Delegated Mechanisms

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Examples of Private contracting

- the transfer of private property
- CEO compensation structuring
- personalized insurance provision

Motivation

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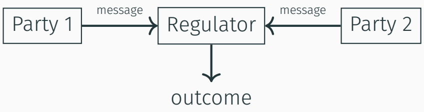
How to regulate without mediating?

A general framework

Bilateral Trade

If regulator intermediates ⇒ Mech Design



The Key Restrictions

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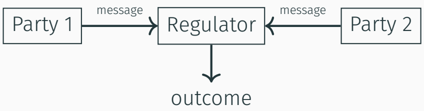
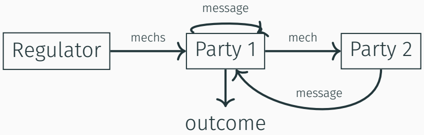


No intermediation ⇒ Delegated Mechs



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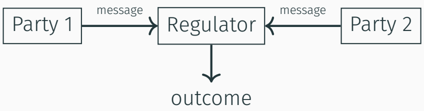

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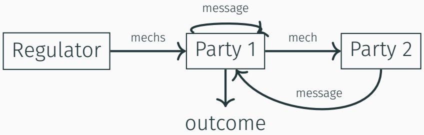
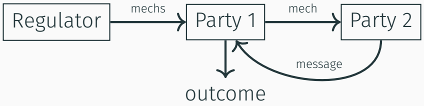


Transparency ⇒ Restricted mechs



Setup

Standard Setup

2 Agents, $i \in \{1, 2\}$

Private type $\theta_i \sim F_i \in \Delta\Theta$

Set of outcomes X

Payoff: $u_i(\theta_i, x)$

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$M = M_1 \times M_2$ set of messages

$g : M \to \Delta X$ outcome function

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i-Mechanism, $\mathcal{M} = (M_{-i}, g)$

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$\mathcal{C}$ induces a **regulated game**:

Agent i picks an i-mechanism from $[\mathcal{M}^\lambda]_{\lambda \in \Lambda}$

Agent $-i$ sends a message from M^λ_{-i}

Definition

A social choice function is implementable through delegation if, for all type profiles, (θ_1, θ_2), $f(\theta_1, \theta_2)$ is a PBE outcome in a regulated game.

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Theorem

A social choice function is implementable through delegation if and only if it is implementable in a grand mechanism in dominant strategies for one of the agents (DSIC-1).

Proof Sketch

Regulated
Game

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Regulated Game

$$\theta_i \xrightarrow{\quad \sigma_i \quad} \Delta\left((\mathcal{M}^{\lambda})_{\lambda \in \Lambda}\right)$$

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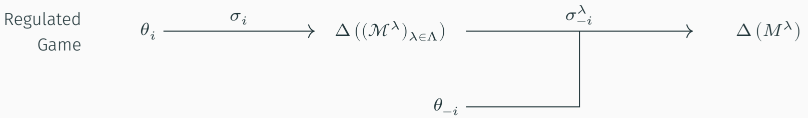

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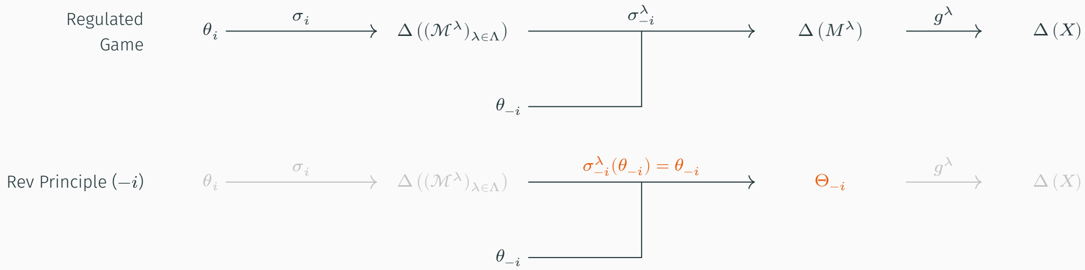

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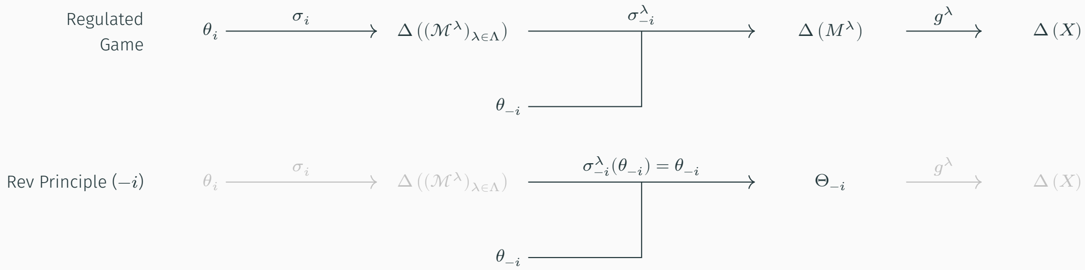

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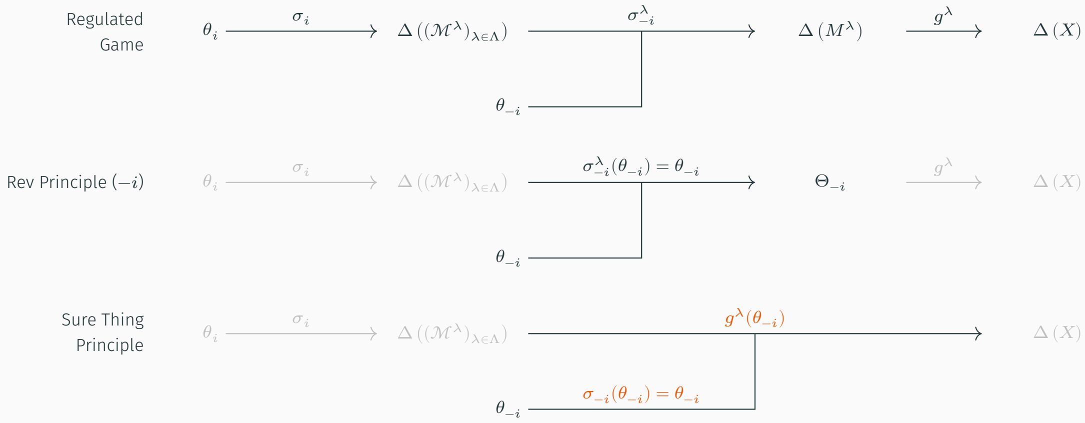

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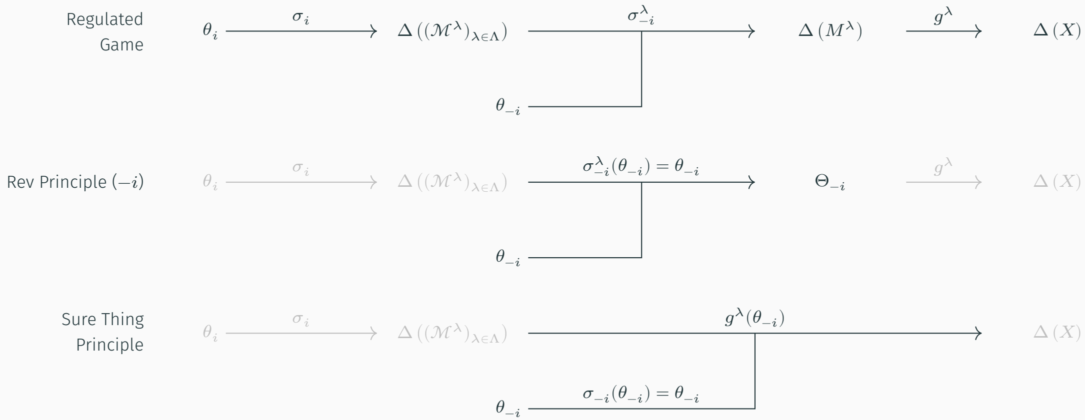

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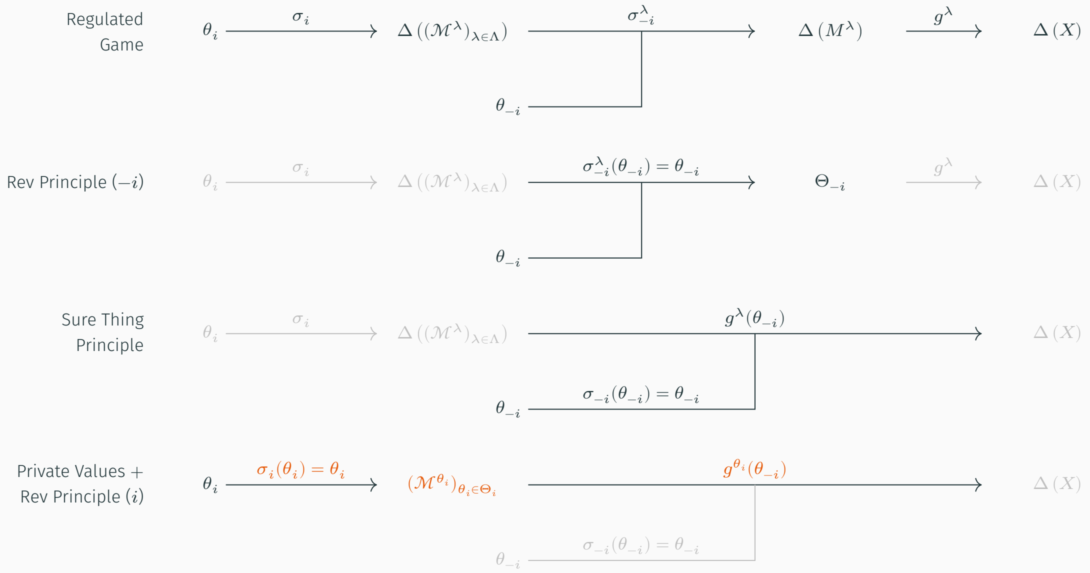

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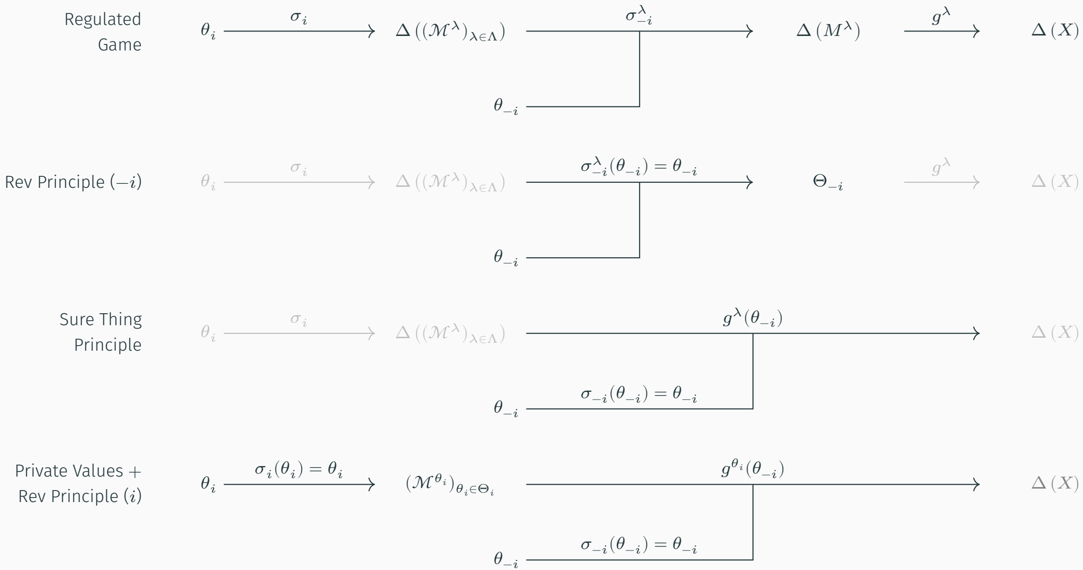

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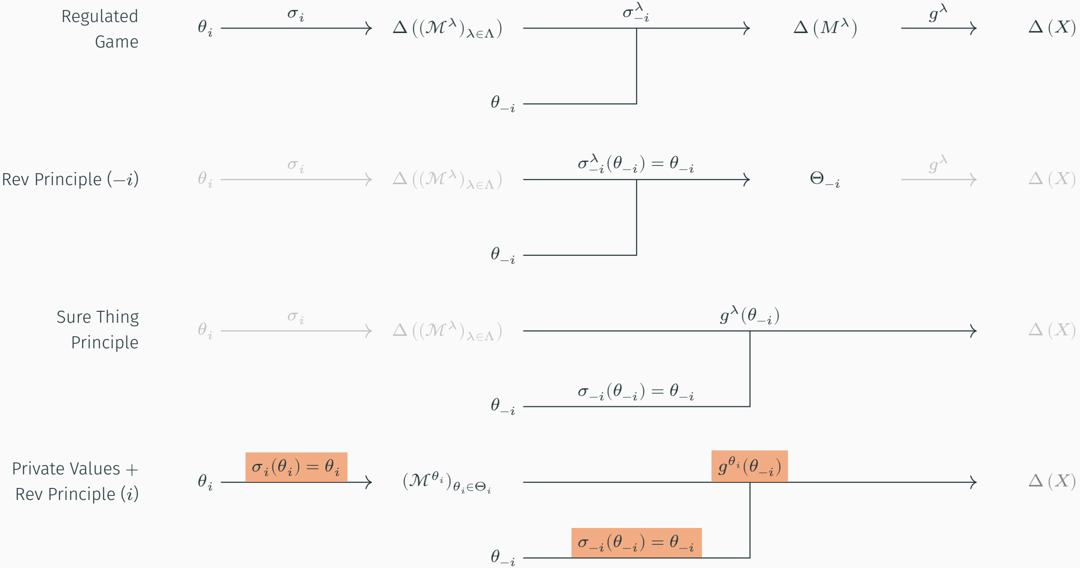

Bilateral Trade

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Outcomes probability of trade, q ; transfers, t_i

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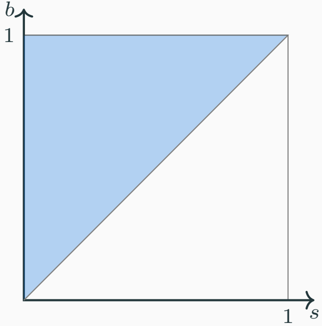

Figure 2: Efficient Allocation

Myerson-Satterthwaite (MS) optimal mechanism:

$$\max \mathbb{E}[(b-s)q(b,s)]$$

$s.t.$ Buyer BIC

Seller BIC

Buyer Ex-Post IR

Seller Interim IR

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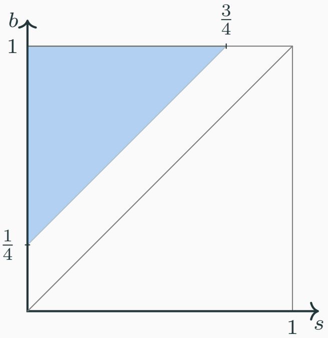

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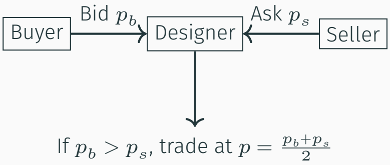

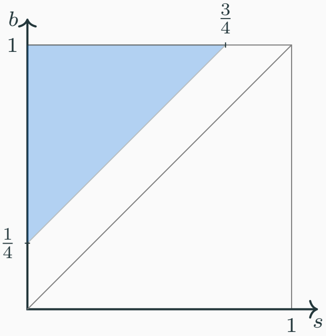

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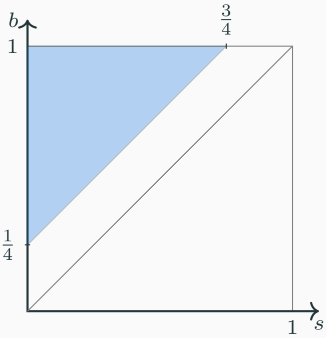

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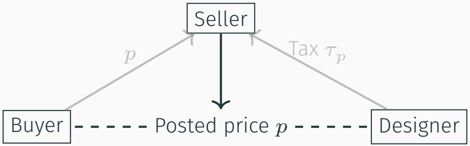

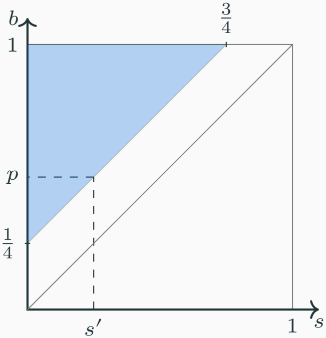

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Optimal Delegated Mechanism (DM):

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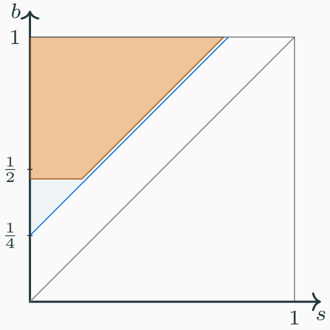

Figure 4: DM vs MS Allocation

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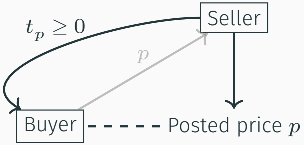


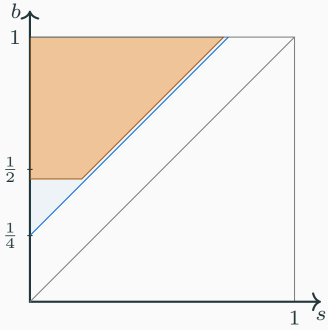
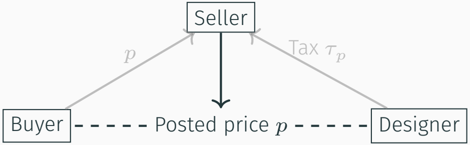


Figure 4: DM vs MS Allocation

Start from MS implementation Interim BB + Buyer DSIC



Sellers want to deviate to high prices $\Rightarrow \tau_p > 0$

Conversely, for low prices $\Rightarrow \tau_p < 0$

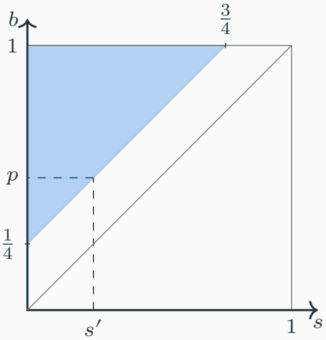


Figure 5: MS Allocation

Delegation Ex-Post BB + Buyer DSIC

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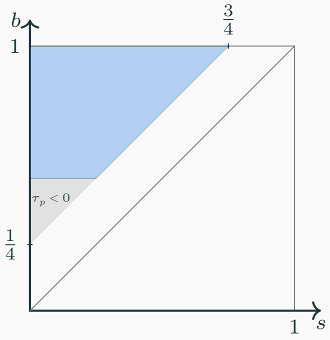

Figure 5: MS Allocation — Subsidized Sellers

Delegation Ex-Post BB + Buyer DSIC

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Add a price floor — transfer tax to buyers

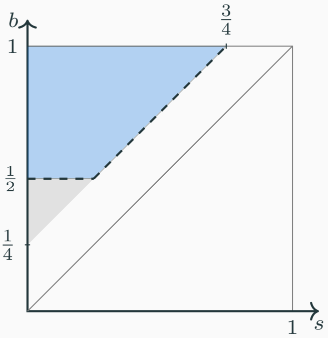

Figure 5: A Feasible Allocation

Delegation Ex-Post BB + Buyer DSIC

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Efficiency gain by reducing price floor

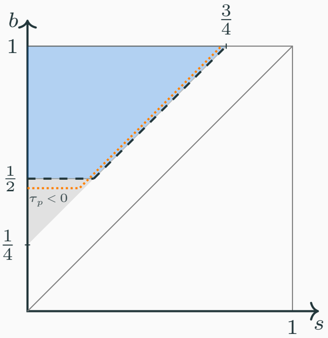

Figure 5: Optimal Allocation

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- Optimal delegation of prices (Amador Bagwell 13, 22)
- In this setting: price caps — bunching for weak sellers

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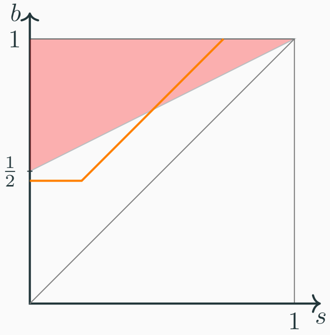

Figure 6: AB allocation (monopoly pricing)

Conclusion

Final Remarks

- Framework for regulating private contracting without mediation
  - Implementable through delegation ⇔ DSIC-1
  - Amenable to the tools of standard mechanism design
- In bilateral trade
  - Bunching among the strongest sellers
  - Improvement on pure price caps
- Going forward
  - Who gets contracting rights?
  - Multi-player interactions: sequential delegation
  - Correlated values

A Partial Literature

- Mechanism Design (& Monopoly Regulation). Baron and Myerson (1982), Myerson Satterthwaite (1983), Akbarpour and Li (2018), Guo Shmaya (2023)
- Delegation. Holmstrom (1980), Alonso Matouschek (2008), Ambrus and Egorov (2009), Koessler Martimort (2012), Amador Bagwell (2013, 2020, 2022)
- Hierarchical Contracting. Tirole (1988), Melumad et al. (1995) — No ability to "regulate" the set of contracts that can be used downstream. Monitoring plays a key role.
- Delegated Contracting.

  Hiriart and Martimort (2012); Malladi (2022) — Regulator restricts set of payments that evaluator uses in downstream MH / Screening problem

  Bhaskar et al. (2023); Mitchell (2023); Bhaskar McClellan (2024) — Delegated implementation of centralized mechanism design problem.
- Also relevant: incomplete contracts, limited commitment, informed principal

Optimal Buyer-Delegated Mechanism

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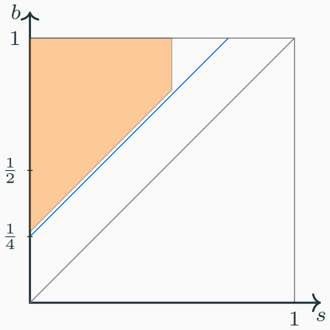

Figure 7: Buyer Optimal DM