Traditionally, economic theorists have proposed models that disregard any information bias with the purpose of simplifying a situation. This paper analyzes competitive markets in context of the exchange of commodities where at least one of the parties does not have full or equal knowledge. When the concept of imperfect information is applied to various economics models, many of their market equilibriums cease to exist. In context of the insurance market, an equilibrium that disregards information bias often does not exist or would lead to the bankruptcy of a given firm. Although when imperfect information is considered, it is found that there are three equilibria that each rely on a price and quantity, rather than a price alone. Further calculations show that if individuals were willing and able to reveal their information, everyone could be made better off. In absence of high-risk consumers, both the low-risk individuals and firm would benefit, thus identifying high-risk individuals as inducers of one-sided externalities on the system. This parasitic relationship caused by the imperfect information is directly responsible for the pitfall of many economic theories.

Consider a situation in which an individual has income $W$ and is subject to an accident with risk $p$ and cost $d$. The individual can protect himself against an outcome of $W - d$ by purchasing insurance at a premium of $\alpha_1$ and pay out $\alpha_2$, effectively transferring the risk onto a firm. This means that his possible outcomes in the two states (no accident, accident) are:

Without Insurance: $(W, W - d)$  
With Insurance: $(W - \alpha_1, W - d + \alpha_2)$

The insurance market is founded on the trade of premium ($\alpha_1$) and payouts ($\alpha_2$). The main principle upon which this market exists is that the individual is risk averse and would prefer to consume less in period 1 if it meant guaranteeing greater consumption in period 2. Under this assumption, it is the consumers’ objective to maximize total utility $V$, where $W_1$ is income without an accident, $W_2$ is the income if such event occurs, $U(W)$ is the utility as a function of income and $p$ is the probability of the accident.

$$V(p, W_1, W_2) = (1 - p)U(W_1) + pU(W_2)$$
This means that a contract offered by an insurance company will be worth:

\[ V(p, \alpha) = V(p, W - \alpha_1, W - d + \alpha_2) \]

Of all the possible contracts, the individual will chose the one that maximizes \( V(p, \alpha) \). This means that it must be that \( V(p, \alpha) \geq V(p, 0) = V(p, W, W - d) \). Although, it is assumed that the individual always purchases insurance because they are risk averse, making preferences such that \( U'' < 0 \), meaning \( V(p, \alpha) \) is quasi-concave. A similar analysis reveals the actions of a typical firm.

It is assumed that the firm is risk neutral and is only concerned with profit given by the equation:

\[ \pi(p, \alpha) = (1 - p)\alpha_1 - p\alpha_2 = \alpha_1 - p(\alpha_1 + \alpha_2) \]

Firms are considered risk neutral due to their ability to sell any number of contracts that they think will generate expected profit. It is also assumed that the market is competitive, with free entry and exit, meaning:

\[ \alpha_1(1 - p) - \alpha_2 p = 0 \]

This indicates that the firm operates under competitive conditions that drive profits to 0, allowing the possibility of an identifiable equilibrium.

Imperfect information is induced into the market via parameter \( p \). It is assumed that individuals know their accident probabilities, while the firm does not. This means that the firm cannot discriminate among customers based on their characteristics. The idea is that high-risk individuals with high values of \( p \) will demand more insurance than lower-risk individuals with lower values of \( p \). It is in the firm’s best interest to know its customers’ characteristics to determine at what premium \( \alpha_1 \) to sell the insurance. One way in which these characteristics are revealed is through the self-selection mechanism. This is where the firm forces the customer to make decisions in such a way that his characteristics are revealed through his choices. With this model in mind, it is possible to identify the equilibria of an insurance market with imperfect information.
Figure 1 represents two individuals with accident probability $p^H$ for high-risk individuals (U^H) and $p^L$ for low risk individuals (U^L). They both have the same endowment $E$. The $45^\circ$ line represents equal income in both periods. Line FE shows the possible actuarially fair rates, with a slope of $(1-p)/p$. This type of a market can only have two equilibria: pooling equilibria in which both groups buy the same contract and separating equilibria where each purchases a different contract. An analysis without the consideration of imperfect information would conclude that equilibrium lies on some point $\alpha$ on the line FE. Although, this is invalid since the low risk consumer can increase their utility by consuming at point $\beta$, where the high risk cannot. This means there must be a separating equilibrium in which each type purchases a separate contract.

Figure 2 shows the separating equilibrium. The low-risk contracts lie on line EL, where the high-risk contracts are one line EH. Notice how the low risk individuals can never be offered a fully insured package at an actuarially fair price, shown by the point $\beta$. Remember that the firm cannot distinguish among its consumers. So if plan $\beta$ were offered, both high and low-risk individuals
would consume at that point, causing the firm to have negative profits. This means that the point $\beta$ is not an equilibrium. Thus, the packages offered to the low-risk group must be contained between points $E$ and $\alpha^L$. This way the high-risk consumers have no incentive to deviate from the contract directed at them. The high-risk individuals will always prefer to fully ensure with a high-risk contract rather than under insure with a low risk contract. Thus, the set $(\alpha^H, \alpha^L)$ is the only possible equilibrium for a competitive market with low and high risk customers. Although, this does not take into account the proportion of high/low risk consumers.

If the proportion is such that the average consumer lies along the line $EF$, the previous equilibrium will hold true. Although, if there are proportionately less high-risk consumers, the line formed by the group as a whole will be $F'E$. In the event $F'E$ is the true proportion, a new equilibrium, $\gamma$, is possible. Since the firm makes profit at this new point and both individuals are benefited, the package will be consumed by both high and low risk individuals. This point $\gamma$ is contained to the area bound by the $U^L$ curve, the $F'E$ proportion line and the 45° line. Although, this point depends on the proportion of high and low risk consumers, meaning in the long run no equilibrium will be reached. The direct reason that this is not an equilibrium is because other firms fail to take into account their own actions on the market. A firm may enter the market and offer a set of contracts that offsets a previously defined equilibrium by attracting low-risk individuals to that firm and away from others. As the proportions of consumers between firms shift, the line $F'E$ changes for other firms such that not point $\gamma$ exists, thus causing the firm to have negative profits. As in any Nash equilibrium, the firm fails to evaluate the consequences of its actions, offering profit-maximizing packages that cause both other firms and itself to reach a long run equilibrium of bankruptcy. Thus, in this situation there is not equilibrium.

A key part of this analysis is to note that the high-risk individuals always exert a negative externality on the low-risk individuals and firms. The externalities are directly dissipated through the losses on part of the low-risk individuals, but the high-risk individuals are not benefited had they been in isolation. If the high-risk consumers would admit to their high accident probabilities, an equilibrium closer to that of a Pareto efficient outcome would be possible, where all individuals would be made better of without making anyone worse off. Although, it is impossible for a competitive market to entirely reach Pareto optimality.
The optimal contract sets will always have solution \((\alpha^H, \alpha^L)\) as shown in the first example. Although, when a contract bundle \(Y\) is possible, we must account for the idea that the high-risk consumers are ‘subsidized’ so to speak by the lower-risk individuals. Hence, the high-risk individuals will always fully ensure so that \(V(p^H, \alpha^H) = U(W - p^Hd + a)\) where \(a\) represents the per capita subsidy of the high-risk by the low-risk. This subsidy decreases \(W_o\) for each low-risk consumer by \(\gamma a\). This extra charge evens out when the low-risk individuals purchase the only contract that is available to them. This additional cost is the cost associated with the externality imposed on the low-risk consumers by the high-risk individuals. This means that \(\alpha^L = (\alpha_1 + \gamma a, \alpha_2 - \gamma a)\) where \(\alpha_1 = \alpha_2 p^L/(1 - p^L)\). Mathematically, the optimal contact is found by choosing \(\alpha\) and \(\alpha^2\) to maximize the equation

\[
U(X)(1 - p^L) + U(Z)p^L
\]
given the constraints

\[
U(Y) \geq U(X)(1 - p^L) + U(Z)p^L
\]
\[
a > 0
\]
where

\[
X = W_o - \gamma a - \alpha_2 p^L/(1 - p^L)
\]
\[
Y = W_o - p^Hd + a
\]
\[
Z = W_o - d - \gamma a + \alpha_2
\]

These equations essentially outline the argument above. The solution is such that when \(a = 0\), and no subsidy is given to the high-risk consumers, there is a separating equilibrium with solution \((\alpha^H, \alpha^L)\) as seen in Figure 2. Although, once \(a > 0\), the problem become more complex, as seen by the analysis of the presence of some point \(Y\) that possibly satisfies a pooling equilibrium. The result is a condition that determines \(X\), \(Y\), and \(Z\) given some proportion of high and low-risk consumers.

\[
\frac{(p^H - p^L)\gamma}{p^L(1 - p^L)} > \frac{U'(Y)[U'(Z) - U'(X)]}{U'(X)U'(Z)}
\]

Although, it is difficult to assign numerical values to many of the variables considered in this analysis, such as a consumer’s utility curve or the probability of incurring an accident. On top of this, such analysis does not take into consideration the aspects of insurance beyond that of analyzing the
presence of high and low risk consumers. Such factors are the idea of signaling and manifestation of moral hazard.

Signaling is the idea that each type of consumer has the ability to share information with the firm that would impact the type of equilibrium reached. In our case, signaling would entail the low-risk individuals proving their low-risk status in order to attain a contract that would better suit their characteristics. By signaling, the expected cost to the firm for offering insurance to the individual decreases, allowing the firm to offer an insurance package more tailored to the needs of the individual, thus increase their utility. Although, it must be that the cost of signaling for the high-risk individual is greater than the benefits received from a low-risk contract, or the high-risk groups will falsely signal to the firm in order to attain the low risk contract. Moral hazard on the other hand is idea that once an individual is fully ensured, they are less cautious and thereby more likely to induce the bad outcome, meaning an unaccounted for increase in $p$. Although most insurance companies have avoided this problem by not offering full coverage while at the same time only agreeing to pay the first set portion of the total cost of the accident. This way there is always some cost to the individual from being under insured and even if the individual has two insurance policies, together they will only cover the first portion of the cost, rather than generating additive coverage for the whole cost of the accident. These are just two examples of additional examples of imperfect information.

With the consideration of imperfect information, single price equilibrium in a conventional competitive analysis is no longer valid. Where equilibrium existed, it consisted of contracts that specified both price and quantity. The form of equilibrium in a model of perfect information depended on a plethora of assumptions, which when reevaluated under the model of imperfect information reached an entirely different equilibrium. Extending this analysis, the new approach proposes the question as to whether many of the other theoretical speculations of economic models really give any insight to actual examples where issues like imperfect information cannot be ignored. There are many other economic models that would warrant reconsideration with the concept of information bias, like that of the financial and labor markets, in which imperfect and asymmetric information play a key role in the final equilibrium. Such an analysis would reveal a greater understanding for why many of the highly valued economic models have crucial pitfalls when applied to the real world.
Bibliography
