Behavioral Characterizations of Naivete (Ahn, Iijima, Le Yaouanq, Sarver)

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Motivation and Overview

- Absolute naivete: Papers like O'Donoghue and Rabin (1999, 2001) apply naivete to particular decision problems. Ahn, Iijima, Le Yaouanq, and Sarver (2019) aim to understand the behavioral implications of naivete more broadly.
- Comparative naivete:
  - When it comes to comparing the naivete of two different individuals, several parametric statistics for quasi-hyperbolic discounting have been suggested. We will see why some of these don't seem reasonable when we consider the resulting behavior in certain decision problems.
  - To avoid such pitfalls, we will instead start with behavioral (non-parametric) definitions of comparative naivete (think Arrow–Pratt but for naivete instead of risk aversion).
- Stochastic Temptations: We will also discuss the random Strotz representation and extend our definition of naivete to random choice.

Outline

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Examples and Motivation

- Some Parametric Proposals and Potential Pitfalls
- A Nonparametric Approach

Results for Deterministic Choice

- Absolute Naivete
- Comparative Naivete
- Application to QH Discounting

General Results (Stochastic Choice)

- Examples and Motivation
- Absolute Naivete
- Comparative Naivete
- Application to Random QH Discounting

Example: Naive Quasi-Hyperbolic Discounting

Period 0 and Period 1 preferences for Naive Quasi-Hyperbolic (QH) Discounting of consumption streams $c = (c_1, c_2, \dots)$:

$$u(c) = w(c_1) + \sum_{t=2}^{\infty} \delta^{t-1} w(c_t)$$

$$v(c) = w(c_1) + \beta \sum_{t=2}^{\infty} \delta^{t-1} w(c_t)$$

$$\hat{v}(c) = w(c_1) + \hat{\beta} \sum_{t=2}^{\infty} \delta^{t-1} w(c_t)$$

- Period 0 commitment preferences: $u(c)$
- Actual period 1 preferences: $v(c)$
- Anticipated period 1 preferences: $\hat{v}(c)$

Example: Naive Quasi-Hyperbolic Discounting

Parametric Proposals for Sophistication and Naivete

- Usual parametric condition for sophistication: $\hat{\beta} = \beta$
- Usual parametric condition for (partial) naivete: $\hat{\beta} \geq \beta$
- Some proposals for comparative naivete (1 more naive than 2):
  - $\hat{\beta}_1 - \beta_1 \geq \hat{\beta}_2 - \beta_2$
  - $\frac{1-\hat{\beta}_1}{1-\beta_1} \leq \frac{1-\hat{\beta}_2}{1-\beta_2}$
  - $\hat{\beta}_1 \geq \hat{\beta}_2 \geq \beta_2 \geq \beta_1$

- All of these proposals for comparative naivete appear intuitively plausible, and all capture the idea that:
  - $\uparrow \hat{\beta}$ (holding β fixed) $\implies$ greater naivete
  - $\downarrow \beta$ (holding $\hat{\beta}$ fixed) $\implies$ greater naivete
- But some generate counterintuitive predictions: We will see that an agent ranked as more naive according to some of these statistics may nonetheless engage in behavior that seems patently more sophisticated.

Consumption-Savings Application

Example (Consumption-Savings Problem)

- Two risk-neutral QH discounters facing two-period problem.
- $\delta_i = 1$ and $\hat{\beta}_1 = 0.9$, $\beta_1 = 0$ and $\hat{\beta}_2 = 0.98$, $\beta_2 = 0.9$

$$u_1(c_1, c_2) = c_1 + c_2 \qquad u_2(c_1, c_2) = c_1 + c_2$$
$$\hat{v}_1(c_1, c_2) = c_1 + 0.9c_2 \qquad \hat{v}_2(c_1, c_2) = c_1 + 0.98c_2$$
$$v_1(c_2, c_2) = c_1 \qquad v_2(c_1, c_2) = c_1 + 0.9c_2$$

- The first two parametric statistics from above suggest that 1 is (strictly) more naive than 2:
  - $\hat{\beta}_1 - \beta_1 = 0.9 > 0.08 = \hat{\beta}_2 - \beta_2$
  - $\frac{1-\hat{\beta}_1}{1-\beta_1} = \frac{0.1}{1} = 0.1 < 0.2 = \frac{0.02}{0.1} = \frac{1-\hat{\beta}_2}{1-\beta_2}$
- The agents are unranked according to the third parametric condition $(\hat{\beta}_1 \geq \hat{\beta}_2 \geq \beta_2 \geq \beta_1)$ since $\hat{\beta}_1 < \hat{\beta}_2$.

Consumption-Savings Application

Example (Consumption-Savings Problem, continued)

$$u_1(c_1, c_2) = c_1 + c_2 \qquad u_2(c_1, c_2) = c_1 + c_2$$
$$\hat{v}_1(c_1, c_2) = c_1 + 0.9c_2 \qquad \hat{v}_2(c_1, c_2) = c_1 + 0.98c_2$$
$$v_1(c_2, c_2) = c_1 \qquad v_2(c_1, c_2) = c_1 + 0.9c_2$$

- Individuals have unit wealth and the option to commit to forced savings plan in period 0:
  - Forced savings plan requires them to save all consumption until period 2, and earns 2% interest: $y = \{(0, 1.02)\}$
  - If plan is declined, individuals can save in period 1 and earn 3% interest for period 2: $x = \{(c_1, 1.03(1 - c_1) : c_1 \in [0, 1]\}$.
  - Notice that the forced savings plan earns lower interest than the unrestricted account.

Example (Consumption-Savings Problem, continued)

$$u_1(c_1, c_2) = c_1 + c_2 \qquad u_2(c_1, c_2) = c_1 + c_2$$
$$\hat{v}_1(c_1, c_2) = c_1 + 0.9c_2 \qquad \hat{v}_2(c_1, c_2) = c_1 + 0.98c_2$$
$$v_1(c_2, c_2) = c_1 \qquad v_2(c_1, c_2) = c_1 + 0.9c_2$$

Forced savings plan: $y = \{(0, 1.02)\}$
Unrestricted account: $x = \{(c_1, 1.03(1 - c_1) : c_1 \in [0, 1]\}$.

- Since $\beta_i \times 1.03 < 1$ for $i = 1, 2$, actual period 1 choice from x is immediate consumption for both individuals: $(1, 0)$.
- Since $\hat{\beta}_1 \times 1.03 < 1$, individual 1 correctly anticipates choosing $(1, 0)$ from x, and therefore selects y in period 0 $\implies$ $u_1 = 1.02$
- Since $\hat{\beta}_2 \times 1.03 > 1$, individual 2 incorrectly anticipates choosing $(0, 1.03)$ from x, and therefore selects x in period 0 $\implies$ $u_2 = 1$
- So, individual 1 fares better than 2.
    - The statistics $\hat{\beta}_i - \beta_i$ and $\frac{1-\hat{\beta}_i}{1-\beta_i}$ erroneously rank 1 as more naive.
    - Third condition ($\hat{\beta}_1 \geq \hat{\beta}_2 \geq \beta_2 \geq \beta_1$) correctly does not rank these individuals (note that 2 fares better in other decision problems).

Outline

Underdemand for Commitment

- Instead of starting from a parametric restrictions and checking if they deliver intuitive predictions, consider the opposite approach:
  - Start instead with the behavior that seems to most reasonably capture increases in naivete.
  - Then determine the implied parametric restrictions for various models.
- What is a natural candidate for such a behavioral definition?
  - Underdemand for commitment: Say 1 is more naive than 2 if she rejects more beneficial commitments (formal definitions later).
  - In the QH discounting model, we will see that 1 has greater underdemand for commitment than 2 $\iff \hat{\beta}_1 \geq \hat{\beta}_2 \geq \beta_2 \geq \beta_1$.
  - Intuition: As in the example, whenever $\hat{\beta}_1 < \hat{\beta}_2$ there are decision problems where a beneficial commitment taken up by 1 is erroneously turned down by 2. Relatedly, whenever $\beta_1 > \beta_2$ there are commitments turned down by both that would only be valuable for 1.

Other Criteria?

- Are there any other plausible options for a behavioral definition of greater naivete?
  - It has been established that consumer naivete can be leveraged by a monopolist to extract more consumer surplus.
  - Consumers may "overvalue" the surplus they will receive from a contract and hence overpay for it.
  - The next example, adapted from DellaVigna and Malmendier (2004), shows how consumer overvaluation relates to their potential for exploitation by a monopolist.

Monopoly Profit Application

Example (Monopoly Profit)

- A monopolist produces a service whose consumption results in delayed benefits, for example, a fitness club offers access to exercise that provides future health benefits.
- The firm offers a two-part tariff at period 0 that specifies (L, p) where L is a fixed payment like monthly dues for gym membership and p is the price of using the service like a per-visit fee at the gym.
- Timeline:
  - Date 0: Consumer decides whether to accept the contract. If accepts, commits to payment L (in date 1).
  - Date 1: Consumer decides whether to use the service. If so, pays p at date 1 and gets (delayed) benefit b at date 2.
- Firm marginal cost of service is c (assume $b > c$).

Example (Monopoly Profit, continued)

- QH consumer $(\hat{\beta}, \beta, \delta)$. Assume $\delta = 1$.
- Consumer problem:
  - Anticipates using service in date 1 $\iff \hat{\beta} b \geq p$.
  - Accepts the contract at date 0 $\iff -L + (b - p)\mathbb{1}_{\hat{\beta} b \geq p} \geq 0$.
- Firm problem:
  - Knows $(\hat{\beta}, \beta)$ and offers (L, p) to maximize: $L + (p - c)\mathbb{1}_{\beta b \geq p}$
  - Given p, will raise L until consumer participation constraint is binding: $L = (b - p)\mathbb{1}_{\hat{\beta} b \geq p}$
  - Thus, firm profit as a function of p (given optimal L) is:

$$\begin{aligned}\pi(p) &= (b - p)\mathbb{1}_{\hat{\beta} b \geq p} + (p - c)\mathbb{1}_{\beta b \geq p} \\ &= \underbrace{(b - p)\mathbb{1}_{\hat{\beta} b \geq p > \beta b}}_{\text{overvaluation}} + \underbrace{(b - c)\mathbb{1}_{\beta b \geq p}}_{\text{social surplus}}\end{aligned}$$

- Notice that $\hat{\beta} > \beta$ allows the monopoly to potentially extract more than the entire consumer surplus.

Example (Monopoly Profit, continued)

$$\pi(p) = \underbrace{(b-p)\mathbb{1}_{\hat{\beta}b \geq p > \beta b}}_{\text{overvaluation}} + \underbrace{(b-c)\mathbb{1}_{\beta b \geq p}}_{\text{social surplus}}$$

- Case 1 — $\beta b \geq c$:
  - Setting $\beta b \geq p$ gives profit $b - c$
  - Setting $\hat{\beta} b \geq p > \beta b \geq c$ gives profit $b - p$ (less than $b - c$)
  - Thus, firm sets $\beta b \geq p$
  - Consumer uses service and gets zero surplus.
- Case 2 — $c > \beta b$:
  - Setting $\beta b \geq p$ gives profit $b - c$
  - Setting $\hat{\beta} b \geq p > \beta b$ gives profit $b - p$
  - Firm sets $p = \beta b + \varepsilon$ for some small $\varepsilon > 0$ and gets profit of approximately $b - \hat{\beta} b$ (greater than $b - c$)
  - Consumer accepts contract, but doesn't use service; obtains negative surplus: $-L \approx -(b - \hat{\beta} b) < 0$ (exploitation)

Overvaluation

- The previous example suggest that we may also wish to compare naivete by comparing the extent to which different individuals overvalue menus.
  - Remember that selecting a contract as in the previous example is a special case of choosing a menu.
  - Overvaluation is the difference between anticipated and actual indirect utility from a menu (e.g., contract).
- Preview of Results: For deterministic QH discounting, we will see that the following conditions are equivalent:
  1. 1 has greater underdemand for commitment than 2
  2. 1 has greater overvaluations than 2
  3. $\hat{\beta}_1 \geq \hat{\beta}_2 \geq \beta_2 \geq \beta_1$
- Interestingly, once we consider stochastic temptation (e.g., random β and $\hat{\beta}$), condition 1 becomes strictly stronger than condition 2.

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Framework: Two Pieces of Choice Data

- C — consequences (compact metric)
- $\triangle(C) \ni p, q$ — lotteries
- $\mathcal{K}(\triangle(C)) \ni x, y,$ — menus (nonempty compact subsets of ΔC)
- $u, v : \triangle(C) \to \mathbb{R}$ — expected-utility functions
    - Write $u \approx v$ if u is a positive affine transformation of v

- Ex Ante Choice: Preference relation $\succsim$ on $\mathcal{K}(\triangle(C))$
    - $\{p\} \succsim \{q\}$ — commitment (welfare, normative) preferences
- Ex Post Choice: Choice function $\mathcal{C} : \mathcal{K}(\triangle(C)) \to \triangle(C)$
    - Note that $\mathcal{C}$ is a *function*, not a *correspondence*. Think of $\mathcal{C}(x) = p$ a one selection from the set of possible optimal choices from x.
    - Definitions/results easily extend to choice correspondences, but this approach is simpler and shows robustness to how ties are broken.

Absolute Naivete: Simple Example

Example

Two options: p and q

Time inconsistency:

- Prefer p if choosing in advance: $\{p\} \succ \{q\}$.
- Choose q if given the option ex post: $\mathcal{C}(\{p, q\}) = q$.
- E.g., preference to maintain a healthy diet, exercise more, decrease spending, or engage timely effort in a difficult task that goes unfulfilled ex post.

Additional information needed to determine if sophisticated or naive:

$$\begin{aligned} \{p, q\} \succ \{q\} &\implies \text{(incorrectly) anticipates choosing } p \\ &\implies \text{naive} \end{aligned}$$

Absolute Naivete for Deterministic Choice

In the previous example, we inferred naivete from

$$\{p, q\} \succ \{q\} \quad \text{and} \quad \mathcal{C}(\{p, q\}) = q.$$

This approach generalizes to any menu:

Definition

An individual is sophisticated if $x \sim \{\mathcal{C}(x)\}$ for all menus x.

An individual is naive if $x \succsim \{\mathcal{C}(x)\}$ for all menus x.

An individual is strictly naive if she is naive and not sophisticated.

- Note that this approach only makes sense if individuals are "consequentialist," meaning utility is not affected by unchosen alternatives. ▸ More on Consequentialism
- Temptation with costly self-control as in Gul and Pesendorfer (2001) would obviously violate consequentialism. See Ahn, Iijima, and Sarver (2020) for alternative characterizations of naivete for preferences with self-control.

Related Empirical Evidence

Several empirical papers have considered similar two-stage primitives and found evidence of naivete. Some examples:

x	$\mathcal{C}(x)$	References
gym membership	burn \$70/month	DellaVigna and Malmendier (2006)
open credit card	carry balance after teaser rate ends	Shui and Ausubel (2005)
smoking cessation contract	smoke and lose money	Giné, Karlan, and Zinman (2010)
postpone 401(k) sign up	postpone again tomorrow	Madrian and Shea (2001)

- As in the case of postponing participation in 401(k)s above, note that procrastination is one example of our comparative. ▸ More on Procrastination

Strotz Representation

$B_w(x) \equiv \text{argmax}_{p \in x}\, w(p)$

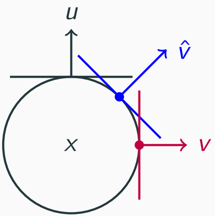
Definition

A Strotz representation of $(\succsim, \mathcal{C})$ is a triple $(u, v, \hat{v})$ of nontrivial (nonconstant) expected-utility functions such that $U : \mathcal{K}(\triangle(C)) \to \mathbb{R}$ defined by

$$U(x) = \max_{p \in B_{\hat{v}}(x)} u(p)$$

represents $\succsim$ and

$$\mathcal{C}(x) \in B_u(B_v(x)).$$



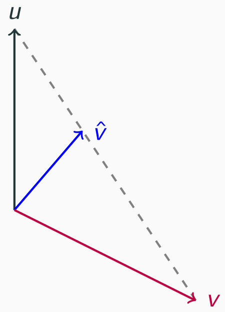
Comparing Strotz Parameters

Definition

Let $u, v, \hat{v}$ be expected-utility functions. Then $\hat{v}$ is more u-aligned than v (denoted $\hat{v} \gg_u v$) if either

$$\hat{v} \approx \alpha u + (1-\alpha) v \quad \text{for some} \quad \alpha \in [0,1],$$

or $v \approx -u$.



Absolute Naivete and Strotz

Theorem

Suppose $(\succsim, \mathcal{C})$ has a Strotz representation $(u, v, \hat{v})$. Then the individual is naive if and only if $\hat{v} \gg_u v$ (and is sophisticated if and only if $\hat{v} \approx v$).

Proof Sketch

"If" direction:

- Lemma: If $v \gg_u v'$ then $\max_{p \in B_v(x)} u(p) \geq \max_{q \in B_{v'}(x)} u(q)$.
- Therefore, $\hat{v} \gg_u v$ implies

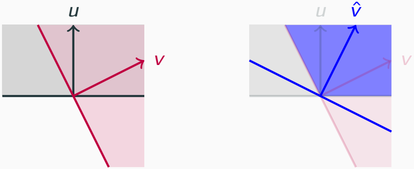
$$U(x) = \max_{p \in B_{\hat{v}}(x)} u(p) \geq \max_{q \in B_v(x)} u(q) = u(\mathcal{C}(x)) \iff x \succsim \{\mathcal{C}(x)\}$$

"Only if" direction:

- Step 1: For any lotteries p, q,

$$\begin{aligned}[u(p) > u(q) \text{ and } v(p) > v(q)] &\implies [\{p\} \succ \{q\} \text{ and } \mathcal{C}(\{p,q\}) = p] \\ &\implies \{p,q\} \succsim \{p\} \succ \{q\} \quad \text{(naivete)} \\ &\implies \hat{v}(p) \geq \hat{v}(q)\end{aligned}$$

- Step 2: By Farkas' Lemma $\hat{v} = au + bv + c$ for $a, b \geq 0$.



Outline

Comparative Naivete: Underdemand for Commitment

- Our first suggested method for comparing naivete was based on comparing the uptake of beneficial commitment devices.

Definition

Individuals 1 is more naive than individual 2 if, for all menus x and lotteries p,

$$x \succ_2 \{p\} \succ_2 \{\mathcal{C}_2(x)\} \implies x \succ_1 \{p\} \succ_1 \{\mathcal{C}_1(x)\}$$

- $\{p\} \succ_i \{\mathcal{C}_i(x)\}$ — commitment to $\{p\}$ is beneficial.
- $x \succ_i \{p\}$ — individual declines this commitment.
- 1 more naive than 2 if declines more beneficial commitments.

Comparative Naivete: Overvaluations

- Our second suggested method for comparing naivete was based on comparing overvaluation of menus (e.g., contracts).

Definition

Suppose $(\succsim, \mathcal{C})$ has a Strotz representation $(u, v, \hat{v})$. The *coefficient of overvaluation* of a menu x is defined by:

$$OV(x) = \underbrace{\max_{p \in B_{\hat{v}}(x)} u(p)}_{\text{believed indirect utility}} - \underbrace{\max_{p \in B_v(x)} u(p)}_{\text{actual indirect utility}}.$$

- Not directly observable in our framework, but could be elicited from valuations of menus at the ex ante stage if transfers are incorporated, e.g., compare amount would pay for x with amount would pay for $\{\mathcal{C}(x)\}$.

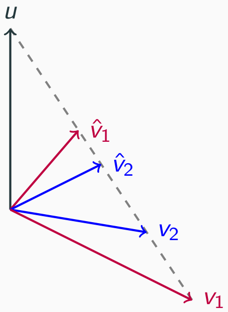
- For comparisons of overvaluations to be meaningful across individuals, it is necessary to normalize so that $u_1 = u_2$.

Comparative Naivete and Strotz

Theorem

Suppose $(\succsim_1, \mathcal{C}_1)$ and $(\succsim_2, \mathcal{C}_2)$ are naive and have Strotz representations $(u, v_1, \hat{v}_1)$ and $(u, v_2, \hat{v}_2)$. Then the following are equivalent:

1. *Individual 1 is more naive than individual 2.*
2. *$OV_1(x) \geq OV_2(x)$ for all menus x.*
3. *$\hat{v}_1 \gg_u \hat{v}_2 \gg_u v_2 \gg_u v_1$ or $\hat{v}_2 \approx v_2$ (individual 2 is sophisticated).*



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Quasi-Hyperbolic Discounting: Absolute Naivete

Example (quasi-hyperbolic discounting)

$C = \prod_{n=1}^{\infty} [a, b]$ with typical element $(c_1, c_2, \dots)$.

$$u(c) = \sum_{t=1}^{\infty} \delta^{t-1} w(c_t).$$

$$v_\beta(c) = w(c_1) + \beta \sum_{t=2}^{\infty} \delta^{t-1} w(c_t).$$

Quasi-hyperbolic discounting is a Strotz representation $(u, v_\beta, v_{\hat{\beta}})$. By our theorem for absolute naivete:

$$\begin{aligned} \text{naive} &\iff v_{\hat{\beta}} \gg_u v_\beta \\ &\iff v_{\hat{\beta}} = \alpha u + (1-\alpha) v_\beta \\ &\iff \hat{\beta} = \alpha 1 + (1-\alpha)\beta \geq \beta. \end{aligned}$$

Similarly, sophisticated if and only if $\hat{\beta} = \beta$.

Quasi-Hyperbolic Discounting: Comparative Naivete

Example (quasi-hyperbolic discounting)

$$u(c) = \sum_{t=1}^{\infty} \delta^{t-1} w(c_t), \qquad v_\beta(c) = w(c_1) + \beta \sum_{t=2}^{\infty} \delta^{t-1} w(c_t).$$

By our theorem for comparative naivete, if 1 and 2 are naive and have QH representations $(w, \beta_1, \hat{\beta}_1, \delta)$ and $(w, \beta_2, \hat{\beta}_2, \delta)$, then 1 is more naive than 2 if and only if either

1. $\hat{\beta}_1 \geq \hat{\beta}_2 \geq \beta_2 \geq \beta_1$ (equivalently, $v_{\hat{\beta}_1} \gg_u v_{\hat{\beta}_2} \gg_u v_{\beta_2} \gg_u v_{\beta_1}$), or
2. 2 is sophisticated ($\hat{\beta}_2 = \beta_2$).

- Our leading example showed that comparisons like $\hat{\beta}_1 - \beta_1 \geq \hat{\beta}_2 - \beta_2$ and $\frac{1-\hat{\beta}_1}{1-\beta_1} \leq \frac{1-\hat{\beta}_2}{1-\beta_2}$ can lead to counterintuitive implications for behavior.
- By our comparative naivete theorem, we see that the ordering generated by the condition $\hat{\beta}_1 \geq \hat{\beta}_2 \geq \beta_2 \geq \beta_1$ always leads to sensible differences in behavior when it comes to uptake of commitment opportunities.

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Example: Frequency Naivete

$$u(c) = \sum_{t=1}^{\infty} \delta^{t-1} w(c_t)$$

$$v(c) = \begin{cases} u(c) & \text{with prob } \theta \\ w(c_1) + \beta \sum_{t=2}^{\infty} \delta^{t-1} w(c_t) & \text{with prob } 1 - \theta \end{cases}$$

$$\hat{v}(c) = \begin{cases} u(c) & \text{with prob } \hat{\theta} \\ w(c_1) + \beta \sum_{t=2}^{\infty} \delta^{t-1} w(c_t) & \text{with prob } 1 - \hat{\theta} \end{cases}$$

Used in a number of applications, for example: Eliaz and Spiegler (2006), Heidhues and Koszegi (2010), Duflo, Kremer, and Robinson (2011)

Example: Frequency Naivete

$$u(c) = \sum_{t=1}^{\infty} \delta^{t-1} w(c_t)$$

$$v(c) = \begin{cases} u(c) & \text{with prob } \theta \\ w(c_1) + \beta \sum_{t=2}^{\infty} \delta^{t-1} w(c_t) & \text{with prob } 1-\theta \end{cases}$$

$$\hat{v}(c) = \begin{cases} u(c) & \text{with prob } \hat{\theta} \\ w(c_1) + \beta \sum_{t=2}^{\infty} \delta^{t-1} w(c_t) & \text{with prob } 1-\hat{\theta} \end{cases}$$

Possible Parametric Restrictions for Sophistication and Naivete

- Sophisticated: $\hat{\theta} = \theta$
- Naive: $\hat{\theta} \geq \theta$
- Comparative naivete (1 more naive than 2):
  - $\hat{\theta}_1 \geq \hat{\theta}_2 \geq \theta_2 \geq \theta_1$?
  - $\hat{\theta}_1 - \theta_1 \geq \hat{\theta}_2 - \theta_2$?
  - Something else?

How Do We Define Naivete and Sophistication Behaviorally?

- When choice is random, what are the right behavioral definitions of naivete and sophistication?

Example

Two options: p and q.

- Prefer p if choosing ex ante: $\{p\} \succ \{q\}$.
- Suppose p is actually chosen with probability α from $\{p, q\}$ at the ex post stage.
- Then, our previous definitions are extended as follows:
  - Sophisticated if $\{p, q\} \sim \{\alpha p + (1-\alpha)q\}$
  - Strictly Naive if $\{p, q\} \succ \{\alpha p + (1-\alpha)q\}$ (overestimates the probability that p will be chosen)
  - Underdemand for commitment and overvaluation are defined similarly to before, but now using the ex post distribution over outcomes $\alpha p + (1-\alpha)q$ in the place of the deterministic choice $\mathcal{C}(x)$.

Generalizing the Example: Random QH Discounting

$$u(c) = \sum_{t=1}^{\infty} \delta^{t-1} w(c_t), \qquad v_\beta(c) = w(c_1) + \beta \sum_{t=2}^{\infty} \delta^{t-1} w(c_t).$$

Random QH Discounting:

- Distribution of β is random.
- Actual distribution is $F(\beta)$.
- Anticipated (believed) distribution is $\hat{F}(\beta)$.

We will show that:

- Sophisticated $\iff$ $\hat{F} = F$
- Naive $\iff$ $\hat{F} \geq_{FOSD} F$ (that is, $\hat{F}(\beta) \leq F(\beta)$ for all $\beta \in [0, 1]$)
- Notice that these conditions reduce to $\hat{\theta} = \theta$ and $\hat{\theta} \geq \theta$ (respectively) in the special case of the previous example.

Generalizing the Example: Random QH Discounting

We will show that two conditions are relevant for comparative naivete:

- 1 has greater underdemand for commitment than 2 iff either $\hat{F}_2 = F_2$ or
$$\hat{F}_1(\beta) \leq \hat{F}_2(\beta) \leq F_2(\beta) \leq F_1(\beta) \quad \forall \beta \tag{1}$$

- 1 has greater overvaluations than 2 iff
$$F_1(\beta) - \hat{F}_1(\beta) \geq F_2(\beta) - \hat{F}_2(\beta) \quad \forall \beta \tag{2}$$

- Note that (1) $\implies$ (2). (For degenerate/deterministic distributions, (1) $\iff$ (2).)

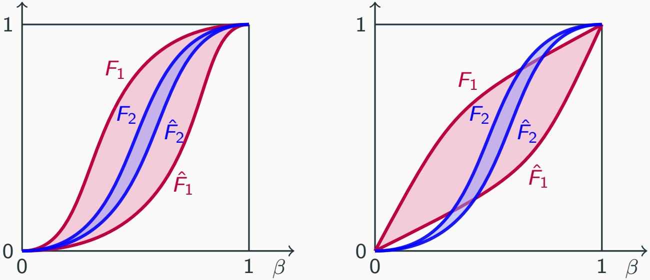


Figure 1: Left: (1) and (2) satisfied. Right: (2) but not (1).

Example: Back to Binary Distributions

$$u(c) = \sum_{t=1}^{\infty} \delta^{t-1} w(c_t)$$

$$v(c) = \begin{cases} u(c) & \text{with prob } \theta \\ w(c_1) + \beta \sum_{t=2}^{\infty} \delta^{t-1} w(c_t) & \text{with prob } 1-\theta \end{cases}$$

$$\hat{v}(c) = \begin{cases} u(c) & \text{with prob } \hat{\theta} \\ w(c_1) + \beta \sum_{t=2}^{\infty} \delta^{t-1} w(c_t) & \text{with prob } 1-\hat{\theta} \end{cases}$$

In the special case of distributions with binary support $\{\beta, 1\}$, the conditions become:

- Sophisticated $\iff$ $\hat{F} = F$ $\iff$ $\hat{\theta} = \theta$
- Naive $\iff$ $\hat{F} \geq_{FOSD} F$ $\iff$ $\hat{\theta} \geq \theta$
- 1 has greater underdemand for commitment than 2 iff either $\hat{F}_2 = F_2$ $(\hat{\theta} = \theta)$ or
  $$\hat{F}_1 \geq_{FOSD} \hat{F}_2 \geq_{FOSD} F_2 \geq_{FOSD} F_1 \iff \hat{\theta}_1 \geq \hat{\theta}_2 \geq \theta_2 \geq \theta_1$$
- 1 has greater overvaluations than 2 iff
  $$F_1(\beta) - \hat{F}_1(\beta) \geq F_2(\beta) - \hat{F}_2(\beta) \quad \forall \beta \iff \hat{\theta}_1 - \theta_2 \geq \hat{\theta}_2 - \theta_2$$

Outline

Framework: Incorporating Stochastic Ex Post Choice

- C — consequences (compact metric)
- $\triangle(C) \ni p, q$ — lotteries
- $\mathcal{K}(\triangle(C)) \ni x, y,$ — menus (nonempty compact subsets of ΔC)
- $u, v : \triangle(C) \to \mathbb{R}$ — expected-utility functions

- Ex Ante Choice: Preference relation $\succsim$ on $\mathcal{K}(\triangle(C))$
- Ex Post Choice: Random choice rule $\lambda : \mathcal{K}(\triangle(C)) \to \triangle(\triangle(C))$
  - Every menu x is mapped to a probability distribution $\lambda^x \in \triangle(\triangle(C))$
  - $\lambda^x(y)$ — probability of choosing a lottery in set $y \subset x$
  - Deterministic choice is the special case of degenerate choice rules that put probability one on a single option for each x, that is, $\lambda^x(\{p\}) = 1 \iff \mathcal{C}(x) = p$.

Absolute Naivete for Random Choice

Average choice from a menu x:

$$m(\lambda^x) = \int_{\triangle(C)} p \, d\lambda^x(p).$$

Reduction of compound lottery, **not** expected value of consumption c.

Example

Suppose p is chosen with probability α from $x = \{p, q\}$ ex post.

- That is, $\lambda^x(\{p\}) = \alpha$.
- $m(\lambda^x)$ is the mixture lottery $\alpha p + (1 - \alpha) q$.

Example

Deterministic choice is a degenerate random choice rule:

$$\mathcal{C}(x) = p \iff \lambda^x(\{p\}) = 1.$$

In this case, $m(\lambda^x) = \mathcal{C}(x)$.

Absolute Naivete for Random Choice

Definition

An individual is sophisticated if $x \sim \{m(\lambda^x)\}$ for all menus x.

An individual is naive if $x \succsim \{m(\lambda^x)\}$ for all menus x.

An individual is strictly naive if she is naive and not sophisticated.

Example

Suppose p is chosen with probability α from $x = \{p, q\}$ ex post.

- Sophisticated if $\{p, q\} \sim \{\alpha p + (1 - \alpha)q\}$.
- Strictly naive if $\{p, q\} \succ \{\alpha p + (1 - \alpha)q\}$.

Random Strotz Representation

$\mathcal{V}$ — space of all continuous expected-utility functions.

Definition

A random Strotz representation of $(\succsim, \lambda)$ is a triple $(u, \mu, \hat{\mu})$ of a nontrivial expected-utility function u and nontrivial probability measures μ and $\hat{\mu}$ over $\mathcal{V}$ with finite-dimensional support such that $U : \mathcal{K}(\triangle(C)) \to \mathbb{R}$ defined by

$$U(x) = \int_{\mathcal{V}} \max_{p \in B_v(x)} u(p)\, d\hat{\mu}(v)$$

represents $\succsim$ and

$$\lambda^x(y) = \mu(p_x^{-1}(y))$$

for some measurable selection function $p_x : \mathcal{V} \to x$ with

$$p_x(v) \in B_u(B_v(x)) \quad \forall v \in \mathcal{V}.$$

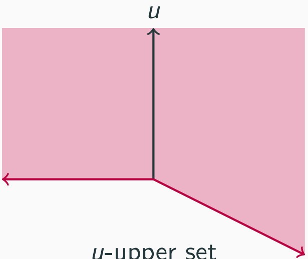
▸ Definition of Finite-Dimensional Support

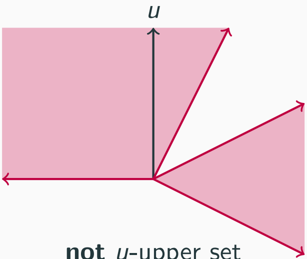
- Equivalently: $\lambda^x(y) = \mu(\{v \in \mathcal{V} : p_x(v) \in y\})$
- Deterministic Strotz: $\mu = \delta_v$ and $\hat{\mu} = \delta_{\hat{v}}$ (Dirac probability measures)

Comparing Random Strotz Parameters

Definition

Let u be an expected-utility function. A measurable set $\mathcal{U} \subset \mathcal{V}$ is a u-upper set if, for any $v \in \mathcal{U}$ and $v' \in \mathcal{V}$, if $v' \gg_u v$ then $v' \in \mathcal{U}$.





Definition

Let u be an expected-utility function, and let $\mu, \hat{\mu}$ be probability measures over $\mathcal{V}$. Then $\hat{\mu}$ is more u-aligned than μ, written as $\hat{\mu} \gg_u \mu$, if $\hat{\mu}(\mathcal{U}) \geq \mu(\mathcal{U})$ for all u-upper sets $\mathcal{U}$.

Absolute Naivete and Random Strotz

Theorem

Suppose $(\succsim, \lambda)$ has a random Strotz representation $(u, \mu, \hat{\mu})$. Then the individual is naive if and only if $\hat{\mu} \gg_u \mu$ (and is sophisticated if and only if $\hat{\mu} \approx \mu$).

Proof uses a comparative result from Dekel and Lipman (2012). They showed that for any μ_1, μ_2,

$$\int \max_{p \in B_v(x)} u(p)\, d\mu_2(v) \geq \int \max_{p \in B_v(x)} u(p)\, d\mu_1(v) \;\; (\forall x) \iff \mu_2 \gg_u \mu_1.$$

We show that naivete implies

$$\int \max_{p \in B_v(x)} u(p)\, d\hat{\mu}(v) = U(x) \geq U(\{m(\lambda^x)\}) = \int \max_{p \in B_v(x)} u(p)\, d\mu(v).$$

Special Case: Back to Deterministic Strotz

- Our first theorem for absolute naivete for the deterministic Strotz representation $(u, v, \hat{v})$ follows as a corollary of this theorem.
- Let $\mu = \delta_v$ and $\hat{\mu} = \delta_{\hat{v}}$.
- Naive $\iff \hat{\mu} \gg_u \mu \iff \hat{v} \gg_u v$.

Outline

Examples and Motivation

- Some Parametric Proposals and Potential Pitfalls
- A Nonparametric Approach

Results for Deterministic Choice

- Absolute Naivete
- Comparative Naivete
- Application to QH Discounting

General Results (Stochastic Choice)

- Examples and Motivation
- Absolute Naivete
- Comparative Naivete
- Application to Random QH Discounting

Comparative Naivete: Underdemand for Commitment

Definition

Individuals 1 is more naive than individual 2 if, for all menus x and lotteries p,

$$x \succ_2 \{p\} \succ_2 \{m(\lambda_2^x)\} \implies x \succ_1 \{p\} \succ_1 \{m(\lambda_1^x)\}$$

- $\{p\} \succ_i \{m(\lambda_i^x)\}$ — commitment to $\{p\}$ is beneficial.
- $x \succ_i \{p\}$ — individual declines this commitment.
- 1 more naive than 2 if declines more beneficial commitments.

Comparative Naivete and Random Strotz

Theorem

Suppose $(\succsim_1, \lambda_1)$ *and* $(\succsim_2, \lambda_2)$ *are naive and have random Strotz representations* $(u, \mu_1, \hat{\mu}_1)$ *and* $(u, \mu_2, \hat{\mu}_2)$*. Then individual 1 is more naive than individual 2 if and only if*

$$\hat{\mu}_1 \gg_u \hat{\mu}_2 \gg_u \mu_2 \gg_u \mu_1$$

or $\hat{\mu}_2 \approx \mu_2$ *(individual 2 is sophisticated).*

Comparative Naivete: Overvaluations

Definition

Suppose $(\succsim, \lambda)$ has a random Strotz representation $(u, \mu, \hat{\mu})$. The coefficient of over-valuation of menu x is defined by:

$$OV(x) = \underbrace{\int_{\mathcal{V}} \max_{p \in B_v(x)} u(p)\, d\hat{\mu}(v)}_{\text{believed indirect utility}} - \underbrace{\int_{\mathcal{V}} \max_{p \in B_v(x)} u(p)\, d\mu(v)}_{\text{actual indirect utility}}$$

- For deterministic choice:

$$1 \text{ more naive than } 2 \iff OV_1(x) \geq OV_2(x) \quad \forall x.$$

- For random choice the latter condition is strictly weaker:

$$1 \text{ more naive than } 2 \implies OV_1(x) \geq OV_2(x) \quad \forall x.$$

- So, we need a different behavioral foundation for comparing overvaluations.

Characterizing Overvaluations

Definition

Fix any p, q such that $\{q\} \succ \{p\}$. The probability premium of menu x is defined by:

$$P(x; p, q) = \sup \left\{ \alpha \in [0,1] : (1-\alpha)x + \alpha\{p\} \succsim (1-\alpha)\{m(\lambda^x)\} + \alpha\{q\} \right\}.$$

Theorem

Suppose $(\succsim_1, \lambda_1)$ and $(\succsim_2, \lambda_2)$ are naive and have random Strotz representations $(u, \mu_1, \hat{\mu}_1)$ and $(u, \mu_2, \hat{\mu}_2)$. Then, fixing any p, q with $u(q) > u(p)$, the following are equivalent:

1. $P_1(x; p, q) \geq P_2(x; p, q)$ *for all menus* x.
2. $OV_1(x) \geq OV_2(x)$ *for all menus* x.
3. $\hat{\mu}_1(\mathcal{U}) - \mu_1(\mathcal{U}) \geq \hat{\mu}_2(\mathcal{U}) - \mu_2(\mathcal{U})$ *for all u-upper sets* $\mathcal{U}$.

$\hat{\mu}_1 \gg_u \hat{\mu}_2 \gg_u \mu_2 \gg_u \mu_1$ (more naive) $\implies$ $\hat{\mu}_1 - \mu_1 \gg_u \hat{\mu}_2 - \mu_2$ (greater overvaluations)

Outline

Application: Random Quasi-Hyperbolic Discounting

$$u(c) = \sum_{t=1}^{\infty} \delta^{t-1} w(c_t). \tag{3}$$

$$v_\beta(c) = w(c_1) + \beta \sum_{t=2}^{\infty} \delta^{t-1} w(c_t). \tag{4}$$

Definition

A random quasi-hyperbolic (RQH) representation of $(\succsim, \lambda)$ is a quadruple $(w, F, \hat{F}, \delta)$ of a continuous nontrivial function $w : [a, b] \to \mathbb{R}$, a scalar $\delta \in (0, 1)$, and cumulative distribution functions $F, \hat{F}$ on $[0, 1]$ such that

$$\int_0^1 \max_{p \in B_{v_\beta}(x)} u(p) \; d\hat{F}(\beta)$$

represents $\succsim$ and

$$\lambda^x(y) = \int_0^1 \mathbb{1}_{[p_x(\beta) \in y]} \, dF(\beta)$$

for some measurable selection function $p_x : [0, 1] \to x$ with $p_x(\beta) \in B_u(B_{v_\beta}(x))$ for all $\beta \in [0, 1]$.

Application: Random Quasi-Hyperbolic Discounting

Corollary

Suppose $(\succsim, \lambda)$ has a RQH representation $(w, F, \hat{F}, \delta)$. Then the individual is naive if and only if $\hat{F} \geq_{FOSD} F$ (and is sophisticated if and only if $\hat{F} = F$).

Corollary

Suppose $(\succsim_1, \lambda_1)$ and $(\succsim_2, \lambda_2)$ are naive and have RQH representations $(w, F_1, \hat{F}_1, \delta)$ and $(w, F_2, \hat{F}_2, \delta)$. Then:

1. *Individual 1 is more naive than individual 2 if and only if*

$$\hat{F}_1 \geq_{FOSD} \hat{F}_2 \geq_{FOSD} F_2 \geq_{FOSD} F_1$$

   or $\hat{F}_2 = F_2$ (individual 2 is sophisticated).

2. *Individual 1 has greater overvaluations than 2 if and only if*

$$F_1(\beta) - \hat{F}_1(\beta) \geq F_2(\beta) - \hat{F}_2(\beta), \qquad \forall \beta \in [0, 1].$$

Outline

Importance of "Consequentialism"

Example (costly self-control)

- $\{p\} \succ \{q\}$
- $\mathcal{C}(\{p, q\}) = p$
- Knows she will choose p from $\{p, q\}$ ("sophisticated")
- Also knows she will exert costly self-control to resist q
- $\{p, q\} \nsim \{p\}$
- We (incorrectly) infer she is "not sophisticated"

- Self-control costs imply value of a menu less than ultimate choice.
- In the presence of costly self-control, our definition of naivete is sufficient, but not necessary, for overly optimistic beliefs.

Return

Naivete and Procrastination

O'Donoghue and Rabin (1999, 2001): Naivete can lead to procrastination in completing tasks with immediate costs and delayed rewards.

The converse is also true:

- Let d_1, d_2, d_3 denote doing it now, tomorrow, or in two periods.
- Choice in first period:
  - $\{d_1\}$ (commit by doing it now) or
  - $\{d_2, d_3\}$ (have option to do it tomorrow or delay again)
- Procrastination:
  - $\{d_2, d_3\} \succ \{d_1\} \succ \{d_3\}$ in first period and
  - $\mathcal{C}(\{d_2, d_3\}) = d_3$ in the second.
- Procrastination implies strict naivete since $\{d_2, d_3\} \succ \{\mathcal{C}(\{d_2, d_3\})\}$.

Return

Finite-Dimensional Support

- Formally, $\mathcal{V}$ is the set of all continuous Bernoulli utilities $v : C \to \mathbb{R}$.
- With slight abuse of notation, also denote the corresponding expected-utility function on C by v:

$$v(p) = \int_C v(c)\, dp(c).$$

- $\mathcal{V}$ is a vector space.

Definition

A probability measure μ on $\mathcal{V}$ has finite-dimensional support if there exists a finite set of expected-utility functions $\{v_1, \ldots, v_n\} \subset \mathcal{V}$ such that $\text{supp}(\mu) \subset \text{span}(\{v_1, \ldots, v_n\})$.

- We are unaware of any applications of the random Strotz model that don't have finite-dimensional support.
  - For example, random QH discounting has two-dimensional support.
  - If C is finite, any $\mu \in \triangle(\mathcal{V})$ has finite-dimensional support.

Return

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