Expectations-Based Reference Points (Kőszegi and Rabin)

Todd Sarver

Econ 885 – Duke University

Outline

Reference Points and Reference Lotteries

Decision Trees and Plans

- Making plans you will actually follow: PE
- Choosing the best plan you will follow: PPE
- Application: Endowment Effect
- Application: Shopping

Choosing Between Decision Trees

- A general procedure
- Special case of full commitment: CPE

Connections to Other Models

Kőszegi and Rabin (2006)

Kőszegi and Rabin (2006) developed a model of expectations-based reference points:

Utility for riskless outcome: $u(c|r)$.

$c = (c_1, c_2, \dots, c_K) \in \mathbb{R}^K$ is consumption.

$r = (r_1, r_2, \dots, r_K) \in \mathbb{R}^K$ is "reference level".

Stochastic consumption: $U(F|r) = \int u(c|r)\, dF(c)$.

Stochastic reference point: $U(F|G) = \iint u(c|r)\, dG(r)\, dF(c)$.

- G is a reference lottery. For example, $c = \$50$ and r is \$0 with probability 0.5 and \$100 with probability 0.5. Interpretation: "mixed emotions for outcomes that compare differently with different counterfactuals" (footnote 3 in the paper).

$$U(F|G) = \iint u(c|r)\, dG(r)\, dF(c).$$

Utility is derived from "consumption utility" and "gain-loss utility":

$$u(c|r) \equiv m(c) + n(c|r).$$

Separability: $m(c) \equiv \sum_{k=1}^{K} m_k(c_k)$ and $n(c|r) \equiv \sum_{k=1}^{K} n_k(c_k|r_k)$.

- Separability of gain-loss utility will be important for obtaining the endowment effect.

Evaluation of gains and losses depends on the associated changes in consumption utility:

$$n_k(c_k|r_k) \equiv \mu(m_k(c_k) - m_k(r_k))$$

This ensures some degree of consistency between consumption utility and gain-loss utility.

$$U(F|G) = \iint \left[\sum_k m_k(c_k) + \sum_k \mu(m_k(c_k) - m_k(r_k)) \right] dG(r)\, dF(c).$$

Most of the analysis and applications in K-R is centered around the special case of:

$$\mu(x) = \begin{cases} \eta x & \text{if } x > 0 \\ \eta \lambda x & \text{if } x \leq 0. \end{cases}$$

We will focus on this case.

Theorem

Given the piecewise linear form of μ, the following hold.

1. *For all F, G, G' such that the marginals of G' first-order stochastically dominate the marginals of G in each dimension, $U(F|G) \geq U(F|G')$.*
2. *For any F, F' that do not generate the same distribution of outcomes in all dimensions, $U(F|F') \geq U(F'|F') \implies U(F|F) > U(F'|F)$.*

General assumptions about "universal gain-loss function" $\mu(\cdot)$:

A0. $\mu(x)$ is continuous for all x, twice differentiable for $x \neq 0$, and $\mu(0) = 0$.

A1. $\mu(x)$ is strictly increasing.

A2. If $y > x > 0$, then $\mu(y) + \mu(-y) < \mu(x) + \mu(-x)$.

A3. $\mu''(x) \leq 0$ for $x > 0$, and $\mu''(x) \geq 0$ for $x < 0$.

A4. $\mu'(0-)/\mu'(0+) = \lambda > 1$.

- A2 and A4 capture loss aversion (and first-order risk aversion).
- A3 captures diminishing sensitivity: the marginal change in gain-loss sensations is greater for changes that are close to one's reference level.

A3'. For all $x \neq 0$, $\mu''(x) = 0$. [Piecewise linear version from previous slide]

Theorem

If μ satisfies A0–A4, then the following hold.

1. *For all F, G, G' such that the marginals of G' first-order stochastically dominate the marginals of G in each dimension, $U(F|G) \geq U(F|G')$.*
2. *For any $c, c' \in \mathbb{R}^K$, $c \neq c'$, $u(c|c') \geq u(c'|c') \implies u(c|c) > u(c'|c)$.*
3. *Suppose that μ satisfies A3'. Then, for any F, F' that do not generate the same distribution of outcomes in all dimensions,*
   $U(F|F') \geq U(F'|F') \implies U(F|F) > U(F'|F)$.

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Connections to Other Models

Simple Case: Deterministic Choice Sets

Suppose the individual is has a choice set $D = \{F, G\}$.

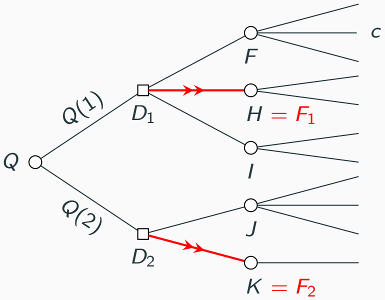
- If she plans to choose F, then F becomes her reference lottery.
- When choosing from D, she follows through on her plan if $U(F|F) \geq U(G|F)$.
- Otherwise, if $U(G|F) > U(F|F)$, then this plan is not feasible.
- In this case, the theorem implies $U(G|G) > U(F|G)$, so choosing G is a feasible (equilibrium) plan.

What if the choice set is not certain? (E.g., unknown prices, wages, or other relevant conditions.)

Timeline for General Case

Lotteries Q over choice sets D of lotteries F (i.e., trees):

- Stage 1:
  - Stage 1a: Make a contingent plan $\{F_l\}_{l \in \mathbb{R}}$ from sets D_l in the support of Q.
  - Stage 1b: Induced distribution of outcomes $G = \int F_l \, dQ(l)$ becomes reference lottery.



In this example, $G(c) = Q(1)H(c) + Q(2)K(c)$

Timeline for General Case

Lotteries Q over choice sets D of lotteries F (i.e., trees):

- Stage 1:
  - Stage 1a: Make a contingent plan $\{F_l\}_{l \in \mathbb{R}}$ from sets D_l in the support of Q.
  - Stage 1b: Induced distribution of outcomes $G = \int F_l \, dQ(l)$ becomes reference lottery.
- Stage 2:
  - Stage 2a: First-stage lottery Q resolves yielding D_l.
  - Stage 2b: Individual must decide whether to follow plan F_l or deviate to another $F_l' \in D_l$ given reference lottery G.
  - Stage 2c: Second-stage lottery F_l (or F_l') resolves.[1]

[1] In many applications of this model, the F_l are degenerate (i.e., deterministic), so the only uncertainty is about the constraint set.

Personal Equilibrium

$D_l \subset \triangle(\mathbb{R}^K)$ is a choice set for each index $l \in \mathbb{R}$.

$Q \in \triangle(\mathbb{R})$ is a distribution over indices (typically finite support, e.g., $l = 1, 2$)

Definition

A selection $\{F_l \in D_l\}_{l \in \mathbb{R}}$ is a personal equilibrium (PE) if for all $l \in \mathbb{R}$ and $F_l' \in D_l$,

$$U\left(F_l \middle| \int F_l dQ(l)\right) \geq U\left(F_l' \middle| \int F_l dQ(l)\right)$$

(Kőszegi and Rabin (2007) call this an unacclimating personal equilibrium (UPE).)

- Choice set D_l is uncertain.
- Individual makes a contingent plan $\{F_l \in D_l\}_{l \in \mathbb{R}}$.
- This forms a compound lottery over outcomes. The reduction of this lottery is the reference lottery: $G = \int F_l dQ(l)$.
- PE: she will not deviate from her plan once choice set D_l is realized.

Personal Equilibrium as a constraint

- Personal Equilibrium does not describe an "optimal" plan for an individual.
- PE instead describes a constraint on the set of plans: A plan is a PE iff she will be able to follow through on that plan.
- If each D_l is convex, a PE exists by Theorem 1 in Koszegi (2010): This constraint set is nonempty.

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Connections to Other Models

Preferred Personal Equilibrium

There may be multiple personal equilibria. They can be ranked according to ex-ante expected utility.

Definition

A selection $\{F_l \in D_l\}_{l \in \mathbb{R}}$ is a preferred personal equilibrium (PPE) if it is a PE, and

$$U\big(\int F_l dQ(l) \big| \int F_l dQ(l)\big) \geq U\big(\int F_l' dQ(l) \big| \int F_l' dQ(l)\big)$$

for all PE selections $\{F_l' \in D_l\}_{l \in \mathbb{R}}$.

Think of PE as a constraint on the contingent plans that the individual can make (is willing to follow through on). The PPE gives the highest ex-ante expected utility subject to this constraint.

Theorem

Suppose Q puts probability one on some menu D that is the set of all convex combinations of a set D^* of deterministic outcomes. If A3' holds, then a lottery is a PPE if and only if it puts probability 1 on an outcome that is a solution to $\max_{c \in D^*} m(c)$.

Interpretation: Loss aversion does not enter into the model when there is no uncertainty.

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Connections to Other Models

Two-Dimensional Consumption Space

Assume $m(c) = c_1 + c_2$, and

$$U((c_1, c_2)|(r_1, r_2)) = \sum_{i=1}^{2} \Big[c_i + \mu(c_i - r_i) \Big]$$

where

$$\mu(x) = \begin{cases} \eta x & \text{if } x > 0 \\ \eta\lambda x & \text{if } x \leq 0. \end{cases}$$

Thus, for $c = (c_1, c_2)$ and $r = (r_1, r_2)$, we have:

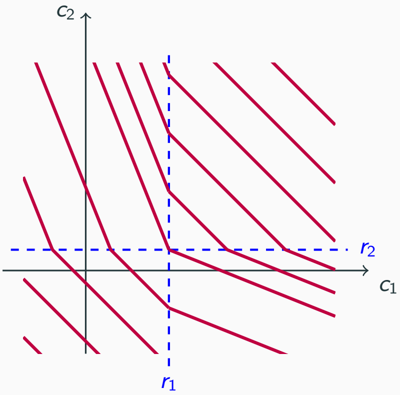
$$U(c|r) = \begin{cases} c_1 + c_2 + \eta(c_1 - r_1) + \eta(c_2 - r_2) & \text{if } c_1 \geq r_1, c_2 \geq r_2 \\ c_1 + c_2 + \eta(c_1 - r_1) + \eta\lambda(c_2 - r_2) & \text{if } c_1 \geq r_1, c_2 < r_2 \\ c_1 + c_2 + \eta\lambda(c_1 - r_1) + \eta(c_2 - r_2) & \text{if } c_1 < r_1, c_2 \geq r_2 \\ c_1 + c_2 + \eta\lambda(c_1 - r_1) + \eta\lambda(c_2 - r_2) & \text{if } c_1 < r_1, c_2 < r_2 \end{cases}$$

Illustration

$$U(c|r) = \begin{cases} c_1 + c_2 + \eta(c_1 - r_1) + \eta(c_2 - r_2) & \text{if } c_1 \geq r_1, c_2 \geq r_2 \\ c_1 + c_2 + \eta(c_1 - r_1) + \eta\lambda(c_2 - r_2) & \text{if } c_1 \geq r_1, c_2 < r_2 \\ c_1 + c_2 + \eta\lambda(c_1 - r_1) + \eta(c_2 - r_2) & \text{if } c_1 < r_1, c_2 \geq r_2 \\ c_1 + c_2 + \eta\lambda(c_1 - r_1) + \eta\lambda(c_2 - r_2) & \text{if } c_1 < r_1, c_2 < r_2 \end{cases}$$

The MRS between c_1 and c_2 is:

- 1 if $c_1 > r_1$ and $c_2 > r_2$, or if $c_1 < r_1$ and $c_2 < r_2$
- $\frac{1+\eta}{1+\eta\lambda} < 1$ if $c_1 > r_1$ and $c_2 < r_2$
- $\frac{1+\eta\lambda}{1+\eta} > 1$ if $c_1 < r_1$ and $c_2 > r_2$



Endowment Effect

- The endowment effect states that an individual's value for a good is higher when she anticipates consuming that good.
- For example, Kahneman, Knetsch, and Thaler (1990) randomly gave out consumption objects (such as mugs and pens) to half of the subjects in an experiment, and they found evidence that values for these objects were higher among those who were given the object.
- By the previous observations, the Kőszegi and Rabin (2006) model can explain this pattern (see also Tversky and Kahneman (1991)):
  - Take $r_1 = 0$ (not endowed with the good) and $r_2 = 0$ (no payments) to get the no-endowment reservation price:
    $$U((1,-p)|(0,0)) = U((0,0)|(0,0)) \implies p = \frac{1+\eta}{1+\eta\lambda} < 1$$
  - Take $r_1 = 1$ (endowed with the good) and $r_2 = 0$ (no payments) to get the with-endowment reservation price:
    $$U((1,0)|(1,0)) = U((0,p)|(1,0)) \implies p = \frac{1+\eta\lambda}{1+\eta} > 1$$

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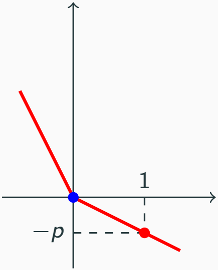

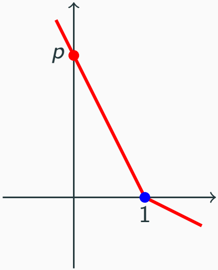

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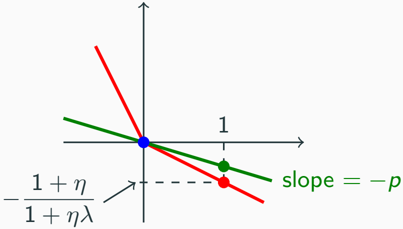
Connections to Other Models

Shoe Shopping

Assumptions:

- $m(c) = c_1 + c_2$
- $c_1 \in \{0, 1\}$ indicates whether the consumer gets a pair of shoes.
- $c_2 = -p$ if the consumer pays price p.

The consumer will strictly prefer to buy at price p with reference point of not buying if and only if



$$\begin{aligned} & U((1,-p)|(0,0)) > U((0,0)|(0,0)) \\ \iff & 1 - p + \eta - \eta\lambda p > 0 \\ \iff & p < \frac{1+\eta}{1+\eta\lambda} \end{aligned} \tag{1}$$

The consumer is willing to buy at price p with reference point of buying at price $\hat{p}$ if and only if

$$\begin{aligned} & U((0,0)|(1,-\hat{p})) \leq U((1,-p)|(1,-\hat{p})) \\ \iff & -\eta\lambda + \eta\hat{p} \leq \begin{cases} 1 - p - \eta\lambda(p - \hat{p}) & \text{if } p > \hat{p} \\ 1 - p + \eta(\hat{p} - p) & \text{if } p \leq \hat{p} \end{cases} \\ \iff & 0 \leq \begin{cases} 1 + \eta\lambda - (1+\eta\lambda)p + (\eta\lambda - \eta)\hat{p} & \text{if } p > \hat{p} \\ 1 + \eta\lambda - (1+\eta)p & \text{if } p \leq \hat{p} \end{cases} \end{aligned} \tag{2}$$

In particular, taking $\hat{p} = p$,

$$U((1,-p)|(1,-p)) \geq U((0,0)|(1,-p)) \iff p \leq \frac{1+\eta\lambda}{1+\eta}$$

It is also interesting to note that with a reference point $(1, -\hat{p})$, the individual is always willing to buy if $p \leq 1$ This follows because the worst-case scenario in Equation (2) is when $\hat{p} = 0$.

Summing up

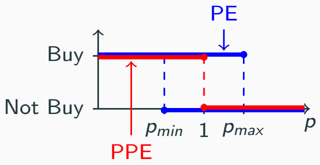
Combining these observations, we have

$$U((1,-p)|(0,0)) > U((0,0)|(0,0)) \iff p < p_{min} \equiv \frac{1+\eta}{1+\eta\lambda}$$

$$U((1,-p)|(1,-p)) \geq U((0,0)|(1,-p)) \iff p \leq p_{max} \equiv \frac{1+\eta\lambda}{1+\eta}.$$

If p is known in advance, this gives the following characterization of PE from $D = \{(0,0),(1,-p)\}$:

$$PE = \begin{cases} \{(1,-p)\} & \text{if } p < p_{min} \\ \{(1,-p),(0,0)\} & \text{if } p_{min} \leq p \leq p_{max} \\ \{(0,0)\} & \text{if } p > p_{max} \end{cases}$$



- The PPE is to buy if $p < 1$ and not buy if $p > 1$ and both if $p = 1$.
- Note that PPE boils down to maximizing $m(c)$ (this shouldn't be a surprise: recall the last theorem).

Surprise Price Changes

Suppose the consumer believes the price will be $p_L \leq 1$ with probability one. K-R ask the "out-of-equilibrium" question of whether the individual would then buy if the price were an instead a higher price p_H. This can also be interpreted as examining her behavior in a probability zero event. The consumer is willing to buy at this price if and only if

$$\begin{aligned} U((1,-p_H)|(1,-p_L)) \geq U((0,0)|(1,-p_L)) \\ \iff 1 - p_H - \eta\lambda(p_H - p_L) \geq -\eta\lambda + \eta p_L \\ \iff (1+\eta\lambda)p_H \leq 1 + \eta\lambda + \eta(\lambda - 1)p_L \\ \iff p_H \leq 1 + p_L \frac{\eta(\lambda-1)}{1+\eta\lambda}. \end{aligned}$$

The expectation of purchasing shoes creates an "attachment effect" that increases willingness to pay. In the PPE where the consumer plans to buy at a price p_L, she can also be induced to buy at a price $p_H > 1$.

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Connections to Other Models

Timeline

Consider adding the choice of two-stage lottery Q, R, etc.:

- Stage 0: Select a two-stage lottery Q *(this choice not modeled by KR)*
- Stage 1:
  - Stage 1a: Make a contingent plan $\{F_I\}_{I\in\mathbb{R}}$ from sets D_I in the support of Q.
  - Stage 1b: Induced distribution of outcomes $G = \int F_I \, dQ(I)$ becomes reference lottery.
- Stage 2:
  - Stage 2a: First-stage lottery Q resolves yielding D_I.
  - Stage 2b: Individual must decide whether to follow plan F_I or deviate to another $F_I' \in D_I$ given reference lottery G.
  - Stage 2c: Second-stage lottery F_I (or F_I') resolves.

How do we model the choice between Q and R?

Choice between two-stage lotteries

Consider a pair of two-stage lotteries:

- Q is a distribution over choice sets D_l.
- R is a distribution over choice sets E_l.

Choosing between Q and R:

- The best feasible plan (PE) for Q is the PPE $\{F_l \in D_l\}_{l\in\mathbb{R}}$.
- The best feasible plan (PE) for R is the PPE $\{G_l \in E_l\}_{l\in\mathbb{R}}$.
- Q is preferred to R iff

$$U\big(\int F_l dQ(l)\big|\int F_l dQ(l)\big) \geq U\big(\int G_l dR(l)\big|\int G_l dR(l)\big)$$

For example, if I expect that the shoe store may increase prices with some probability, I may prefer to stay home and thereby commit to not buying (R) than to go to the store (Q). (E.g., see the discussion on p. 1149–50 in Kőszegi and Rabin (2006).)

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Connections to Other Models

Committing early on

- Stage 0: Select a two-stage lottery Q
- Stage 1:
  - Stage 1a: Make a contingent plan $\{F_l\}_{l \in \mathbb{R}}$ from sets D_l in the support of Q.
  - Stage 1b: Induced distribution of outcomes $G = \int F_l \, dQ(l)$ becomes reference lottery.
- Stage 2:
  - Stage 2a: First-stage lottery Q resolves yielding D_l.
  - Stage 2b: Individual must decide whether to follow plan F_l or deviate to another $F'_l \in D_l$ given reference lottery G.
  - Stage 2c: Second-stage lottery F_l (or F'_l) resolves.

Suppose Q is degenerate and puts probability one on a set D that contains only F (and R similarly corresponds to full commitment to G). The timeline above reduces to:

- Stage 0: Choose between available lotteries F, G, ...
- Stage 1: Chosen lottery becomes reference lottery.
- Stage 2: Lottery resolves.

Committing early on

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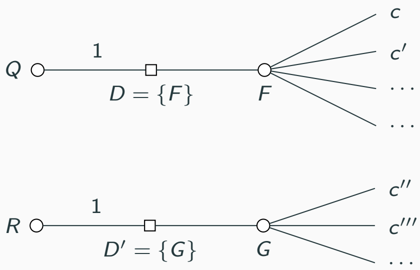

Choice-Acclimating Personal Equilibrium

$D \subset \triangle(\mathbb{R}^K)$ is a choice set.

Kőszegi and Rabin (2007) introduced the following concept for when the agent *commits to the choice early on*.

Definition

A lottery $F \in D$ is a choice-acclimating personal equilibrium (CPE) if for all $F' \in D$,

$$U(F|F) \geq U(F'|F').$$

- In words, consuming F with F as a reference lottery is better than consuming F' with F' as a reference lottery.
- We'll see next that CPE overlaps with two different models from non-expected-utility theory.

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Connections to Other Models

CPE and Quadratic Utility

- Assume there is only one dimension of consumption ($k = 1$).
- Write u instead of m, and assume it is increasing.
- Assume $\eta = 1$ (this is just a normalization).

Let $V_{CPE}(F) = U(F|F)$ denote the CPE value function:

$$V_{CPE}(F) = \iint \Big[u(x) + \mu(u(x) - u(y))\Big] \, dF(x) \, dF(y),$$

$$\mu(z) = \begin{cases} z & \text{if } z > 0 \\ \lambda z & \text{if } z \leq 0. \end{cases}$$

Clearly, CPE is a special case of Quadratic Utility.

Quadratic Utility (Chew, Epstein, and Segal (1991))

$$V_Q(F) = \iint \phi(x, y) \, dF(x) \, dF(y).$$

Properties of CPE

Before describing other connections, we need a few properties of CPE.

Example (CPE may not be FOSD monotone)

Assume $u(x) = x$. Fix any $x > 0$ and let

$$F = \begin{pmatrix} 0 & 1-\varepsilon \\ x & \varepsilon \end{pmatrix}$$

$$\begin{aligned} V_{CPE}(F) &= \varepsilon^2 x + (1-\varepsilon)\varepsilon(2x) + (1-\varepsilon)\varepsilon(-\lambda x) \\ &= \Big(\varepsilon + (1-\varepsilon)(2-\lambda)\Big)\varepsilon x \end{aligned}$$

If $\lambda > 2$, then for $\varepsilon > 0$ sufficiently small, $V_{CPE}(F) < 0$.

Violation of FOSD monotonicity.

Properties of CPE

$$V_{CPE}(F) = \iint \Big[u(x) + \mu(u(x) - u(y))\Big]\, dF(x)\, dF(y),$$

$$\mu(z) = \begin{cases} z & \text{if } z > 0 \\ \lambda z & \text{if } z \le 0. \end{cases}$$

Theorem (FOSD)

V_{CPE} respects FOSD iff $0 \le \lambda \le 2$.

Theorem (Risk Aversion)

1. *If $\lambda < 1$, then V_{CPE} does not respect SOSD (is not risk-averse).*
2. *If $1 \le \lambda \le 2$ and u is concave, then V_{CPE} respects SOSD (is risk-averse).*

CPE and RDU

Rank-Dependent Utility

$$V_{RDU}(F) = \int u(x)\, d(g \circ F)(x)$$

A CPE representation is monotone if it respects FOSD (i.e., $0 \leq \lambda \leq 2$).

Theorem

Any preference with a monotone CPE representation (u, λ) has an RDU representation (u, g_λ) where $g_\lambda(z) = \lambda z + (1 - \lambda) z^2$.

Note that g_λ is concave iff $\lambda \geq 1$, which matches our prior conditions for risk aversion for RDU.

Details, details. . .

Note that these slides use the cumulative formula for the probability distortion for RDU, whereas Masatlioglu and Raymond (2016) use the decumulative formula.

Some expressions will therefore be slightly different than in their paper. Their decumulative distortion function was

$$w_\lambda(z) = (2-\lambda)z + (\lambda-1)z^2.$$

A couple things that are true, but that you should work through and verify for yourself:

1. For any decumulative distortion function w, the corresponding cumulative distortion function is $g(z) = 1 - w(1-z)$.
2. In this particular case, $g_\lambda(z) = 1 - w_\lambda(1-z) = \lambda z + (1-\lambda)z^2$.

A Tight Characterization

What we know so far:

- Monotone CPE $\Longrightarrow$ QU
- Monotone CPE $\Longrightarrow$ RDU

It turns out that the combination of QU and RDU also implies CPE, giving a tight characterization. The following is the main result from Masatlioglu and Raymond (2016).

Theorem

A preference has both QU and RDU representations iff it has a monotone CPE representation.

Kahneman, D., J. L. Knetsch, and R. H. Thaler (1990): "Experimental Tests of the Endowment Effect and the Coase Theorem", *Journal of Political Economy*, 98, 1325–1348 (page 20).

Kőszegi, B. and M. Rabin (2006): "A Model of Reference-Dependent Preferences", *Quarterly Journal of Economics*, 121, 1133–1165 (pages 3, 20, 30).

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Masatlioglu, Y. and C. Raymond (2016): "A Behavioral Analysis of Stochastic Reference Dependence", *American Economic Review*, 106, 2760–2782 (pages 40, 41).

Tversky, A. and D. Kahneman (1991): "Loss Aversion in Riskless Choice: A Reference-Dependent Model", *Quarterly Journal of Economics*, 106, 1039–1061 (page 20).