\widetilde{\omega}] = \frac{\tau_x\widetilde{x}_i+\tau_\omega\widetilde{\omega}}{\tau_x+\tau_\omega+\tau_a}, \\
var[\widetilde{a}|\widetilde{x}_i,\widetilde{\omega}] &= \frac{1}{\tau_x+\tau_\omega+\tau_a},
\end{aligned}
$$

$$
\begin{aligned}
E[\widetilde{f}|\widetilde{x}_i,\widetilde{y}_i,\widetilde{\omega},\widetilde{\eta}] &= E[\widetilde{f}|\widetilde{y}_i,\widetilde{\eta}] = \frac{\tau_y\widetilde{y}_i+\tau_\eta\widetilde{\eta}}{\tau_y+\tau_\eta+\tau_f}, \\
var[\widetilde{a}|\widetilde{y}_i,\widetilde{\eta}] &= \frac{1}{\tau_y+\tau_\eta+\tau_f}, \\
E[\widetilde{\varepsilon}_P|\widetilde{x}_i,\widetilde{y}_i,\widetilde{\omega},\widetilde{\eta}] &= \frac{\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}{\phi_y}E[\widetilde{\xi}] = 0, \\
var[\widetilde{\varepsilon}_P|\widetilde{x}_i,\widetilde{y}_i,\widetilde{\omega},\widetilde{\eta}] &= \frac{1}{\tau_p}.
\end{aligned}
$$

Solving for the equilibrium - optimal trading strategy

- Therefore

$$
\begin{aligned}
& E[\widetilde{V}|\widetilde{x}_i,\widetilde{y}_i,\widetilde{\omega},\widetilde{\eta}] \\
= \quad & \frac{\beta(1-\beta)}{c} e^{\frac{1}{2}\frac{1}{\tau_p+\tau_\eta+\tau_f}} E[e^{2\widetilde{a}} e^{\frac{(2\tau_p+\tau_\eta+\tau_f)f+\tau_p\widetilde{\varepsilon}_P+\tau_\eta\widetilde{\eta}}{\tau_p+\tau_\eta+\tau_f}} |\widetilde{x}_i,\widetilde{y}_i,\widetilde{\omega},\widetilde{\eta}] \\
= \quad & \frac{\beta(1-\beta)}{c} \exp(\frac{1}{2}\frac{1}{\tau_p+\tau_\eta+\tau_f} + 2\frac{\tau_x\widetilde{x}_i+\tau_\omega\widetilde{\omega}}{\tau_x+\tau_\omega+\tau_a} \\
& + \frac{2}{\tau_x+\tau_\omega+\tau_a} \\
& + \frac{2\tau_p+\tau_\eta+\tau_f}{\tau_p+\tau_\eta+\tau_f}\frac{\tau_y\widetilde{y}_i+\tau_\eta\widetilde{\eta}}{\tau_y+\tau_\eta+\tau_f} \\
& + \frac{1}{2}(\frac{2\tau_p+\tau_\eta+\tau_f}{\tau_p+\tau_\eta+\tau_f})^2\frac{1}{\tau_y+\tau_\eta+\tau_f} \\
& + \frac{1}{2}(\frac{\tau_p}{\tau_p+\tau_\eta+\tau_f})^2\frac{1}{\tau_p} + \frac{\tau_\eta\widetilde{\eta}}{\tau_p+\tau_\eta+\tau_f}).
\end{aligned}
$$

Solving for the equilibrium - optimal trading strategy

- Can similarly reduce $E[\widetilde{V}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}]$ to $\exp(b_0^V + b_x^V \widetilde{x}_i + b_y^V \widetilde{y}_i + b_\omega^V \widetilde{\omega} + b_\eta^V \widetilde{\eta})$, where

$$
\begin{aligned}
b_0^V &= \log \frac{\beta(1-\beta)}{c} + \frac{2\tau_p + \tau_\eta + \tau_f}{2(\tau_p + \tau_\eta + \tau_f)^2} + \frac{2}{\tau_x + \tau_\omega + \tau_a} \\
&\quad + \frac{1}{2}\left(\frac{2\tau_p + \tau_\eta + \tau_f}{\tau_p + \tau_\eta + \tau_f}\right)^2 \frac{1}{\tau_y + \tau_\eta + \tau_f}, \\
b_x^V &= \frac{2\tau_x}{\tau_x + \tau_\omega + \tau_a}, \; b_y^V = \frac{2\tau_p + \tau_\eta + \tau_f}{\tau_p + \tau_\eta + \tau_f} \frac{\tau_y}{\tau_y + \tau_\eta + \tau_f}, \\
b_\omega^V &= \frac{2\tau_\omega}{\tau_x + \tau_\omega + \tau_a}, \\
b_\eta^V &= \frac{2\tau_p + \tau_\eta + \tau_f}{\tau_p + \tau_\eta + \tau_f} \frac{\tau_\eta}{\tau_y + \tau_\eta + \tau_f} + \frac{\tau_\eta}{\tau_p + \tau_\eta + \tau_f}.
\end{aligned}
$$

Solving for the equilibrium - optimal trading strategy

- Therefore $E[\widetilde{V}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}] > E[\widetilde{P}|\widetilde{x}_i, \widetilde{y}_i, \widetilde{\omega}, \widetilde{\eta}]$ is equivalent to

  $$b_0^V + b_x^V \widetilde{x}_i + b_y^V \widetilde{y}_i + b_\omega^V \widetilde{\omega} + b_\eta^V \widetilde{\eta} > b_0^P + b_x^P \widetilde{x}_i + b_y^P \widetilde{y}_i + b_\omega^P \widetilde{\omega} + b_\eta^P \widetilde{\eta},$$

  which is equivalent to

  $$(b_x^V - b_x^P)\widetilde{x}_i + (b_y^V - b_y^P)\widetilde{y}_i + (b_\omega^V - b_\omega^P)\widetilde{\omega} + (b_\eta^V - b_\eta^P)\widetilde{\eta} > b_0^P - b_0^V.$$

- Comparing this with the conjectured speculator's trading strategy, buy if and only if $\widetilde{x}_i + \phi_y \widetilde{y}_i + \phi_\omega \widetilde{\omega} + \phi_\eta \widetilde{\eta} > g$, matching coefficients results in

  $$\begin{aligned} \phi_y &= \frac{b_y^V - b_y^P}{b_x^V - b_x^P}, \; \phi_\omega = \frac{b_\omega^V - b_\omega^P}{b_x^V - b_x^P}, \\ \phi_\eta &= \frac{b_\eta^V - b_\eta^P}{b_x^V - b_x^P}, \; g = \frac{b_0^P - b_0^V}{b_x^V - b_x^P}. \end{aligned}$$

- The key coefficient of interest is ϕ_y so we focus on ϕ_y.

Solving for the equilibrium - optimal trading strategy

- Recall that

$$\phi_y = \frac{b_y^V - b_y^P}{b_x^V - b_x^P} = \frac{\frac{\tau_y}{\tau_y+\tau_\eta+\tau_f}\left(\frac{2\tau_p+\tau_\eta+\tau_f}{\tau_p+\tau_\eta+\tau_f} - \frac{\phi_y}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\right)}{\frac{\tau_x}{\tau_x+\tau_\omega+\tau_a}\left(2 - \frac{1}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\right)}.$$

This equation is only a function of ϕ_y as τ_p is also a function of ϕ_y only. This results in an equation that ϕ_y must satisfy, i.e., ϕ_y satisfies

$$\phi_y = \frac{\frac{\tau_y}{\tau_y+\tau_\eta+\tau_f}\left(1 + \frac{\frac{\phi_y^2\tau_x\tau_y\tau_\xi}{\tau_y+\phi_y^2\tau_x}}{\tau_\eta+\tau_f+\frac{\phi_y^2\tau_x\tau_y\tau_\xi}{\tau_y+\phi_y^2\tau_x}} - \frac{\phi_y}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\right)}{\frac{\tau_x}{\tau_x+\tau_\omega+\tau_a}\left(2 - \frac{1}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\right)}.$$

Solving for the equilibrium - optimal trading strategy

- Also note that we need $b_x^V - b_x^P > 0$, which is equivalent to $\frac{2\tau_x}{\tau_x+\tau_\omega+\tau_a} > \frac{1}{\lambda\sqrt{\tau_x^{-1}+\phi_y^2\tau_y^{-1}}}\frac{\tau_x}{\tau_x+\tau_\omega+\tau_a}$, and can be reduced to $\phi_y^2 > \tau_y(\frac{1}{4\lambda^2} - \frac{1}{\tau_x})$, which will be automatically satisfied when $\lambda > \frac{\sqrt{\tau_x}}{2}$.
- When $\lambda > \frac{\sqrt{\tau_x}}{2}$, the equation is satisfied for some $\phi_y > 0$. To see this, note that when $\phi_y \to 0$, the left hand side $\to 0$ but the right hand side $\to \frac{\frac{\tau_y}{\tau_y+\tau_\eta+\tau_f}}{\frac{\tau_x}{\tau_x+\tau_\omega+\tau_a}(2-\frac{1}{\lambda\sqrt{\tau_x^{-1}}})} > 0$ as $\lambda > \frac{\sqrt{\tau_x}}{2}$ so $\frac{1}{\lambda\sqrt{\tau_x^{-1}}} < 2$. When $\phi_y \to +\infty$, the left hand side $\to +\infty$ but the right hand side $\to \frac{\frac{\tau_y}{\tau_y+\tau_\eta+\tau_f}(1+\frac{\tau_y\tau_\xi}{\tau_\eta+\tau_f+\tau_y\tau_\xi})-\frac{1}{\lambda\sqrt{\tau_y^{-1}}}}{2\frac{\tau_x}{\tau_x+\tau_\omega+\tau_a}}$ which is finite. Therefore by intermediate value theorem a solution always exists.

Solving for the equilibrium - optimal trading strategy

- This completes the majority of Proposition 1. Proposition 1 went on to show that the solution is unique when λ is sufficiently large (by illustrating monotonicity of the derivative of the left hand side minus right hand side with respect to ϕ_y)

The effect of disclosure - real efficiency calculation

- Focus on the effect of disclosure on real efficiency, defined as $RE \equiv E[\widetilde{A}\widetilde{F}K^* - \frac{c}{2}(K^*)^2]$.
- By law of iterated expectations,

$$\begin{aligned} RE &\equiv E[\widetilde{A}\widetilde{F}K^* - \frac{c}{2}(K^*)^2] \\ &= E[E[\widetilde{A}\widetilde{F}K^* - \frac{c}{2}(K^*)^2|\widetilde{a}, \widetilde{\eta}, \widetilde{s}_P]] \\ &= E[\widetilde{A}K^* E[\widetilde{F}|\widetilde{\eta}, \widetilde{s}_P] - \frac{c}{2}(K^*)^2]. \end{aligned}$$

The effect of disclosure - real efficiency calculation

- Recall that $K^* = \frac{\beta}{c}\widetilde{A}E[\widetilde{F}|\widetilde{a}, \widetilde{P}, \widetilde{\omega}, \widetilde{\eta}]$ so we have

$$\begin{aligned} RE &= \frac{\beta}{c}(1-\frac{\beta}{2})E[(\widetilde{A}E[\widetilde{F}|\widetilde{\eta}, \widetilde{s}_P])^2] \\ &= \frac{\beta}{c}(1-\frac{\beta}{2})E[\widetilde{A}^2]E[(E[\widetilde{F}|\widetilde{\eta}, \widetilde{s}_P])^2] \end{aligned}$$

- Note that $E[\widetilde{A}^2] = E[e^{2\widetilde{a}}] = e^{2E[\widetilde{a}]+\frac{1}{2}4var(\widetilde{a})} = e^{\frac{2}{\tau_a}}$.
- $E[\widetilde{F}|\widetilde{\eta}, \widetilde{s}_P] = E[e^{\widetilde{f}}|\widetilde{\eta}, \widetilde{s}_P] = e^{E[\widetilde{f}|\widetilde{\eta},\widetilde{s}_P]+\frac{1}{2}var[\widetilde{f}|\widetilde{\eta},\widetilde{s}_P]}$. Therefore $(E[\widetilde{F}|\widetilde{\eta}, \widetilde{s}_P])^2 = e^{2E[\widetilde{f}|\widetilde{\eta},\widetilde{s}_P]+var[\widetilde{f}|\widetilde{\eta},\widetilde{s}_P]}$. Therefore

$$\begin{aligned} & E[(E[\widetilde{F}|\widetilde{\eta}, \widetilde{s}_P])^2] \\ = & E[e^{2E[\widetilde{f}|\widetilde{\eta},\widetilde{s}_P]+var[\widetilde{f}|\widetilde{\eta},\widetilde{s}_P]}] \\ = & e^{2E[E[\widetilde{f}|\widetilde{\eta},\widetilde{s}_P]]+4\asymp\frac{1}{2}var[E[\widetilde{f}|\widetilde{\eta},\widetilde{s}_P]+var[\widetilde{f}|\widetilde{\eta},\widetilde{s}_P]} \\ = & e^{2E[\widetilde{f}]+2var[E[\widetilde{f}|\widetilde{\eta},\widetilde{s}_P]+var[\widetilde{f}|\widetilde{\eta},\widetilde{s}_P]} \\ = & e^{2var[E[\widetilde{f}|\widetilde{\eta},\widetilde{s}_P]+var[\widetilde{f}|\widetilde{\eta},\widetilde{s}_P]} \end{aligned}$$

The effect of disclosure - real efficiency calculation

- Using the variance decomposition equation,

$$var(E[\tilde{f}|\tilde{\eta},\tilde{s}_P]) = var(\tilde{f}) - var(\tilde{f}|\tilde{\eta},\tilde{s}_P) = \frac{1}{\tau_f} - var(\tilde{f}|\tilde{\eta},\tilde{s}_P),$$

resulting in

$$\begin{aligned} RE &= \frac{\beta}{c}(1-\frac{\beta}{2})\exp[\frac{2}{\tau_a}+\frac{2}{\tau_f} - var(\tilde{f}|\tilde{\eta},\tilde{s}_P)] \\ &= \frac{\beta}{c}(1-\frac{\beta}{2})\exp[\frac{2}{\tau_a}+\frac{2}{\tau_f} - \frac{1}{\tau_\eta+\tau_p+\tau_f}]. \end{aligned}$$

Note that *RE* increases in τ_p -> the more informative the price, the higher the real efficiency due to feedback effect (the manager learns more about $\tilde{f}$ from prices). Also *RE* increases in τ_η -> more precise public disclosure about $\tilde{f}$ also results in the manager learning more about $\tilde{f}$ and thus improve real efficiency.

The effect of disclosure on real efficiency

- An increase in τ_η has two effects: $\frac{dRE}{d\tau_\eta} = \frac{1}{(\tau_\eta+\tau_P+\tau_f)^2}(1 + \frac{\partial \tau_P}{\partial \tau_\eta})$.
  - Direct effect: RE increases in τ_η
  - Indirect effect: RE increases in τ_P, which decreases in τ_η when λ is large
- To see how indirect effect is negative, note that $\tau_P = \frac{\phi_y^2 \tau_x \tau_y \tau_\zeta}{\tau_y + \phi_y^2 \tau_x}$ so $\frac{\partial \tau_P}{\partial \tau_\eta} = \frac{2\tau_P \tau_y}{\phi_y(\tau_y + \phi_y^2 \tau_x)} \frac{\partial \phi_y}{\partial \tau_\eta}$.

The effect of disclosure on real efficiency

- When $\lambda \longrightarrow +\infty$, the equation of ϕ_y reduces to

$$\phi_y \to \frac{b_y^V}{b_x^V} = \frac{\frac{\tau_y}{\tau_y+\tau_\eta+\tau_f}\left(1+\frac{\frac{\phi_y^2\tau_x\tau_y\tau_\xi}{\tau_y+\phi_y^2\tau_x}}{\tau_\eta+\tau_f+\frac{\phi_y^2\tau_x\tau_y\tau_\xi}{\tau_y+\phi_y^2\tau_x}}\right)}{\frac{2\tau_x}{\tau_x+\tau_\omega+\tau_a}}, \text{ or}$$

$$\phi_y\frac{2\tau_x}{\tau_x+\tau_\omega+\tau_a} = \frac{\tau_y}{\tau_y+\tau_\eta+\tau_f}\left(1+\frac{\tau_P}{\tau_\eta+\tau_f+\tau_P}\right).$$

- Intuitively, when $\lambda \longrightarrow +\infty$, market is very liquid so price does not change much -> traders use their information to update cash flows other than prices -> so the relative weight ϕ_y is determined by the extent they use $\widetilde{y}_i$ to forecast cash flow relative to the extent they use $\widetilde{x}_i$, i.e., $\frac{b_y^V}{b_x^V}$.

- Take derivative of both sides of the equation with respect to τ_η results in

$$
\begin{aligned}
& \frac{\partial \phi_y}{\partial \tau_\eta} \frac{2\tau_x}{\tau_x + \tau_\omega + \tau_a} \\
= \quad & -\frac{\tau_y}{(\tau_y + \tau_\eta + \tau_f)^2} (1 + \frac{\tau_P}{\tau_\eta + \tau_f + \tau_P}) \\
& -\frac{\tau_y}{\tau_y + \tau_\eta + \tau_f} \frac{\tau_P}{(\tau_\eta + \tau_f + \tau_P)^2} \\
& +\frac{\tau_y}{\tau_y + \tau_\eta + \tau_f} \frac{\tau_\eta + \tau_f}{(\tau_\eta + \tau_f + \tau_P)^2} \frac{\partial \tau_P}{\partial \tau_\eta}.
\end{aligned}
$$

The effect of disclosure on real efficiency

- Note that

$$\frac{\partial \tau_P}{\partial \tau_\eta} = \frac{2\tau_P \tau_y}{\phi_y(\tau_y + \phi_y^2 \tau_x)} \frac{\partial \phi_y}{\partial \tau_\eta},$$

and

$$\frac{2\tau_x}{\tau_x + \tau_\omega + \tau_a} = \frac{1}{\phi_y} \frac{\tau_y}{\tau_y + \tau_\eta + \tau_f} (1 + \frac{\tau_P}{\tau_\eta + \tau_f + \tau_P}).$$

The effect of disclosure on real efficiency

- Insert into the expression of the previous slide results in

$$
\begin{aligned}
& \frac{\partial \phi_y}{\partial \tau_\eta} \frac{1}{\phi_y} \frac{\tau_y}{\tau_y + \tau_\eta + \tau_f} [\frac{\tau_\eta + \tau_f}{\tau_\eta + \tau_f + \tau_P} \\
& + \frac{2\tau_P}{\tau_\eta + \tau_f + \tau_P} (1 - \frac{\tau_\eta + \tau_f}{\tau_\eta + \tau_f + \tau_P} \frac{\tau_y}{\tau_y + \phi_y^2 \tau_x})] \\
= \quad & -\frac{\tau_y}{(\tau_y + \tau_\eta + \tau_f)^2} (1 + \frac{\tau_P}{\tau_\eta + \tau_f + \tau_P}) - \\
& \frac{\tau_y}{\tau_y + \tau_\eta + \tau_f} \frac{\tau_P}{(\tau_\eta + \tau_f + \tau_P)^2}.
\end{aligned}
$$

This results in $\frac{\partial \phi_y}{\partial \tau_\eta} < 0$.

The effect of disclosure on real efficiency

- We can further refine the expression of $\frac{\partial \phi_y}{\partial \tau_\eta}$ as

$$
\begin{aligned}
\frac{\partial \phi_y}{\partial \tau_\eta} &= \frac{-\phi_y\left[\frac{1}{\tau_y+\tau_\eta+\tau_f}\left(1+\frac{\tau_P}{\tau_\eta+\tau_f+\tau_P}\right)+\frac{\tau_P}{(\tau_\eta+\tau_f+\tau_P)^2}\right]}{\frac{\tau_\eta+\tau_f}{\tau_\eta+\tau_f+\tau_P}+\frac{2\tau_P}{\tau_\eta+\tau_f+\tau_P}\left(1-\frac{\tau_\eta+\tau_f}{\tau_\eta+\tau_f+\tau_P}\frac{\tau_y}{\tau_y+\phi_y^2\tau_x}\right)} \\
&= \frac{-\phi_y\left[\frac{\tau_\eta+\tau_f+2\tau_P}{\tau_y+\tau_\eta+\tau_f}+\frac{\tau_P}{(\tau_\eta+\tau_f+\tau_P)}\right]}{\tau_\eta+\tau_f+2\tau_P-(\tau_\eta+\tau_f)\frac{\tau_y}{\tau_y+\phi_y^2\tau_x}} \\
&= \frac{-\phi_y\left[\frac{1}{\tau_y+\tau_\eta+\tau_f}+\frac{\tau_P}{(\tau_\eta+\tau_f+2\tau_P)(\tau_\eta+\tau_f+\tau_P)}\right]}{1-\frac{\tau_\eta+\tau_f}{\tau_\eta+\tau_f+2\tau_P}\frac{\tau_y}{\tau_y+\phi_y^2\tau_x}}.
\end{aligned}
$$

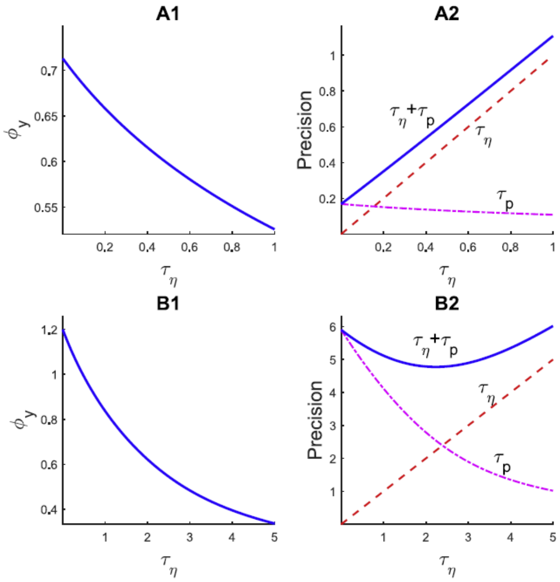
The effect of disclosure on real efficiency

- So
$$\frac{\partial \phi_y}{\partial \tau_\eta} = \frac{-\phi_y\left[\frac{1}{\tau_y+\tau_\eta+\tau_f} + \frac{\tau_P}{(\tau_\eta+\tau_f+2\tau_P)(\tau_\eta+\tau_f+\tau_P)}\right]}{1 - \frac{\tau_\eta+\tau_f}{\tau_\eta+\tau_f+2\tau_P}\frac{\tau_y}{\tau_y+\phi_y^2\tau_x}}.$$
- Intuition: first, increasing τ_η resulting in speculators putting higher weight on $\widetilde{\eta}$ and lower weight on $\widetilde{y}_i$, reducing b_y^V and thus ϕ_y (recall that $\phi_y \rightarrow \frac{b_y^V}{b_x^V}$), which is reflected in the bracket terms inside the numerator; second, increasing τ_η resulting in speculators trading more aggressively on $\widetilde{x}_i$ as their profit from trading on $\widetilde{y}_i$ is reduced) and thus increasing b_x^V, which is reflected in the denominator.

The effect of disclosure on real efficiency

- So when would the direct effect dominate and when would the indirect effect dominate?
- When $\tau_\eta \to +\infty$, then $\frac{\partial \phi_y}{\partial \tau_\eta} \to 0$. so the direct effect dominate (part c of Proposition 2)
- When $\tau_\eta \to 0$, then $\frac{\partial \phi_y}{\partial \tau_\eta} \to \frac{-\phi_y[\frac{1}{\tau_y+\tau_f}(1+\frac{\tau_P}{\tau_f+\tau_P})+\frac{\tau_P}{(\tau_f+\tau_P)^2}]}{\frac{\tau_f}{\tau_f+\tau_P}+\frac{2\tau_P}{\tau_f+\tau_P}\frac{\phi_y^2\tau_x}{\tau_y+\phi_y^2\tau_x}}$. When $\tau_\xi \to 0$, $\tau_P \to 0$ so $\frac{\partial \phi_y}{\partial \tau_\eta} \to -\frac{\phi_y}{\tau_y+\tau_f}$ and $\frac{\partial \tau_P}{\partial \tau_\eta} \to 0$ so the direct effect dominate; when $\tau_\xi \to +\infty$, $\tau_P \to +\infty$ so $\frac{\partial \phi_y}{\partial \tau_\eta} \to \frac{-\phi_y \frac{1}{\tau_y+\tau_\eta+\tau_f}}{2\frac{\phi_y^2\tau_x}{\tau_y+\phi_y^2\tau_x}}$ and $\frac{\partial \tau_P}{\partial \tau_\eta} \to +\infty$ so the indirect effect dominate (part d of Proposition 2)
- Intuition: when τ_η is very precise decision maker does not need to learn much from prices so direct effect dominates; when τ_η is small and τ_ξ is large, decision maker has the potential to learn much from prices so indirect effect dominates.

The effect of disclosure on real efficiency



The effect of disclosure on real efficiency

- Now consider the effect of τ_ω on real efficiency. Note that since the firm knows $\widetilde{a}$, the effect of τ_ω on real efficiency is completely through indirect effect, i.e., how it affects τ_P ('s informativeness of $\widetilde{f}$) through trading.
- Note that $\frac{\partial \tau_P}{\partial \tau_\omega} = \frac{2\tau_P \tau_y}{\phi_y(\tau_y + \phi_y^2 \tau_x)} \frac{\partial \phi_y}{\partial \tau_\omega}$.
- Again when $\lambda \longrightarrow +\infty$, $\phi_y \frac{2\tau_x}{\tau_x + \tau_\omega + \tau_a} = \frac{\tau_y}{\tau_y + \tau_\eta + \tau_f}(1 + \frac{\tau_P}{\tau_\eta + \tau_f + \tau_P})$ so

$$
\begin{aligned}
& \frac{\partial \phi_y}{\partial \tau_\omega} \frac{2\tau_x}{\tau_x + \tau_\omega + \tau_a} - \phi_y \frac{2\tau_x}{(\tau_x + \tau_\omega + \tau_a)^2} \\
= & \frac{\tau_y}{\tau_y + \tau_\eta + \tau_f} \frac{\tau_\eta + \tau_f}{(\tau_\eta + \tau_f + \tau_P)^2} \frac{\partial \tau_P}{\partial \tau_\omega} \\
= & \frac{\tau_y}{\tau_y + \tau_\eta + \tau_f} \frac{\tau_\eta + \tau_f}{(\tau_\eta + \tau_f + \tau_P)^2} \frac{2\tau_P \tau_y}{\phi_y(\tau_y + \phi_y^2 \tau_x)} \frac{\partial \phi_y}{\partial \tau_\omega}.
\end{aligned}
$$

The effect of disclosure on real efficiency

- Again $\frac{2\tau_x}{\tau_x+\tau_\omega+\tau_a} = \frac{1}{\phi_y}\frac{\tau_y}{\tau_y+\tau_\eta+\tau_f}(1+\frac{\tau_P}{\tau_\eta+\tau_f+\tau_P})$. Insert into the equation of the previous slide results in

$$\begin{aligned}
&\frac{\partial\phi_y}{\partial\tau_\omega}\frac{1}{\phi_y}\frac{\tau_y}{\tau_y+\tau_\eta+\tau_f}(\frac{\tau_\eta+\tau_f}{\tau_\eta+\tau_f+\tau_P} \\
&+\frac{2\tau_P}{\tau_\eta+\tau_f+\tau_P}-\frac{2\tau_P}{\tau_\eta+\tau_f+\tau_P}\frac{\tau_\eta+\tau_f}{\tau_\eta+\tau_f+\tau_P}\frac{\tau_y}{\tau_y+\phi_y^2\tau_x}) \\
= \quad & \phi_y\frac{2\tau_x}{(\tau_x+\tau_\omega+\tau_a)^2} \\
= \quad & \frac{1}{\tau_x+\tau_\omega+\tau_a}\frac{\tau_y}{\tau_y+\tau_\eta+\tau_f}(1+\frac{\tau_P}{\tau_\eta+\tau_f+\tau_P}).
\end{aligned}$$

- Therefore $\frac{\partial\phi_y}{\partial\tau_\omega} > 0$ so real efficiency always increases in τ_ω.

The effect of disclosure on real efficiency

- To better view the intuition, rewrite $\frac{\partial \phi_y}{\partial \tau_\omega}$ as

$$\frac{\partial \phi_y}{\partial \tau_\omega} = \frac{\frac{\phi_y}{\tau_x+\tau_\omega+\tau_a}\frac{\tau_\eta+\tau_f+2\tau_P}{\tau_\eta+\tau_f+\tau_P}}{\frac{\tau_\eta+\tau_f+2\tau_P}{\tau_\eta+\tau_f+\tau_P} - \frac{2\tau_P}{\tau_\eta+\tau_f+\tau_P}\frac{\tau_\eta+\tau_f}{\tau_\eta+\tau_f+\tau_P}\frac{\tau_y}{\tau_y+\phi_y^2\tau_x}}$$

$$= \frac{\frac{\phi_y}{\tau_x+\tau_\omega+\tau_a}}{1 - \frac{2\tau_P}{\tau_\eta+\tau_f+2\tau_P}\frac{\tau_\eta+\tau_f}{\tau_\eta+\tau_f+\tau_P}\frac{\tau_y}{\tau_y+\phi_y^2\tau_x}}$$

- Again two effects: first, increasing τ_ω resulting in speculators putting higher weight on $\widetilde{\omega}$ and lower weight on $\widetilde{x}_i$, reducing b_x^V and thus increasing ϕ_y (recall that $\phi_y \to \frac{b_y^V}{b_x^V}$), which is reflected in the $\frac{1}{\tau_x+\tau_\omega+\tau_a}$ term in the numerator; second, increasing τ_ω resulting in speculators trading more aggressively on $\widetilde{y}_i$ as their profit from trading on $\widetilde{x}_i$ is reduced) and thus increasing b_y^V, further amplified by that the decision maker will learn more from price when the speculator trades more aggressively on $\widetilde{y}_i$, which is reflected in the denominator

Implications of GY

- Information disclosure is more beneficial when the real decision maker already knows a lot about disclosed information and may not be beneficial when, absent such disclosure, the market does a good job of aggregating
  - Stress test: regulators disclose stress test results and creditors make real decisions -> disclose more about loan quality rather than network externalities (e.g., systematic risk)
  - Credit ratings: credit rating agencies disclose ratings and creditors make real decisions -> disclose more about quality of the firm's products rather than competition and market interactions
  - Firm disclosure: disclose information that are hard facts that the firm knows a lot and not easily aggregated from the market (e.g., firm specific information such as revenues, net income) rather than noisy and easily aggregated from the market (e.g., synergies from future acquisitions that depend a lot more on macro and industry factors)

Summary of feedback effect with disclosure

- Disclosure implies that either the manager or some other third party has information that the market does not know
- The effect of disclosure on feedback effect operates through affecting speculators' trading and thus private information acquisition activities.
- If the manager's information and the speculator's information signals are substitutes (complements), then more disclosure crowds out (crowds in) private information acquisition and has negative (positive) real consequences through feedback.

Next class (March 21st, after spring break)

- Verifiable disclosure (but managers can choose to strategically disclose)
- Since the market relies on the firms' disclosure without private information of its own, the price process is much simpler.
- Since the model relates to manager's private information that the market does not know, it has more relevant implications for empirical accounting research on capital market consequences of managerial disclosure.