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A Game-Theoretic Analysis of Skyscrapers, Helsley and Strange (2008)

Skyscrapers have received little attention from urban economists in the past according to Helsley and Strange (2008). What has been discussed relates to their place in the standard urban model, otherwise known as the monocentric city model. In this model, skyscrapers are attributed to the phenomenon of increasing land prices as one approaches the city center. Due to these higher prices, buildings are built up in order to save land costs. In their study, Helsley and Strange (2008) argue that higher land prices are not the only reason for the stratospheric height of these manmade marvels. To builders, the height (and relative height) of their building carries importance for issues of publicity and pride.

In order to capture this importance of building the highest building in any market, the study uses game theory to simulate a skyscraper building contest and then continues on to explain an equation that models overbuilding. Building profits are still a determinant in the builder's payoff function (as they are in the monocentric city model); however in this particular case whether or not the builder has succeeded in constructing the tallest building becomes part of the function as well. They describe their model as an "all-pay auction" (Helsley and Strange, 2008). That is, builders spend their resources and he or she who bids the highest takes the prize. This prize does come at a cost; its value is partially diminished through the bad economics of skyscrapers. The article proposes two situations: simultaneous and sequential construction. In the simultaneous game theoretic model, no contestant gains any value from competing in the game except for the builder who constructs the highest skyscraper. He or she will enjoy the value of this prize; however dissipation occurs for reasons that will be discussed. In the sequential model, the cost comes before construction "where the leader builds a tall-enough building to deter competitors" (Helsley and Strange, 2008). They use the story of the Empire State Building as evidence for this model.

Introduction of Variables

The model first introduces a situation in which two risk-neutral builders exist, $i = 1, 2$. Both builders own land on which to build, however we assume that builder 1 possesses a better location. This causes $h_1^* > h_2^*$, where h_i^* is defined as profit maximizing building height. Because value is given to height in this situation, the model adds some exogenous variable $v > 0$ to include building height. This is multiplied by the indicator variable, δ, which is given a value of 1 if the builder succeeds in building the tallest building in his or her market or 0 if he or she does not succeed. This gives the equation

$$\delta v + \pi_i (h_i),$$

where π_i is builder i's profit and h_i is building height. Building height can be expressed as equation

$$v + \pi_i (h_i^P) = \pi_i (h_i^*).$$

The variable h_i^P represents what Helsley and Strange (2008) call a "pre-emption"; that is, "if a rival builder j chose height $h_j \geq h_i^P$, builder i would concede the contest because it would never be in the builder's interest to choose a height that would win". In other words, when $h_i > h_i^P$, $v + \pi_i (h_i^P) < \pi_i (h_i^*)$. This expression also explains the prize dissipation experienced in a skyscraper contest. The contest payout will be less than the profit-maximized payout as shown above. In the following two games, h_1^* is assumed to be less than h_2^P. This is because there would be no competition in the market otherwise.

The Sequential Game Theoretic Model

In the sequential model, h_i is chosen sequentially. The model proposes that builder 1 goes first with the strategy to choose a height such that $h_1 \geq h_2^P$ so that builder 2 will surrender to building at his profit maximization height ($h_2 = h_2^*$). His or her pre-emption will not surpass builder 1's actual building height. If builder 1 chooses a height such that $h_1 \leq h_2^* \leq h_2^P$, the equation $h_2 = h_2^*$ still holds because builder 2 will continue to build at his or her profit-maximizing height since this will win the competition and be economically rational. If $h_2^* \leq h_1 \leq h_2^P$ then builder 2 will just top builder 1's height in order to win the competition. Therefore, Helsley and Strange (2008) propose it is better for builder 1 to win the contest by surpassing h_2^P to achieve $v + \pi_1 (h_2^P)$. In the sequential game theoretic model, a significant cost is the choice to build high enough that no other competitor will choose to build higher. This can lead to overbuilding, which will be discussed later.

The Simultaneous Game Theoretic Model and Overbuilding

The simultaneous model considers a game in which two builders choose building height simultaneously. Helsley and Strange (2008) propose that in this case, a positive probability weight (ϕ) on is placed on building height h_i. This represents the probability that builder i will build at a height less than or equal to h. The payoff for this situation can be expressed as $\pi(h^*)$, the profit maximizing situation. The model proposes that builder i's payoff is equal to $\phi(h_i)v + \pi(h_i)$. This expression is claimed to represent the probability that a builder wins choosing a height multiplied by the value of the prize in addition to the value of the building. This gives the general equation

$$\phi_j(h_i)v + \pi(h_i) = \pi(h^*)$$

for which $j = 1, 2$ and h_i is an element of h^* and h^P. This can be rearranged to be

$$\phi_j(h_i) = [\pi(h^*) - \pi(h_i)]/v.$$

Helsley and Strange (2008) then differentiate with respect to h_i, resulting in

$$\phi(h_i) = -\pi'(h_i)/v.$$

The expected building height can be calculated by integrating by parts the derivation of this equation. The equation is set up as

$$E[h] = \int_{h^*}^{h^P} -(\pi'(h)/v)h\,dh.$$

Helsley and Strange (2008) claim that integration by parts yields

$$E[h] = h^* + \frac{1}{v}\left[\int_{h^*}^{h^P} \pi(h)dh - \pi(h^P)(h^P - h^*)\right] > h^*,$$

knowing $v + \pi_i(h_i^P) = \pi_i(h_i^*)$. Through this equation, it is easy to see that overbuilding occurs. Expected building height, $E[h]$, is greater than profit maximizing building height h^* (the expression in brackets remains positive as $\pi(h)$ is decreasing on the interval $[h^*, h^P]$ according to Helsley and Strange (2008)).

Implications

Helsley and Strange (2008) present new work in the study of these urban marvels. They come to the conclusion that skyscrapers are not economical when they are built in a contest to reach the highest altitude. Their research includes plenty of historical data, citing stories of the construction of many of the world's tallest skyscrapers. Some of them, such as the Burj Dubai, fulfill the proposed overbuilding prophecy. It has been hard for them to find enough tenants for their space. The game theoretic model alerts builders of the poor economics of building for height. The authors stress that this can impact the real estate market and its cycles, contributing to "increases in vacancies and declines in rents, leading to subsequent slowdowns" (2008). In fact, they cite that, when built, the Empire State Building, the Manhattan Company Building, and the Chrysler Building brought a whopping 4,000,000 extra square feet of commercial space to New York City (20% of the city's stock) (2008). Perhaps further research might be done to measure the impact of the construction of a skyscraper on a city's real estate market. How does it change prices? Also how does pricing within a skyscraper itself change?

Bibliography

Robert Helsley and William Strange, 2008, "A game-theoretic analysis of skyscrapers," Journal of Urban Economics 64: 49-64.