# Overview of tax incidence and Harberger 1962

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# Outline

Overview of tax incidence (based on Fullerton and Metcalf 2002)

Simple examples of tax incidence in a partial equilibrium model

Static models of tax incidence - Harberger (1962)

# What is tax incidence

- Fundamental question: which party bears the tax burden of a particular tax (income tax, corporate tax, sales tax, etc.)? In other words, how does a particular tax policy affect the welfare of everybody in the society?
- Where can the burden lie:
  - Higher product prices (burden goes to the consumers)
  - Lower wages (burden goes to the employees)
  - lower rates to other production factors (capital, land, etc., burden goes to the owners of those factors)

# What is tax incidence

- Types of models:
  - Partial equilibrium models (a tax is imposed but agnostic about how the tax revenue will be used; only consider the choice of a firm without considering the choice of consumers)
  - General equilibrium models (usually not tractable and have to rely on numerical solutions)

# General insights

- In partial equilibrium models, tax buden depends on the factor supply elasticity relative to factor demand elasticity.
  - The higher supply elasticity relative to demand elasticity, the less the burden falls on the supplier of factor and the more the burden falls on the demander of the factor.
- A most commonly used general equilibrium model consists of two products, two sectors (a corporate sector and a non-corporate sector) and two production factors (capital and labor)
  - Economists usually model corporate income taxes as a tax on capital factor in the corporate sector.
  - The main question is whether burden falls only on capital or labor shares some burden (Harberger 1962 is the first attempt).

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# Incidence of a wage tax

- Consider a firm that hires labor to produce goods that are sold at constant unit price $p$. The competitive wage $w$ is determined by equating labor supply and labor demand, that is, equilibrium wage $w^*$ is determined by $L^S(w^*) = L^D(w^*)$.

- Now considering a wage tax, that is, companies bear a wage cost of $w(1+\tau)$, where $\tau$ is the wage tax rate.
- Of course, labor becomes more expensive for the firm, resulting in firm demanding less labor, which reduces the wage rate, that is, worker will bear some of the tax burden.
- How much burden worker bears depends on the supply and demand elasticity.

# incidence of a wage tax- continued

- Recall the definition of demand and supply elasticity: $\eta^S = \frac{d \ln L^S}{d \ln w} = \frac{\frac{dL^S}{L^S}}{\frac{dw}{w}} = \frac{\widehat{L}^S}{\widehat{w}} \geq 0$ and $\eta^D = \frac{d \ln L^D}{d \ln[w(1+\tau)]} = \frac{\frac{dL^D}{L^D}}{\frac{dw}{w} + \frac{d\tau}{1+\tau}} = \frac{\widehat{L}^D}{\widehat{w} + \widehat{\tau}} \leq 0$. Therefore $\widehat{L}^S = \eta^S \widehat{w}$ and $\widehat{L}^D = \eta^D (\widehat{w} + \widehat{\tau})$.
- $\widehat{L}^S = \widehat{L}^D$ results in $\frac{\widehat{w}}{\widehat{\tau}} = \frac{\eta^D}{\eta^S - \eta^D} \in (-1, 0)$. Therefore some but not all of the burden is shifted to the labor.
- The higher $\eta^S$ is and/or the lower $|\eta^D|$ is, the less the burden is shifted to the labor.
- When $\eta^S \to +\infty$ (perfect elastic supply) or $\eta^D \to 0$ (perfect inelastic demand), labor bears no burden.
- When $\eta^S \to 0$ (perfect elastic supply) or $\eta^D \to -\infty$ (perfect inelastic demand), labor bears complete burden.

# Incidence of a corporate income tax modelled in the usual way

- Consider a firm that hires labor to produce goods that are sold at constant unit price $p$. The competitive wage $w$ is determined by equating labor supply and labor demand, that is, equilibrium wage $w^*$ is determined by $L^S(w^*) = L^D(w^*)$. The production function is $F(L)$.
- Now considering a corporate income tax, that is, companies obtain an after-tax profit of $(1-\tau)[F(L) - wL]$. This formulation will also endogenizes the labor demand function.
- First order condition results in $F_L(L^*) = w$, which implicitly defines the labor demand function $L^D(w)$
- Labor supply is still $L^S(w)$, and market clearing condition $L^D(w^*) = L^S(w^*)$ determines the equilibrium wage.
- In this case both $L^D$ and $L^S$ is independent of $\tau$. Therefore $w^*$ is also independent of $\tau$, that is, the firm bears all the burden.

# Incidence of a corporate income tax modelled in the usual way, continued

- Intuition: pure profit tax reduces the marginal benefit of hiring labor and the marginal cost of hiring labor proportionally, resulting in no distortion to labor choice and thus equilibrium wage.
- Similar logic will apply if the firm uses both capital and labor to produce and tries to maximize an after-tax profit of $(1-\tau)[F(K,L) - rK - wL]$, implying that both wage expense and capital rental expense are fully deductible.
- The insight that pure profit tax results in zero incidence on production factors first illustrated in Stiglitz 1973 Journal of Public Economics.

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# Setup of Harberger 1962

- A two-sector general equilibrium model with two production factors (capital $K$ and labor $L$).
- There are two goods, $X$ and $Y$, produced with the two factors with constant return to scale production function: $X = f(K_x, L_x)$ and $Y = g(K_y, L_y)$.
- Each factor has a fixed supply but can freely move between sectors, $K_x + K_y = \overline{K}$ and $L_x + L_y = \overline{L}$.
- Harberger studies the incidence of a tax on (earnings of) capital in one sector (denoted the corporate sector), that is, how the price of capital and labor changes with the imposition of the tax.
- Assume without loss of generality that before the imposition of the tax, $p_x = p_y = 1$.

# Key equations- equation (1)

- Reduced form of demand for $X$: $E$, the definition of (relative) price elasticity of demand for $X$, is defined as

$$E = \frac{\frac{dX}{X}}{\frac{d(\frac{p_X}{p_Y})}{\frac{p_X}{p_Y}}} = \frac{\frac{dX}{X}}{\frac{\frac{p_Y \, dp_X - p_X \, dp_Y}{p_Y^2}}{\frac{p_X}{p_Y}}} = \frac{\frac{dX}{X}}{\frac{dp_X}{p_X} - \frac{dp_Y}{pY}} = \frac{\frac{dX}{X}}{dp_X - dp_Y},$$

resulting in

$$\frac{dX}{X} = E(dp_X - dp_Y),$$

which is equation (1) in Harberger (1962). Note that $E < 0$.

# Key equations- equation (2)

- Assuming that $f$ is homogenous of degree 1 (Cobb-Douglas production function $K^{\alpha}L^{\beta}$ with $\alpha+\beta=1$ being a special case), then differentiate $X=f(K_x,L_x)$ results in

$$dX=\frac{\partial f}{\partial K_x}dK_x+\frac{\partial f}{\partial L_x}dL_x,$$

  and

$$\frac{dX}{X}=\frac{\frac{\partial f}{\partial L_x}}{f(K_x,L_x)}dL_x+\frac{\frac{\partial f}{\partial K_x}}{f(K_x,L_x)}dK_x. \tag{1}$$

- Since for any $t$,

$$f(tK_x,tL_x)=tf(K_x,L_x)$$

  differentiate with respect to $t$ and setting $t=1$ results in

$$\frac{\partial f}{\partial K_x}K_x+\frac{\partial f}{\partial L_x}L_x=f(K_x,L_x)$$

# Key equations- equation (2), continued

- Denote $f_K = \frac{\frac{\partial f}{\partial K_x} K_x}{f(K_x, L_x)}$ and $f_L = \frac{\frac{\partial f}{\partial L_x} L_x}{f(K_x, L_x)}$ as the share of capital and labor in the production function, respectively, then inserting into equation (1) results in

$$\frac{dX}{X} = f_L \frac{dL_x}{L_x} + f_K \frac{dK_x}{K_x},$$

  which is equation (2) in Harberger (1962).

# Key equations - equation (3),(4) and (3'),(4')

- The elasticity of subsitution (between production factors) for producing $Y$ is defined as

$$S_y = \frac{\frac{d(\frac{K_y}{L_y})}{\frac{K_y}{L_y}}}{\frac{d(\frac{p_k}{p_l})}{\frac{p_k}{p_l}}} = \frac{\frac{\frac{L_y\,dK_y - K_y\,dL_y}{L_y^2}}{\frac{K_y}{L_y}}}{\frac{\frac{p_l\,dp_k - p_k\,dp_l}{p_l^2}}{\frac{p_k}{p_l}}} = \frac{\frac{dK_y}{K_y} - \frac{dL_y}{L_y}}{\frac{dp_k}{p_k} - \frac{dp_l}{p_l}} = \frac{\frac{dK_y}{K_y} - \frac{dL_y}{L_y}}{dp_k - dp_l},$$

  which is equivalent to

$$\frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y(dp_k - dp_l),$$

  that is, equation (3') of Harberger (1962). Note that $S_y < 0$.
- For producing $X$, we similarly have

$$\frac{dK_x}{K_x} - \frac{dL_x}{L_x} = S_x(dp_k + T - dp_l),$$

  where $T$ is the amount of tax per unit of capital, that is, equation (4') of Harberger (1962). Note that $S_x < 0$.

# Further simplifying the system of equations

- We now have four equations and nine unknowns: $dX$, $dp_x$, $dp_y$, $dL_x$, $dL_y$, $dK_x$, $dK_y$, $dp_l$ and $dp_k$. Need five more equations.
- Three are straightforward: since $K_x + K_y$ and $L_x + L_y$ are both constants, we have $dK_y = -dK_x$ and $dL_y = -dL_x$ (equations (5) and (6) in Harberger 1962).
- We can always choose a numeraire good (that is, price set equal to 1). Harberger picks labor as the numeraire good, that is, $p_l$ is always equal to 1, so $dp_l = 0$, that is, equation (9) in Harberger 1962.

# Further simplifying the system of equations, continued

- Two comes from constant return to scale of production function: constant returns to scale implies an economic profit of zero for both sectors, that is, we always have $p_y Y = p_l L_y + p_k K_y$. Differentiating results in

$$p_y dY + Y dp_y = p_l dL_y + L_y dp_l + p_k dK_y + K_y dp_k. \tag{2}$$

- Note that

$$dY = \frac{\partial g}{\partial K_y} dK_y + \frac{\partial g}{\partial L_y} dL_y, \tag{3}$$

and first order condition results in

$$p_y \frac{\partial g}{\partial K_y} = p_k, \tag{4}$$

and

$$p_y \frac{\partial g}{\partial L_y} = p_l. \tag{5}$$

# Further simplifying the system of equations, continued

- Insert equation (4) and (5) into equation (3) results in

$$p_y dY = p_k dK_y + p_l dL_y \tag{6}$$

- Insert equation (6) into equation (2) results in

$$Ydp_y = L_y dp_l + K_y dp_k,$$

which, when dividing by $Y$, results in

$$dp_y = \frac{L_y}{Y} dp_l + \frac{K_y}{Y} dp_k = g_L dp_l + g_K dp_k,$$

which is equation (8) in Harberger (1962). We can similarly get

$$dp_x = f_L dp_l + f_K (dp_k + T),$$

which is equation (7) in Harberger (1962).

# Final results

- We thus have nine equations with nine unknowns and we can solve for the answer to the incidence question, $dp_k$, to be (equation (12) of Harberger (1962)):

$$dp_k = \frac{Ef_K(\frac{K_x}{K_y} - \frac{L_x}{L_y}) + S_x(\frac{f_L K_x}{K_y} + \frac{f_K L_x}{L_y})}{E(g_K - f_K)(\frac{K_x}{K_y} - \frac{L_x}{L_y}) - S_y - S_x(\frac{f_L K_x}{K_y} + \frac{f_K L_x}{L_y})} T.$$

- The sign of the denominator is positive as the last two terms are positive and the first term is positive as $E < 0$ and $g_K > f_K$ if and only if $\frac{K_y}{Y} > \frac{K_x}{X}$, or, equivalently, $\frac{K_y}{K_x} > \frac{Y}{X} > \frac{L_y}{L_x}$ so $(g_K - f_K)(\frac{K_x}{K_y} - \frac{L_x}{L_y}) < 0$, as $\frac{Y}{X}$ has to lie between $\frac{K_y}{K_x}$ and $\frac{L_y}{L_x}$ due to homogenous of degree one.

# Final results, continued

- Therefore $sgn(dp_k) = sgn(Ef_K(\frac{K_x}{K_y} - \frac{L_x}{L_y}) + S_x(\frac{f_L K_x}{K_y} + \frac{f_K L_x}{L_y}))$. The second term is clearly negative.
- $dp_k > 0$ only if $\frac{K_x}{K_y} - \frac{L_x}{L_y} < 0$, that is, labor bears more of the burden if $X$ is more labor intensive (result 1). Intuition: A tax on $K_x$ results in a reduced demand of sector $X$ for both capital and labor, implying that sector $Y$ has to absorb more capital and labor. However, since $Y$ is less labor intensive, $Y$ can only absorb more labor if labor prices fall relatively more, that is, labor bears more of the burden.
- $dp_k \rightarrow 0$ when $S_y \rightarrow -\infty$, that is, labor and capital bear the same burden if sector $Y$ is perfectly inelastic between capital and labor (result 4). Intuition: infinite elasticity makes it costless for sector $Y$ to absorb more capital and labor without imposing undue burden on either factor.

# Final results, continued

- $dp_k \rightarrow -T$ when $S_x \rightarrow -\infty$, that is, capital price reduces by the full tax amount if sector $X$ is perfectly inelastic between capital and labor (result 5). Intuition: infinite elasticity makes it costless for sector $X$ to switch from capital to labor, resulting in capital price reduced by the full tax amount.
- $dp_k \rightarrow -T\frac{K_x}{K_x+K_y}$ when $S_x = S_y = E\frac{X+Y}{Y}$, that is, capital bears the full burden if all elasticities of subsitutions are the same (result 10). Intuition: same elasticities implies that there is no way for capital of the taxed sector to shift the burden to labor, resulting in capital bearing the full burden.

# Takeaways from Harberger (1962)

- First general equilibrium analysis of tax incidence: his subsequent matching to data shows that with reasonable elasticity coefficients, tax burden is more likely to be born entirely by capital.
- Kind of the Modigliani- Miller of tax incidence. Shows under what assumptions tax incidence will be born entirely by capital. Harberger (1962) assumes
  - fixed capital stock
  - closed economy
  - no financing decisions (no debt-equity choice and thus no interest tax shield)
  - no uncertainty
- See section 2 of Fullerton and Metcalf 2002 for a nice discussion of a more general model that incorporates Harberger (1962) as a special case.