BA 932 Session 3 Feedback Effect Without Disclosure

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What we have done

- First glance at how information economics models differ from traditional economic models
- Key difference: prices aggregate information (noisily) from privately informed traders
- Uninformed traders do not have info but try to infer information from informed traders -> by conjecturing informed traders' strategies (and later verifying they are indeed the optimal strategies)
- Two different market structures: perfectly competitive (when everybody takes price as given in choosing their demand) and imperfectly competitive (when some or all traders has to take into account the effect of demand on price when choosing demand)

This class- feedback effect in the absence of disclosure

- Since prices aggregate information, wouldn't it be natural to think that managers/firms can learn from prices to guide their decisions? - The feedback of prices into real decisions
- Such "feedback effect" would not arise in either Grossman and Stiglitz or Kyle - in those models the future firm value is exogenously given and there is no real decisions to be made
- Therefore to model "feedback effect", need a model when the managers/firms make real decisions.
- If the market is more informed and the manager learns from prices, what role does disclose play in modelling feedback effect - next class

Price efficiency and real efficiency

- A knee-jerk reaction to feedback effect is: since managers learn from prices, the more informative price is, the more efficient the real decision will be -> price efficiency and real efficiency move in the same direction
- The flaw in this logic: if managers learn from prices, then price will anticipate such learning; alternatively, if price anticipate that managers ignore prices, then price will anticipate such non-learning -> managers will in turn not learn from prices and the conjecture becomes "self-fulfilling".
- In this case, price is completely informative as it reflects that manager will not learn from prices, but real efficiency suffers.
- Therefore, feedback effect may generate a wedge between price efficiency and real efficiency (Bond, Edmans and Goldstein 2012).

Price efficiency and real efficiency - a (simplified) example adopted from Dow and Gorton (1997)

- A firm endowed with a project that requires investment of 1 at date 1 and generates a random return at date 2.
- The random return, which is the state denoted by θ, is equally likely to be H or L with $H > L$. The firm can choose whether or not to invest (the real decision) based on the stock price at date 1.
- Stock market trading occurs at date 1 in a discrete Kyle setting when the informed trader may not trade - essentially Glosten-Milgrom setting.
- Informed trader who can know perfectly about θ with cost δ enters the market with probability π and chooses to trade -1 or 1 shares; uninformed trader enters the market with probability $1 - \pi$ and is equally likely to trade -1 or 1 shares. Denote the order flow as S.

Price efficiency and real efficiency - an example

- Assumption 1: $\frac{H+L}{2} < 1$, i.e., in the absence of any information, the project is negative NPV (crucial for the wedge between price efficiency and real efficiency)
- Assumption 2: $\frac{1+\pi}{2}H + \frac{1-\pi}{2}L > 1$ (you will see why we need this assumption soon).
- Assumption 3: $\frac{(1-\pi)}{2}(H - L) > \delta$ to make information acquisition profitable.

Price efficiency and real efficiency - an example

- There are two equilibrium
  - Equilibrium 1: informed trader does not acquire information, the firm chooses not to invest in the project, and price equals zero to reflect this.
  - Equilibrium 2: informed trader acquires information, the firm learns from prices and invest in the project if and only if order flow is $+1$.

Price efficiency and real efficiency - an example

- To see why the first one is an equilibrium, note that if there is no information to be learned from the prices, the firm will not invest because the project is negative NPV in the absence of any information (assumption 1). Knowing that the firm will not invest, traders do not acquire any information because information acquisition is costly and no benefit when the firm does not invest.

Price efficiency and real efficiency - an example

- To see why the second one is an equilibrium, note that if traders acquire information, then $P(-1) = 0$ as it comes from either an informed trader who gets bad news or an uninformed trader and the project has negative NPV.
- To calculate $P(+1)$, again use Bayes' rule to calculate

$$\begin{aligned}
\Pr(\theta &= H|S = 1) \\
&= \frac{\Pr(S = 1|\theta = H)\Pr(\theta = H)}{\Pr(S = 1|\theta = H)\Pr(\theta = H) + \Pr(S = 1|\theta = L)\Pr(\theta = L)} \\
&= \frac{\Pr(S = 1|\theta = H)}{\Pr(S = 1|\theta = H) + \Pr(S = 1|\theta = L)} \\
&= \frac{\pi + \frac{1}{2}(1-\pi)}{\pi + \frac{1}{2}(1-\pi) + \frac{1}{2}(1-\pi)} = \frac{1+\pi}{2}.
\end{aligned}$$

Price efficiency and real efficiency - an example

- Therefore if the firm chooses to invest, the expected payoff (minus investment) is
$$\frac{1+\pi}{2}H + \frac{1-\pi}{2}L - 1 > 0,$$
which comes from assumption 2. Therefore the firm will invest when observing order flow of $+1$ and
$$P(S=1) = \frac{1+\pi}{2}H + \frac{1-\pi}{2}L.$$

Price efficiency and real efficiency - an example

- The informed trader's expected trading profit will be

$$[H - P(S = 1)] = \frac{(1-\pi)}{2}(H - L) > \delta,$$

from assumption 3, so the informed trader will acquire information.

- We therefore have two equilibrium but the firm value is lower (0) in equilibrium 1 so the real efficiency is lower.
- In both equilibrium, prices are semi-strong form efficient to the extent that they reflect all available public information so price efficiency is not the same as real efficiency -> self-fulfilling inefficient equilibria may arise.

Price efficiency and real efficiency

- From the above example one see that the wedge between price efficiency and real efficiency is generated through trader's information acquisition activities.
- Usually we think about information acquisition activities as symmetrical: you have incentive to acquire and trade on both good and bad news so long as they are equally informative.
- When incorporating feedback effect, asymmetry in information acquisition and trading may arise, resulting in asymmetric informativeness of stock prices, which is what we talk next (Edmans, Goldstein and Jiang 2015 or EGJ).
- Intuition: if I trade on bad (good) news -> firm learns from bad (good) news and improve firm value -> I make less (more) profit from bad (good) news -> I am less (more) willing to trade on bad news.

EGJ - setup of the firm

- A firm endowed with an asset-in-place needs to make investment decision $d \in \{-1, 0, 1\}$, representing disinvestment, keep the current scale, and further investment, respectively. Choosing $d \in \{-1, 1\}$ incurs a cost $c \geq 0$ (adjustment cost).
- Value of the firm v determined by d and an underlying state $\theta \in \{H, L\}$, i.e., $v(\theta, d)$. Prior probability that $\theta = H$ is $\frac{1}{2}$.

$$\begin{aligned} v(H,0) &= R_H > v(L,0) = R_L, \\ v(H,1) &= R_H + x - c > v(H,0) > v(H,-1) = R_H - x - c, \\ v(L,-1) &= R_L + x - c > v(L,0) > v(L,1) = R_L - x - c. \end{aligned}$$

- The assumptions on v imply that $v(H,1) - v(L,1) > v(H,0) - v(L,0) > v(H,-1) - v(L,-1)$, i.e., information is less useful the lower the level of investment.
- In addition, it is not obvious whether $v(H,-1)$ or $v(L,-1)$ is bigger. Focus on the case when $v(H,-1) > v(L,-1)$, i.e., feedback effect is not too strong.

EGJ - manager's investment decision based on posterior beliefs (from stock price)

- Since choosing $d = \pm 1$ is costly, need the posterior to be sufficiently different from prior to make such decision.
- Clearly when posterior that $\theta = H$ is sufficiently high (low), choose $d = +1$ ($d = -1$).
- Denote the threshold above which to choose $d = +1$ as γ_1, then

$$\gamma_1 R_H + (1-\gamma_1) R_L = \gamma_1 (R_H + x - c) + (1-\gamma_1)(R_H - x - c),$$

resulting in

$$\gamma_1 = \frac{1}{2} + \frac{c}{2x}.$$

EGJ - manager's investment decision based on posterior beliefs (from stock price)

- Similarly, denote the threshold below which to choose $d = -1$ as γ_{-1}, then

$$\gamma_{-1} R_H + (1 - \gamma_{-1}) R_L = \gamma_{-1}(R_H - x - c) + (1 - \gamma_{-1})(R_L + x - c),$$

resulting in

$$\gamma_{-1} = \frac{1}{2} - \frac{c}{2x}.$$

- Note that if $c = 0$, then $\gamma_1 = \gamma_{-1} = \frac{1}{2}$. The higher the cost of adjusting investment, c, is relative to the benefit of adjusting investment, x, the higher γ_1 and the lower γ_{-1} is (i.e., need more extreme beliefs).

EGJ - setup of the financial market

- Again use discrete Kyle model with the informed trader not always present: present with probability $\lambda > 0$.
- The informed trader observes θ perfectly and trades $s \in \{-1, 0, 1\}$ if present. Trading $s = \pm 1$ costs $\kappa > 0$ (crucial as otherwise the informed trader will always trade when present and the asymmetric trading will not be observed)
- A noise trader always presents and trades $z \in \{-1, 0, 1\}$ with equal probability.
- Market maker observes $X = s + z \in \{-2, -1, 0, 1, 2\}$ and sets price $P(X) = E[v|X]$.
- Incorporating feedback makes determining price more tricky, as v depends on d, which in turn depends on information learned from P (or equivalently, X).

EGJ - equilibrium definition

- Similar to all equilibrium definition, everybody chooses his or her strategy to maximize his or her payoff, taking into account others' optimal strategies, and use Bayes' Rule whenever applicable
  - The speculator chooses $s(\theta)$ to maximize expected value of $s(v-p) - |s|\kappa$.
  - The manager chooses d to maximize expected firm value v based on the manager's information, including information learned from prices.
  - The marker maker sets $P(X) = E[v|X]$.
  - There are some technical issues related to off-equilibrium beliefs. Do not worry about them now.

Solving for the model - no feedback benchmark

- When λ is too small, the information contained in prices is not enough to change the posteriors much, when the prices are not fully revealing (due to the investment adjustment cost c).
- The firm will always choose $d = 0$ for those order flows, so equivalent to no feedback.
- What would be the trader's strategies? When κ too small, always trade; when κ too large, always not trade; what about κ in the middle: trading only on good news (BNS) and trading only on bad news (SNB)?

No feedback benchmark- restrictions on posterior beliefs

- When $X = +2 (X = -2)$, it is fully revealing that $\theta = H (\theta = L)$ so $P(+2) = R_H + x - c$ and $P(-2) = R_L + x - c$. Note that when prices are fully revealing, it does not matter whether there is feedback or not as informed trader always makes zero profits in those circumstances.
- When $X = +1$, if the informed trader is always trading, then using Bayes' Rule,

$$
\begin{aligned}
& \Pr(H| + 1) \\
= & \frac{\Pr(X = +1|\theta = H) \Pr(\theta = H)}{\Pr(X = +1|\theta = H) \Pr(\theta = H) + \Pr(X = +1|\theta = L) \Pr(\theta = L)} \\
= & \frac{\Pr(X = +1|\theta = H)}{\Pr(X = +1|\theta = H) + \Pr(X = +1|\theta = L)} \\
= & \frac{\lambda \times \frac{1}{3} + (1-\lambda) \times \frac{1}{3}}{\lambda \times \frac{1}{3} + (1-\lambda) \times \frac{1}{3} + (1-\lambda) \times \frac{1}{3}} = \frac{1}{2-\lambda},
\end{aligned}
$$

- Therefore, if $\frac{1}{2-\lambda} \leq \gamma_1$, no investment will be made when $X = +1$.

No feedback benchmark- restrictions on prior beliefs

- Similarly, When $X = -1$, if the informed trader is always trading, then using Bayes' Rule,

$$
\begin{aligned}
& \Pr(H|-1) \\
= \;& \frac{\Pr(X=-1|\theta=H)\Pr(\theta=H)}{\Pr(X=-1|\theta=H)\Pr(\theta=H)+\Pr(X=-1|\theta=L)\Pr(\theta=L)} \\
= \;& \frac{\Pr(X=-1|\theta=H)}{\Pr(X=-1|\theta=H)+\Pr(X=-1|\theta=L)} \\
= \;& \frac{(1-\lambda)\times\frac{1}{3}}{(1-\lambda)\times\frac{1}{3}+\lambda\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}} = \frac{1-\lambda}{2-\lambda}.
\end{aligned}
$$

- Therefore, if $\frac{1-\lambda}{2-\lambda} \geq \gamma_{-1}$, or, equivalently, $\frac{1}{2-\lambda} \leq \gamma_1$, no investment will be made when $X = -1$.

No feedback benchmark- prices conjecturing that the informed trader always trade

- What about $X = 0$?

$$
\begin{aligned}
\Pr(\theta &= H|X = 0) \\
&= \frac{\Pr(X = 0|\theta = H)\Pr(\theta = H)}{\Pr(X = 0|\theta = H)\Pr(\theta = H) + \Pr(X = 0|\theta = L)\Pr(\theta = L)} \\
&= \frac{\Pr(X = 0|\theta = H)}{\Pr(X = 0|\theta = H) + \Pr(X = 0|\theta = L)} \\
&= \frac{\lambda \times \frac{1}{3} + (1-\lambda) \times \frac{1}{3}}{\lambda \times \frac{1}{3} + (1-\lambda) \times \frac{1}{3} + \lambda \times \frac{1}{3} + (1-\lambda) \times \frac{1}{3}} \\
&= \frac{1}{2} \in (\gamma_{-1}, \gamma_1).
\end{aligned}
$$

- Therefore no investment will be made when $X = 0$, resulting in $v(\theta, d) = R_\theta$ when $X = 0, \pm 1$.

No feedback benchmark- prices conjecturing that the informed trader always trade

- In addition, when the conjecture is that the informed trader will always trade,

$$
\begin{aligned}
P^T(+1) &= \frac{1}{2-\lambda} R_H + \frac{1-\lambda}{2-\lambda} R_L, \\
P^T(0) &= \frac{1}{2} R_H + \frac{1}{2} R_L, \\
P^T(-1) &= \frac{1-\lambda}{2-\lambda} R_H + \frac{1}{2-\lambda} R_L,
\end{aligned}
$$

where the superscript T refers to trading.

No feedback benchmark- prices conjecturing that the informed trader always not trade

- If the market maker's conjecture is that the informed trader will not trade, then no order flows has no information content and market maker sets $P^{NT}(+1) = P^{NT}(0) = P^{NT}(-1) = \frac{R_H + R_L}{2}$, where the superscript NT refers to no-trading.
- Under this conjecture, the only possible order flows are $X \in \{-1, 0, 1\}$. $X = \pm 2$ then is off-equilibrium path. Need a reasonable off-equilibrium belief for $P^{NT}(\pm 2)$ to gauge deviation. Clearly a reasonable off-equlibrium belief is that $X = +2(X = -2)$ corresponds to the positively (negatively) informed speculator buying (selling), resulting in $P^{NT}(+2) = R_H + x - c$ and $P^{NT}(-2) = R_L + x - c$.

No feedback benchmark- prior beliefs when conjecturing the informed trader BNS

- Suppose the conjecture is that the informed trader will buy when observing $\theta = H$ but not sell when observing $\theta = L$ (BNS), then using Bayes' Rule,

$$
\begin{aligned}
& \Pr(H| +1) \\
= & \frac{\Pr(X = +1|\theta = H)\Pr(\theta = H)}{\Pr(X = +1|\theta = H)\Pr(\theta = H) + \Pr(X = +1|\theta = L)\Pr(\theta = L)} \\
= & \frac{\Pr(X = +1|\theta = H)}{\Pr(X = +1|\theta = H) + \Pr(X = +1|\theta = L)} \\
= & \frac{\lambda \times \frac{1}{3} + (1-\lambda) \times \frac{1}{3}}{\lambda \times \frac{1}{3} + (1-\lambda) \times \frac{1}{3} + \lambda \times \frac{1}{3} + (1-\lambda) \times \frac{1}{3}} = \frac{1}{2}.
\end{aligned}
$$

No feedback benchmark- prior beliefs when conjecturing the informed trader BNS

$$
\begin{aligned}
& \Pr(H|-1) \\
= \; & \frac{\Pr(X=-1|\theta=H)\Pr(\theta=H)}{\Pr(X=-1|\theta=H)\Pr(\theta=H)+\Pr(X=-1|\theta=L)\Pr(\theta=L)} \\
= \; & \frac{\Pr(X=-1|\theta=H)}{\Pr(X=-1|\theta=H)+\Pr(X=-1|\theta=L)} \\
= \; & \frac{(1-\lambda)\times\frac{1}{3}}{(1-\lambda)\times\frac{1}{3}+\lambda\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}} = \frac{1-\lambda}{2-\lambda}.
\end{aligned}
$$

No feedback benchmark- prices conjecturing that the informed trader BNS

$$
\begin{aligned}
&\Pr(H|0) \\
= \; & \frac{\Pr(X=0|\theta=H)\Pr(\theta=H)}{\Pr(X=-1|\theta=H)\Pr(\theta=H)+\Pr(X=-1|\theta=L)\Pr(\theta=L)} \\
= \; & \frac{\Pr(X=0|\theta=H)}{\Pr(X=0|\theta=H)+\Pr(X=0|\theta=L)} \\
= \; & \frac{\lambda\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}}{\lambda\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}+\lambda\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}}=\frac{1}{2}.
\end{aligned}
$$

- Under BNS, the possible order flows $X \in \{-1, 0, +1, +2\}$.
- Therefore $P^{BNS}(+2) = R_H + x - c$,
  $P^{BNS}(+1) = P^{BNS}(0) = \frac{1}{2}R_H + \frac{1}{2}R_L$,
  $P^{BNS}(-1) = \frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L$.
- $X = -2$ is the off-equilibrium belief and a reasonable belief is that $P^{BNS}(-2) = R_L + x - c$, i.e., fully revealing.

No feedback benchmark- prior beliefs when conjecturing the informed trader SNB

- Suppose the conjecture is that the informed trader will sell when observing $\theta = L$ but not buy when observing $\theta = H$ (SNB), then using Bayes' Rule,

$$
\begin{aligned}
& \Pr(H|+1) \\
= & \frac{\Pr(X=+1|\theta=H)\Pr(\theta=H)}{\Pr(X=+1|\theta=H)\Pr(\theta=H)+\Pr(X=+1|\theta=L)\Pr(\theta=L)} \\
= & \frac{\Pr(X=+1|\theta=H)}{\Pr(X=+1|\theta=H)+\Pr(X=+1|\theta=L)} \\
= & \frac{\lambda\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}}{\lambda\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}} = \frac{1}{2-\lambda}.
\end{aligned}
$$

No feedback benchmark- prior beliefs when conjecturing the informed trader SNB

$$
\begin{aligned}
& \Pr(H|-1) \\
= \; & \frac{\Pr(X=-1|\theta=H)\Pr(\theta=H)}{\Pr(X=-1|\theta=H)\Pr(\theta=H)+\Pr(X=-1|\theta=L)\Pr(\theta=L)} \\
= \; & \frac{\Pr(X=-1|\theta=H)}{\Pr(X=-1|\theta=H)+\Pr(X=-1|\theta=L)} \\
= \; & \frac{\lambda\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}}{\lambda\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}+\lambda\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}}=\frac{1}{2}.
\end{aligned}
$$

No feedback benchmark- prices conjecturing that the informed trader SNB

$$
\begin{aligned}
& \Pr(H|0) \\
= \; & \frac{\Pr(X=0|\theta=H)\Pr(\theta=H)}{\Pr(X=-1|\theta=H)\Pr(\theta=H)+\Pr(X=-1|\theta=L)\Pr(\theta=L)} \\
= \; & \frac{\Pr(X=0|\theta=H)}{\Pr(X=0|\theta=H)+\Pr(X=0|\theta=L)} \\
= \; & \frac{\lambda\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}}{\lambda\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}+\lambda\times\frac{1}{3}+(1-\lambda)\times\frac{1}{3}}=\frac{1}{2}.
\end{aligned}
$$

- Under SNB, the possible order flows $X \in \{-2, -1, 0, +1\}$.
- Therefore $P^{SNB}(+1) = \frac{1}{2-\lambda}R_H + \frac{1-\lambda}{2-\lambda}R_L$, $P^{SNB}(0) = P^{SNB}(-1) = \frac{1}{2}R_H + \frac{1}{2}R_L$, $P^{SNB}(-2) = R_L + x - c$.
- $X = +2$ is the off-equilibrium belief and a reasonable belief is that $P^{SNB}(+2) = R_H + x - c$, i.e., fully revealing.

No feedback benchmark- comparison between BNS and SNB

- One can see that under BNS, $X = +1$ is completely uninformative but $X = -1$ is (partially) informative whereas under SNB, $X = -1$ is completely uninformative but $X = +1$ is (partially) informative.
- The reason is that under BNS, $X = +1$ is not informative because negatively informed trader does not trade and so the bad news is not reflected in the order flow; $X = -1$ is informative because positively informed trader will trade so $X = -1$ is very likely due to negatively informed trader not trade.
- Similar logic applies to SNB.

No feedback benchmark- when no trading is the unique equilibrium

- When the informed trader does not trade, the expected payoff is 0. Now need to check whether deviating to trading (either partially or fully) generates less than zero.
- If the informed trader deviates to BNS, then the expected payoff (trading profit minus the trading cost) will be

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^{NT}(+2)) + \frac{1}{3}(R_H - P^{NT}(+1)) \\
& + \frac{1}{3}(R_H - P^{NT}(0)) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] = \frac{1}{2}(\frac{R_H - R_L}{3} - \kappa).
\end{aligned}
$$

No feedback benchmark- when no trading is the unique equilibrium

- If the informed trader deviates to SNB, then the expected payoff (trading profit minus the trading cost) will be

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(P^{NT}(-2) - R_L - x + c) + \frac{1}{3}(P^{NT}(+1) - R_L) \\
& + \frac{1}{3}(P^{NT}(0) - R_L) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] = \frac{1}{2}(\frac{R_H - R_L}{3} - \kappa).
\end{aligned}
$$

No feedback benchmark- when no trading is the unique equilibrium

- If the informed trader always trade, then the expected trading profit (gross of trading costs) will be

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^{NT}(+2)) + \frac{1}{3}(R_H - P^{NT}(+1)) \\
& + \frac{1}{3}(R_H - P^{NT}(0)) - \kappa] \\
& + \frac{1}{2}[\frac{1}{3}(P^{NT}(-2) - R_L - x + c) + \frac{1}{3}(P^{NT}(+1) - R_L) \\
& + \frac{1}{3}(P^{NT}(0) - R_L) - \kappa] \\
= \; & \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] \\
& + \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] \\
= \; & \frac{R_H - R_L}{3} - \kappa.
\end{aligned}
$$

No feedback benchmark- when no trading is the unique equilibrium

- Thus, if $\kappa \geq \frac{R_H - R_L}{3} \equiv \kappa_{NT}$, then no trading is the unique equilibrium as every deviation generates smaller expected payoff.

No feedback benchmark - when BNS can be an equilibrium

- If the informed trader chooses BNS, then the informed trader can either deviate to always trading, no trading or SNB.
- The payoff from choosing BNS is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^{BNS}(+2)) + \frac{1}{3}(R_H - P^{BNS}(+1)) \\
& + \frac{1}{3}(R_H - P^{BNS}(0)) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa].
\end{aligned}
$$

No feedback benchmark - when BNS can be an equilibrium

- The payoff to deviating to always trading is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^{BNS}(+2)) + \frac{1}{3}(R_H - P^{BNS}(+1)) \\
& + \frac{1}{3}(R_H - P^{BNS}(0)) - \kappa] \\
& + \frac{1}{2}[\frac{1}{3}(P^{BNS}(-2) - R_L - x + c) + \frac{1}{3}(P^{BNS}(-1) - R_L) \\
& + \frac{1}{3}(P^{BNS}(0) - R_L) - \kappa] \\
= \; & \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] \\
& + \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa]
\end{aligned}
$$

No feedback benchmark - when BNS can be an equilibrium

- The payoff to deviating to choosing SNB is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(P^{BNS}(-2) - R_L - x + c) + \frac{1}{3}(P^{BNS}(-1) - R_L) \\
& + \frac{1}{3}(P^{BNS}(0) - R_L) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa]
\end{aligned}
$$

  and the payoff to deviating to no trading is zero.

- Since $\frac{1-\lambda}{2-\lambda}(R_H - R_L) < \frac{1}{2}(R_H - R_L)$, when $\kappa \in [\frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L), \frac{1}{3}(R_H - R_L))$, BNS is an equilibrium. Denote $\kappa_{NF} \equiv \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L)$, the condition is $\kappa \in [\kappa_{NF}, \kappa_{NT})$.

No feedback benchmark - when SNB can be an equilibrium

- If the informed trader chooses SNB, then the informed trader can either deviate to always trading, no trading or BNS.
- The payoff from choosing SNB is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(P^{SNB}(-2) - R_L - x + c) + \frac{1}{3}(P^{SNB}(-1) - R_L) \\
& + \frac{1}{3}(P^{SNB}(0) - R_L) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa].
\end{aligned}
$$

No feedback benchmark - when SNB can be an equilibrium

- The payoff to deviating to always trading is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^{SNB}(+2)) + \frac{1}{3}(R_H - P^{SNB}(+1)) \\
& + \frac{1}{3}(R_H - P^{SNB}(0)) - \kappa] \\
& + \frac{1}{2}[\frac{1}{3}(P^{SNB}(-2) - R_L - x + c) + \frac{1}{3}(P^{SNB}(-1) - R_L) \\
& + \frac{1}{3}(P^{SNB}(0) - R_L) - \kappa] \\
= \;& \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] \\
& + \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa].
\end{aligned}
$$

No feedback benchmark - when SNB can be an equilibrium

- The payoff to deviating to choosing BNS is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^{SNB}(+2)) + \frac{1}{3}(R_H - P^{SNB}(+1)) \\
& + \frac{1}{3}(R_H - P^{SNB}(0)) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa]
\end{aligned}
$$

and the payoff to deviating to no trading is zero.

No feedback benchmark - when SNB can be an equilibrium

- Since $\frac{1-\lambda}{2-\lambda}(R_H - R_L) < \frac{1}{2}(R_H - R_L)$, when $\kappa \in [\frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L), \frac{1}{3}(R_H - R_L))$, i.e., when $\kappa \in [\kappa_{NF}, \kappa_{NT})$, SNB is an equilibrium.
- We have multiple equilibria here because of self-fulfilling beliefs: if the market maker believes that the informed will buy (sell) but not sell (buy), then $X = +1$ $(X = -1)$ is not very informative but $X = -1(X = +1)$ will be very informative, i.e., closer to the actual firm value, this indeed results in buying (selling) generates more profit than selling (buying), justifying the BNS (SNB) equilibrium.

No feedback benchmark - when always trading can be an equilibrium

- If the informed trader chooses always trading, then the informed trader can either deviate to BNS, SNB or no trading.
- The payoff from always trading is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^T(+2)) + \frac{1}{3}(R_H - P^T(+1)) \\
& + \frac{1}{3}(R_H - P^T(0)) - \kappa] \\
& + \frac{1}{2}[\frac{1}{3}(P^T(-2) - R_L - x + c) + \frac{1}{3}(P^T(-1) - R_L) \\
& + \frac{1}{3}(P^T(0) - R_L) - \kappa] \\
= \; & \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] \\
& + \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] = \kappa_{NF} - \kappa
\end{aligned}
$$

No feedback benchmark - when always trading can be an equilibrium

- The payoff to deviating to choosing BNS is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^T(+2)) + \frac{1}{3}(R_H - P^T(+1)) \\
& + \frac{1}{3}(R_H - P^T(0)) - \kappa] \\
= \; & \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] = \frac{1}{2}(\kappa_{NF} - \kappa).
\end{aligned}
$$

No feedback benchmark - when always trading can be an equilibrium

- The payoff to deviating to choosing SNB is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(P^T(-2) - R_L - x + c) + \frac{1}{3}(P^T(-1) - R_L) \\
& + \frac{1}{3}(P^T(0) - R_L) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] = \frac{1}{2}(\kappa_{NF} - \kappa).
\end{aligned}
$$

  and the payoff to deviating to no trading is zero.
- Therefore when $\kappa < \kappa_{NF}$, trading strategy generates the highest expected payoff and is the unique equilibrium.

No feedback benchmark - summarizing the main results

- When $\kappa < \kappa_{NF}$, the informed trader always trade; when $\kappa \geq \kappa_{NT}$, the informed trader always not trade; when $\kappa \in [\kappa_{NF}, \kappa_{NT})$, both BNS and SNB can be an equilibrium.
- Whenever the informed trader trades, information will be partially reflected in prices, which reduces the trading profit relative the case when prices do not reflect such information. So trade on both positive and negative information when trading cost is sufficiently small, trade on neither when trading cost is sufficiently large, and trade on either positive or negative information when trading cost is in the middle.
- However, can trade on either positive or negative information. In the absence of feedback, the manager does not make any decisions to change firm value that depends on the nature of information incorporated into the prices. So trading on either information generates the same expected profit.

Incorporating feedback - prices

- Now we incorporate feedback and assume that $\frac{1-\lambda}{2-\lambda} \leq \gamma_{-1}$, or, equivalently, $\frac{1}{2-\lambda} \geq \gamma_1$. This implies that in the presence of informed trading (i.e., when $X = \pm 1$ becomes informative), $X = \pm 1$ will lead to manager's decision change, which affects firm value.
- Note that the posterior probabilities of θ stays the same for each of the four conjecture of the informed trader's strategies (as they do not depend on the manager's investment decisions), only the expected payoff will change.

Incorporating feedback - prices

- Therefore under NT: $P^{NT}(+1) = P^{NT}(0) = P^{NT}(-1) = \frac{R_H + R_L}{2}$, and off-equilibrium beliefs $P^{NT}(+2) = R_H + x - c$ and $P^{NT}(-2) = R_L + x - c$ (the same as under no feedback as with NT, $X = \pm 1$ is not informative).
- Under BNS: $P^{BNS}(+2) = R_H + x - c$, $P^{BNS}(+1) = P^{BNS}(0) = \frac{1}{2}R_H + \frac{1}{2}R_L$, $P^{BNS}(-1) = \frac{1-\lambda}{2-\lambda}(R_H - x - c) + \frac{1}{2-\lambda}(R_L + x - c)$, and off-equilibrium beliefs $P^{BNS}(-2) = R_L + x - c$.
  - With feedback, $P^{BNS}(-1)$ is different, as in this case $\frac{1-\lambda}{2-\lambda} \leq \gamma_{-1}$ so the manager will choose $d = -1$, resulting in a payoff of $R_H - x - c$ when $\theta = H$ and a payoff of $R_L + x - c$ when $\theta = L$.

Incorporating feedback - prices

- Under SNB: $P^{SNB}(+1) = \frac{1}{2-\lambda}(R_H + x - c) + \frac{1-\lambda}{2-\lambda}(R_L - x - c)$, $P^{SNB}(0) = P^{SNB}(-1) = \frac{1}{2}R_H + \frac{1}{2}R_L$, $P^{SNB}(-2) = R_L + x - c$, and off-equilibrium beliefs $P^{SNB}(+2) = R_H + x - c$.
  - With feedback, $P^{SNB}(+1)$ is different, as in this case $\frac{1}{2-\lambda} \geq \gamma_1$ so the manager will choose $d = +1$, resulting in a payoff of $R_H + x - c$ when $\theta = H$ and a payoff of $R_L - x - c$ when $\theta = L$.
- Under T: $P^T(+2) = R_H + x - c$,
  $P^T(+1) = \frac{1}{2-\lambda}(R_H + x - c) + \frac{1-\lambda}{2-\lambda}(R_L - x - c)$,
  $P^T(0) = \frac{1}{2}R_H + \frac{1}{2}R_L$,
  $P^T(-1) = \frac{1-\lambda}{2-\lambda}(R_H - x - c) + \frac{1}{2-\lambda}(R_L + x - c)$, and
  $P^{NT}(-2) = R_L + x - c$.
  - With feedback, both $P^T(+1)$ and $P^T(-1)$ are different, as both $X = +1$ and $X = -1$ are informative and the manager chooses $d = +1$ and $d = -1$, respectively.

Incorporating feedback - when no trading is the unique equilibrium

- When the informed trader does not trade, the expected payoff is 0. Now need to check whether deviating to trading (either partially or fully) generates less than zero.
- If the informed trader deviates to BNS, then the expected payoff (trading profit minus the trading cost) will be

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^{NT}(+2)) + \frac{1}{3}(R_H - P^{NT}(+1)) \\
& + \frac{1}{3}(R_H - P^{NT}(0)) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] = \frac{1}{2}(\frac{R_H - R_L}{3} - \kappa).
\end{aligned}
$$

Incorporating feedback - when no trading is the unique equilibrium

- If the informed trader deviates to SNB, then the expected payoff (trading profit minus the trading cost) will be

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(P^{NT}(-2) - R_L - x + c) + \frac{1}{3}(P^{NT}(+1) - R_L) \\
& + \frac{1}{3}(P^{NT}(0) - R_L) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] = \frac{1}{2}(\frac{R_H - R_L}{3} - \kappa).
\end{aligned}
$$

Incorporating feedback - when no trading is the unique equilibrium

- If the informed trader always trade, then the expected trading profit (gross of trading costs) will be

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^{NT}(+2)) + \frac{1}{3}(R_H - P^{NT}(+1)) \\
& + \frac{1}{3}(R_H - P^{NT}(0)) - \kappa] \\
& + \frac{1}{2}[\frac{1}{3}(P^{NT}(-2) - R_L - x + c) + \frac{1}{3}(P^{NT}(+1) - R_L) \\
& + \frac{1}{3}(P^{NT}(0) - R_L) - \kappa] \\
= \; & \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] \\
& + \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] \\
= \; & \frac{R_H - R_L}{3} - \kappa.
\end{aligned}
$$

Incorporating feedback - when no trading is the unique equilibrium

- Thus, if $\kappa \geq \frac{R_H - R_L}{3} \equiv \kappa_{NT}$, then no trading is the unique equilibrium as every deviation generates smaller expected payoff.
- The threshold is the same as that in the no feedback case, as with no trading, prices are completely uninformative so there cannot be any feedback.

Incorporating feedback - when BNS can be an equilibrium

- If the informed trader chooses BNS, then the informed trader can either deviate to always trading, no trading or SNB.
- The payoff from choosing BNS is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^{BNS}(+2)) + \frac{1}{3}(R_H - P^{BNS}(+1)) \\
& + \frac{1}{3}(R_H - P^{BNS}(0)) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa].
\end{aligned}
$$

Incorporating feedback - when BNS can be an equilibrium

- The payoff to deviating to always trading is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^{BNS}(+2)) + \frac{1}{3}(R_H - P^{BNS}(+1)) \\
& + \frac{1}{3}(R_H - P^{BNS}(0)) - \kappa] \\
& + \frac{1}{2}[\frac{1}{3}(P^{BNS}(-2) - R_L - x + c) + \frac{1}{3}(P^{BNS}(-1) - R_L - x + c) \\
& + \frac{1}{3}(P^{BNS}(0) - R_L) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] \\
& + \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa]
\end{aligned}
$$

Incorporating feedback - when BNS can be an equilibrium

- Since $\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) < \frac{1-\lambda}{2-\lambda}(R_H - R_L) < \frac{1}{2}(R_H - R_L)$, when $\kappa \in [\frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{3}\frac{1}{2}(R_H - R_L), \frac{1}{3}(R_H - R_L))$, BNS is an equilibrium. Denote $\kappa_T \equiv \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{3}\frac{1}{2}(R_H - R_L)$, the condition is $\kappa \in [\kappa_T, \kappa_{NT})$.
- Note that $\kappa_T < \kappa_{NF}$ so with feedback BNS is more likely to be an equilibrium. Intuition: incorporating feedback reduces the expected profit from trading on negative information (as firms make decisions to improve their value, firm value increases from R_L to $R_L + x - c$ when observing $X = -1$, which makes deviating to selling even less attractive with feedback.

Incorporating feedback - when SNB can be an equilibrium

- If the informed trader chooses SNB, then the informed trader can either deviate to always trading, no trading or BNS.
- The payoff from choosing SNB is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(P^{SNB}(-2) - R_L - x + c) + \frac{1}{3}(P^{SNB}(-1) - R_L) \\
& + \frac{1}{3}(P^{SNB}(0) - R_L) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa].
\end{aligned}
$$

Incorporating feedback - when SNB can be an equilibrium

- The payoff to deviating to always trading is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^{SNB}(+2)) + \frac{1}{3}(R_H + x - c - P^{SNB}(+1)) \\
& + \frac{1}{3}(R_H - P^{SNB}(0)) - \kappa] \\
& + \frac{1}{2}[\frac{1}{3}(P^{SNB}(-2) - R_L - x + c) + \frac{1}{3}(P^{SNB}(-1) - R_L) \\
& + \frac{1}{3}(P^{SNB}(0) - R_L) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] \\
& + \frac{1}{2}[0 + \frac{1}{3}\frac{1}{2}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa].
\end{aligned}
$$

Incorporating feedback - when SNB can be an equilibrium

- The payoff to deviating to choosing BNS is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^{SNB}(+2)) + \frac{1}{3}(R_H - P^{SNB}(+1)) \\
& + \frac{1}{3}(R_H - P^{SNB}(0)) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa]
\end{aligned}
$$

and the payoff to deviating to no trading is zero.

Incorporating feedback - when SNB can be an equilibrium

- Note that it is not always true that $\frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x) < \frac{1}{2}(R_H - R_L)$. If this holds, then when $\kappa \in [\frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x) + \frac{1}{3}\frac{1}{2}(R_H - R_L), \frac{1}{3}(R_H - R_L))$, SNB is an equilibrium. Denote $\kappa_{SNB} \equiv \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x) + \frac{1}{3}\frac{1}{2}(R_H - R_L)$, the condition is $\kappa \in \left[\kappa_{SNB}, \kappa_{NT}\right)$. Clearly $\kappa_{SNB} > \kappa_{NF}$ so with feedback SNB is less likely to be an equilibrium (and may not exist if $\kappa_{SNB} \geq \kappa_{NT}$, i.e., when $\frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x) \geq \frac{1}{2}(R_H - R_L)$).
- Intuition: incorporating feedback reduces the expected profit from trading on negative information as discussed above but increases the expected profit from deviating to trading on positive information as firms learn from $X = +1$ and choose $d = +1$ to further increase firm value (the firm value increases from R_H to $R_H + x - c$ when $X = +1$).

No feedback benchmark - when always trading can be an equilibrium

- If the informed trader chooses always trading, then the informed trader can either deviate to BNS, SNB or no trading.

No feedback benchmark - when always trading can be an equilibrium

- The payoff from always trading is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^T(+2)) + \frac{1}{3}(R_H + x - c - P^T(+1)) \\
& + \frac{1}{3}(R_H - P^T(0)) - \kappa] \\
& + \frac{1}{2}[\frac{1}{3}(P^T(-2) - R_L - x + c) + \frac{1}{3}(P^T(-1) - R_L - x + c) \\
& + \frac{1}{3}(P^T(0) - R_L) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] \\
& + \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{3}\frac{1}{2}(R_H - R_L) - \kappa] \\
= \quad & \frac{1}{2}(\kappa_T - \kappa) + \frac{1}{2}(\kappa_{SNB} - \kappa)
\end{aligned}
$$

No feedback benchmark - when always trading can be an equilibrium

- The payoff to deviating to choosing BNS is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(R_H + x - c - P^T(+2)) + \frac{1}{3}(R_H + x - c - P^T(+1)) \\
& + \frac{1}{3}(R_H - P^T(0)) - \kappa] \\
= \; & \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L) \\
& + \frac{1}{3}\frac{1}{2}(R_H - R_L + 2x) - \kappa] \\
= \; & \frac{1}{2}(\kappa_{SNB} - \kappa).
\end{aligned}
$$

No feedback benchmark - when always trading can be an equilibrium

- The payoff to deviating to choosing SNB is

$$
\begin{aligned}
& \frac{1}{2}[\frac{1}{3}(P^T(-2) - R_L - x + c) + \frac{1}{3}(P^T(-1) - R_L - x + c) \\
& + \frac{1}{3}(P^T(0) - R_L) - \kappa] \\
= \quad & \frac{1}{2}[0 + \frac{1}{3}\frac{1-\lambda}{2-\lambda}(R_H - R_L) + \frac{1}{3}\frac{1}{2}(R_H - R_L - 2x) - \kappa] \\
= \quad & \frac{1}{2}(\kappa_T - \kappa).
\end{aligned}
$$

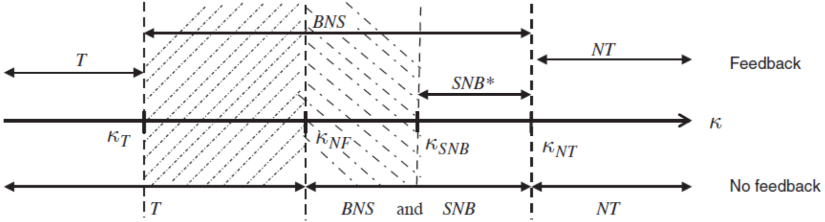
  and the payoff to deviating to no trading is zero.

- Since $\kappa_T < \kappa_{SNB}$, when $\kappa < \kappa_T$, trading strategy generates the highest expected payoff and is the unique equilibrium.

Incorporating feedback - summarizing the main results

- When $\kappa < \kappa_T$, the informed trader always trade; when $\kappa \geq \kappa_{NT}$, the informed trader always not trade; when $\kappa \in [\kappa_T, \kappa_{NT})$, BNS always exists and when $\kappa \in [\kappa_{SNB}, \kappa_{NT})$ (which may be empty), SNB exists as well.
- $\kappa_T < \kappa_{NF} < \kappa_{SNB}$ so the region of BNS expands and the region of SNB shrinks when incorporating feedback.
- Intuition driven by the fact that investment adjustments from learning from stock prices always increase firm value: even better for trading on positive information but makes it worse for trading on negative information -> therefore the asymmetry in trading on positive versus negative information
- This is an endogenous higher cost of short-selling.

Incorporating feedback -graphical illustration



BNS is the unique equilibrium with feedback but does not exist without feedback.

BNS is the unique equilibrium with feedback but coexists with *SNB* without feedback.

*Region disappears if $\kappa_{SNB} \geq \kappa_{NT}$

FIGURE 1. PARAMETER RANGES FOR EQUILIBRIA WITH AND WITHOUT FEEDBACK

EGJ - implications on beliefs

- Since the informed trader is less willing to trade on negative information (i.e., BNS is more pervasive), the manager's posterior belief change is smaller with negative information versus positive information, conditional on the informed trader being present (Proposition 2).
- To see this, we focus on BNS. Denote $q(X) = \Pr(\theta = H|X)$. From the discussion above we have

$$q(-2) = 0, q(-1) = \frac{1-\lambda}{2-\lambda}, \; q(0) = q(+1) = \frac{1}{2} \text{ and } q(+2) = 1.$$

EGJ - implications on beliefs

- Therefore when the positively informed speculator is present (and trades),

$$q^{H,spec} = \frac{1}{3}[q(0) + q(+1) + q(+2)] = \frac{2}{3} > \frac{1}{2},$$

and when the negatively informed speculator is present (and does not trade),

$$q^{L,spec} = \frac{1}{3}[q(-1) + q(0) + q(+1)] = \frac{1-\lambda}{6-3\lambda} + \frac{1}{3} < \frac{1}{2}.$$

- In addition,

$$|q^{H,spec} - \frac{1}{2}| - |\frac{1}{2} - q^{L,spec}| = \frac{1-\lambda}{6-3\lambda} > 0$$

and decreasing in λ as the derivative with respect to λ is $-\frac{1}{3(2-\lambda)^2} < 0$, which is intuitive as when λ increases the manager is more likely to distinguish between a negatively informed trader who chooses not to trade versus an uninformed trader.

EGJ- implications on short-term stock returns (before all uncertainties are realized)

- The magnitude of the short-term stock return upon the positive speculator being present is larger than that of the negative speculator being present (Proposition 3)
- To see this, again focus on BNS. From the discussions above we have

$$
\begin{aligned}
P(+2) &= R_H + x - c,\ P(+1) = P(0) = \frac{R_H + R_L}{2}, \\
P(-1) &= \frac{1-\lambda}{2-\lambda}(R_H - x - c) + \frac{1}{2-\lambda}(R_L + x - c).
\end{aligned}
$$

EGJ- implications on short-term stock returns (before all uncertainties are realized)

- Therefore at the very beginning,

$$
\begin{aligned}
P_0 &= \frac{\lambda}{2}\frac{1}{3}(P(+2)+P(+1)+P(0)) \\
&\quad +(1-\frac{\lambda}{2})\frac{1}{3}(P(-1)+P(0)+P(+1)) \\
&= \frac{3R_H+3R_L-2c+2\lambda x}{6}.
\end{aligned}
$$

- Note that P_0 increases in λ as P_0 incorporates the possibility of firm value increase due to learning from the market so increases in the probability of an informed speculator being present.

EGJ- implications on short-term stock returns (before all uncertainties are realized)

- So the short-term return when a speculator is positively informed is present (and trades) will be

$$\begin{aligned} P_1^{H,spec} - P_0 &= \frac{1}{3}(P(+2) + P(+1) + P(0)) - P_0 \\ &= \frac{R_H - R_L + 2(1-\lambda)x}{6} > 0, \end{aligned}$$

  and the short-term return when a speculator is negatively informed is present (and does not trade) will be

$$\begin{aligned} P_1^{L,spec} - P_0 &= \frac{1}{3}(P(-1) + P(0) + P(+1)) - P_0 \\ &= -\frac{\lambda}{6}[\frac{1}{2-\lambda}(R_H - R_L) + \frac{1-\lambda}{2-\lambda}2x] < 0. \end{aligned}$$

EGJ- implications on short-term stock returns (before all uncertainties are realized)

- The difference in the magnitude of the short-term return is therefore

$$\begin{aligned} & |P_1^{H,spec} - P_0| - |P_1^{L,spec} - P_0| \\ = \ & \frac{(1-\lambda)}{3(2-\lambda)}(R_H - R_L) > 0, \end{aligned}$$

and decreasing in λ as the derivative with respect to λ is proportional to $-\frac{1}{(2-\lambda)^2}$.

Asymmetric trading continued

- EGJ show that when the speculator is always informed but not always present, feedback effect facilitates buying and discourages short-selling.
- However, when the speculator may not be always informed but always present, Goldstein and Gumbel (2008, Review of Economics Studies) show that feedback effect facilitates the uninformed to short-sell but not buy.
- Intuition: even though the speculator is uninformed, the speculator still has information advantage over the manager (as the speculator knows he or she is uninformed but the manager is not). Therefore, the speculator can short-sell, letting the manager believe that this is driven by a negatively informed speculator and chooses corrective actions -> such actions turns out to be value-destroying when the firm's projects do not need any correction in the absence of any info -> price drops and the speculator benefits -> manipulative short-selling.

Asymmetric trading continued

- The negatively informed speculator will not benefit from buying as buying results in the manager believe that this is driven by positively informed speculator and chooses corrective actions -> such actions turns out to be value-destroying when the firm's projects do not need any correction in the absence of any info -> price drops and the speculator losses.
- You will solve a simplified version of this problem in your HW2.
- For the manipulative short-selling to be viable, manager has to believe that the short-selling is driven by a negatively informed speculator -> the probability of an informed speculator being present need to be sufficiently high -> banning all short-selling including informed destroys firm value.
- Gao, Jiang and Lu (2023) show that when introducing strategic complementaries among investors (to be learned next year), banning all short-selling can be beneficial.

Next class

- Introduce firm disclosure to the feedback effect (in this class the manager is always uninformed so there is nothing to disclose)
- So need to introduce manager's information set.
- It turns out that the correlation between manager's information set and the investor's information set plays an important role in the effect of disclosure with feedback effect.