Mixture-Averse Preferences

Todd Sarver

Econ 885 – Duke University

High level overview

Sarver (2018) develops a new class of recursive non-expected-utility preferences:

- Generalizes Epstein-Zin-Kreps-Porteus (EZKP) recursive expected utility. (Epstein and Zin (1989), Kreps and Porteus (1978))
- Main Axiom: ~~Independence~~ $\rightarrow$ Mixture Aversion
- Representation: Agent optimizes risk attitude subject to some constraint/cost.
- Applications to heterogeneous stock market participation and Rabin paradox (with background risk).

Outline

Overview and Intuition

- Mixture Aversion axiom
- Parametric special case of the model used in application
- Attitudes toward risk on the margin

Applications

- Heterogeneous Stock Market Participation
- Rabin paradox

Decision-Theoretic Analysis

- Recursive framework
- Axiom and representation result

Related Literature and Additional Analysis

- Related literature in decision theory
- Related literature on nonparticipation and summary of numerical results

Outline

Overview and Intuition

- Mixture Aversion axiom
- Parametric special case of the model used in application
- Attitudes toward risk on the margin

Applications

- Heterogeneous Stock Market Participation
- Rabin paradox

Decision-Theoretic Analysis

- Recursive framework
- Axiom and representation result

Related Literature and Additional Analysis

- Related literature in decision theory
- Related literature on nonparticipation and summary of numerical results

Overview: Mixture Aversion axiom

Suppose we use current consumption to measure an individual's value for increases in the probability of a better outcome next period (MRS).

Mixture Aversion axiom

Increasing the probability of a good future outcome makes additional increases even more desirable.

Overview: Mixture Aversion axiom

Suppose we use current consumption to measure an individual's value for increases in the probability of a better outcome next period (MRS).

Mixture Aversion axiom

Increasing the probability of a good future outcome makes additional increases even more desirable.

Example

Individual can exert additional effort now to increase the probability of a future promotion.

When would she be more willing to put forth effort:

- when initial chances are low and could be increased slightly?
- when initial probability is already high and could be made certain?

Overview: Mixture Aversion axiom

Suppose we use current consumption to measure an individual's value for increases in the probability of a better outcome next period (MRS).

Mixture Aversion axiom

Increasing the probability of a good future outcome makes additional increases even more desirable.

Example

Individual can exert additional effort now to increase the probability of a future promotion.

When would she be more willing to put forth effort:

- when initial chances are low and could be increased slightly?
- when initial probability is already high and could be made certain?

Mixture aversion $\Longrightarrow$ more willing in second scenario.
Time-separable EU $\Longrightarrow$ same willingness in either case.

Overview: Mixture Aversion axiom

Mixture Aversion axiom

Increasing the probability of a good future outcome makes additional increases even more desirable.

▸ Formal Definition

Connections:

- *Certainty effect (Allais):* individuals assign a premium to increases in probability that lead to certainty.
- *Probabilistic insurance (K-T):* decreasing the probability of insurance payment in the event of a loss leads to more than proportional decrease in willingness to pay for policy.

Overview: Mixture Aversion axiom

Mixture Aversion axiom

Increasing the probability of a good future outcome makes additional increases even more desirable.

▸ Formal Definition

Connections:

- *Certainty effect (Allais):* individuals assign a premium to increases in probability that lead to certainty.
- *Probabilistic insurance (K-T):* decreasing the probability of insurance payment in the event of a loss leads to more than proportional decrease in willingness to pay for policy.

Significant features permitted by the axiom:

1. First-order risk aversion.
2. Aversion to marginal increase in risk may drop with exposure.

Overview: Mixture Aversion axiom

Mixture Aversion axiom

Increasing the probability of a good future outcome makes additional increases even more desirable.

▸ Formal Definition

Connections:

- *Certainty effect (Allais):* individuals assign a premium to increases in probability that lead to certainty.
- *Probabilistic insurance (K-T):* decreasing the probability of insurance payment in the event of a loss leads to more than proportional decrease in willingness to pay for policy.

Significant features permitted by the axiom:

1. First-order risk aversion. (Old)
2. Aversion to marginal increase in risk may drop with exposure. (New)

Overview: Mixture Aversion axiom

Mixture Aversion axiom

Increasing the probability of a good future outcome makes additional increases even more desirable.

▸ Formal Definition

Connections:

- *Certainty effect (Allais):* individuals assign a premium to increases in probability that lead to certainty.
- *Probabilistic insurance (K-T):* decreasing the probability of insurance payment in the event of a loss leads to more than proportional decrease in willingness to pay for policy.

Significant features permitted by the axiom:

1. First-order risk aversion. (Old)
2. Aversion to marginal increase in risk may drop with exposure. (New)

Outline

Overview and Intuition

- Mixture Aversion axiom
- Parametric special case of the model used in application
- Attitudes toward risk on the margin

Applications

- Heterogeneous Stock Market Participation
- Rabin paradox

Decision-Theoretic Analysis

- Recursive framework
- Axiom and representation result

Related Literature and Additional Analysis

- Related literature in decision theory
- Related literature on nonparticipation and summary of numerical results

Overview: Some utility functions

Illustration for consumption-savings problem with i.i.d. gross return R_t, wealth constraint $w_{t+1} = (w_t - c_t)R_{t+1}$.

Value function (Epstein-Zin)

$$\mathcal{V}(w_t) = \max_{c_t, w_{t+1}} \Big\{ u(c_t) + \beta \mathcal{R}(\mathcal{V}(w_{t+1})) \Big\}.$$

$\mathcal{R}(\mathcal{V}(w_{t+1}))$ is the risk adjusted continuation value.

Example (time-separable expected utility)

$$\mathcal{R}(\mathcal{V}(w_{t+1})) = \mathbb{E}_t\big[\mathcal{V}(w_{t+1})\big]$$

Example (Epstein-Zin-Kreps-Porteus expected utility)

$$\mathcal{R}(\mathcal{V}(w_{t+1})) = h^{-1}\mathbb{E}_t\big[h(\mathcal{V}(w_{t+1}))\big]$$

Overview: Some utility functions

Value function (Epstein-Zin)

$$\mathcal{V}(w_t) = \max_{c_t, w_{t+1}} \left\{ u(c_t) + \beta \mathcal{R}(\mathcal{V}(w_{t+1})) \right\}.$$

Example (time-separable expected utility)

$$\mathcal{R}(\mathcal{V}(w_{t+1})) = \mathbb{E}_t \left[\mathcal{V}(w_{t+1}) \right]$$

Example (Epstein-Zin-Kreps-Porteus expected utility)

$$\mathcal{R}(\mathcal{V}(w_{t+1})) = h^{-1} \mathbb{E}_t \left[h(\mathcal{V}(w_{t+1})) \right]$$

Example (new to this paper)

$$\mathcal{R}(\mathcal{V}(w_{t+1})) = \sup_{\theta \in \Theta} \left\{ h_\theta^{-1} \mathbb{E}_t \left[h_\theta(\mathcal{V}(w_{t+1})) \right] - \tau(\theta) \right\}$$

for family of transformations $\{h_\theta\}_{\theta \in \Theta}$ and "cost" function $\tau : \Theta \to \mathbb{R}_+$ with $\inf_{\theta \in \Theta} \tau(\theta) = 0$.

General Representation

Overview: Getting more specific

$$\mathcal{V}(w_t) = \max_{c_t, w_{t+1}} \left\{ u(c_t) + \beta \mathcal{R}(\mathcal{V}(w_{t+1})) \right\}$$

Example (parametric version used in stock market application)

$$u(c) = (1-\beta)\log(c) \qquad h_\theta(x) = -\exp(-\theta x)$$

$$\Theta = \{\theta_L, \theta_H\}, \qquad \theta_H > \theta_L, \qquad \tau(\theta) = \begin{cases} \tau_H = 0 & \text{if } \theta = \theta_H \\ \tau_L > 0 & \text{if } \theta = \theta_L. \end{cases}$$

$$\mathcal{R}(\mathcal{V}(w_{t+1})) = \sup_{\theta \in \Theta} \left\{ -\frac{1}{\theta} \log\left(\mathbb{E}_t \left[\exp(-\theta \mathcal{V}(w_{t+1})) \right] \right) - \tau(\theta) \right\}.$$

Outline

Overview and Intuition

- Mixture Aversion axiom
- Parametric special case of the model used in application
- Attitudes toward risk on the margin

Applications

- Heterogeneous Stock Market Participation
- Rabin paradox

Decision-Theoretic Analysis

- Recursive framework
- Axiom and representation result

Related Literature and Additional Analysis

- Related literature in decision theory
- Related literature on nonparticipation and summary of numerical results

Overview: Attitudes toward increases in risk

$$\mathcal{V}(w_t) = \max_{c_t, w_{t+1}} \left\{ u(c_t) + \beta \mathcal{R}(\mathcal{V}(w_{t+1})) \right\}$$

$$\mathcal{R}(\mathcal{V}(w_{t+1})) = \sup_{\theta \in \Theta} \left\{ -\frac{1}{\theta} \log\left(\mathbb{E}_t \left[\exp(-\theta \mathcal{V}(w_{t+1})) \right] \right) - \tau(\theta) \right\}.$$

$$\Theta = \{\theta_L, \theta_H\}, \qquad \theta_H > \theta_L, \qquad \tau(\theta) = \begin{cases} \tau_H = 0 & \text{if } \theta = \theta_H \\ \tau_L > 0 & \text{if } \theta = \theta_L. \end{cases}$$

- When current risk exposure is small, θ_H is optimal

  $\Longrightarrow$ Less willing to take on additional risk on the margin.

- When current risk exposure is large, θ_L is optimal

  $\Longrightarrow$ More willing to take on additional risk on the margin.

Overview: Attitudes toward increases in risk

$$\mathcal{V}(w_t) = \max_{c_t, w_{t+1}} \left\{ u(c_t) + \beta \mathcal{R}(\mathcal{V}(w_{t+1})) \right\}$$

$$\mathcal{R}(\mathcal{V}(w_{t+1})) = \sup_{\theta \in \Theta} \left\{ -\frac{1}{\theta} \log\left(\mathbb{E}_t \left[\exp(-\theta \mathcal{V}(w_{t+1})) \right] \right) - \tau(\theta) \right\}.$$

$$\Theta = \{\theta_L, \theta_H\}, \qquad \theta_H > \theta_L, \qquad \tau(\theta) = \begin{cases} \tau_H = 0 & \text{if } \theta = \theta_H \\ \tau_L > 0 & \text{if } \theta = \theta_L. \end{cases}$$

- When current risk exposure is small, θ_H is optimal
  $\implies$ Less willing to take on additional risk on the margin.

- When current risk exposure is large, θ_L is optimal
  $\implies$ More willing to take on additional risk on the margin.

Implications:

- Insurance: high willingness to pay for low deductibles.
- Investment: indirect utility from allocation to risky asset can have multiple peaks/solutions.

Outline

Overview and Intuition

- Mixture Aversion axiom
- Parametric special case of the model used in application
- Attitudes toward risk on the margin

Applications

- Heterogeneous Stock Market Participation
- Rabin paradox

Decision-Theoretic Analysis

- Recursive framework
- Axiom and representation result

Related Literature and Additional Analysis

- Related literature in decision theory
- Related literature on nonparticipation and summary of numerical results

Outline

Overview and Intuition

- Mixture Aversion axiom
- Parametric special case of the model used in application
- Attitudes toward risk on the margin

Applications

- Heterogeneous Stock Market Participation
- Rabin paradox

Decision-Theoretic Analysis

- Recursive framework
- Axiom and representation result

Related Literature and Additional Analysis

- Related literature in decision theory
- Related literature on nonparticipation and summary of numerical results

Evidence

Two key facts:

- **Fact 1:** Many households have limited or no participation in equity markets.
- **Fact 2:** Although participation is positively correlated with wealth, a nontrivial fraction of wealthy households hold little or no public (or private) equity.

References: Mankiw and Zeldes (1991), Haliassos and Bertaut (1995), Heaton and Lucas (2000), Campbell (2006), Guiso and Sodini (2013)

Illustration: Endogenous Heterogeneity in Equilibrium

Consider a **static model** for illustration:

- Two states: $Z = \{z^l, z^h\}$. (assume equally likely for simplicity)
- Endowment e: $e(z^l) < e(z^h)$.
- Continuum of consumers: $[0, 1]$.

Illustration: Endogenous Heterogeneity in Equilibrium

Consider a **static model** for illustration:

- Two states: $Z = \{z^l, z^h\}$. (assume equally likely for simplicity)
- Endowment e: $e(z^l) < e(z^h)$.
- Continuum of consumers: $[0, 1]$.

Consumers have identical homothetic CRRA preferences over $c(z)$:

$$\begin{aligned}\mathcal{R}(\log(c)) &= \sup_{\theta \in \Theta} \left\{ -\frac{1}{\theta} \log\Big(\mathbb{E}\big[\exp(-\theta \log(c))\big]\Big) - \tau(\theta) \right\} \\ &= \sup_{\theta \in \Theta} \left\{ \log\left(\mathbb{E}\big[c^{-\theta}\big]^{-\frac{1}{\theta}}\right) - \tau(\theta) \right\}\end{aligned}$$

$$\Theta = \{\theta_L, \theta_H\}, \qquad \theta_H > \theta_L, \qquad \tau(\theta) = \begin{cases} \tau_H = 0 & \text{if } \theta = \theta_H \\ \tau_L > 0 & \text{if } \theta = \theta_L. \end{cases}$$

Illustration: Endogenous Heterogeneity in Equilibrium

Preferences over State-Contingent Consumption

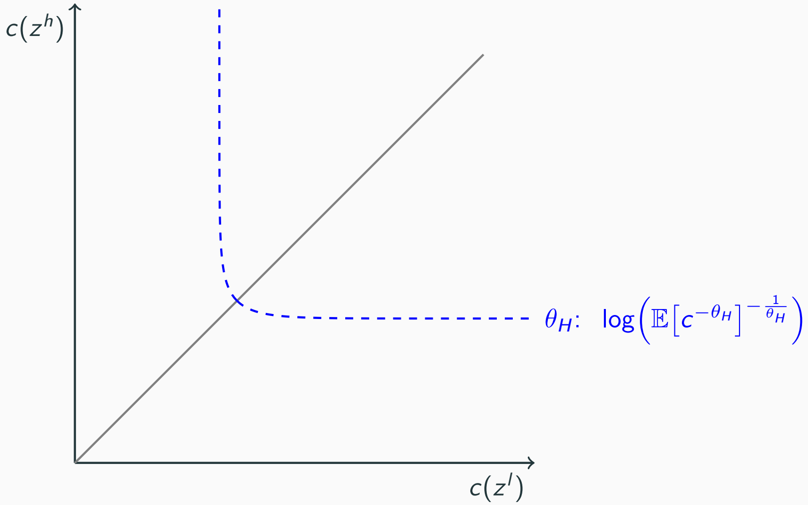


Illustration: Endogenous Heterogeneity in Equilibrium

Preferences over State-Contingent Consumption

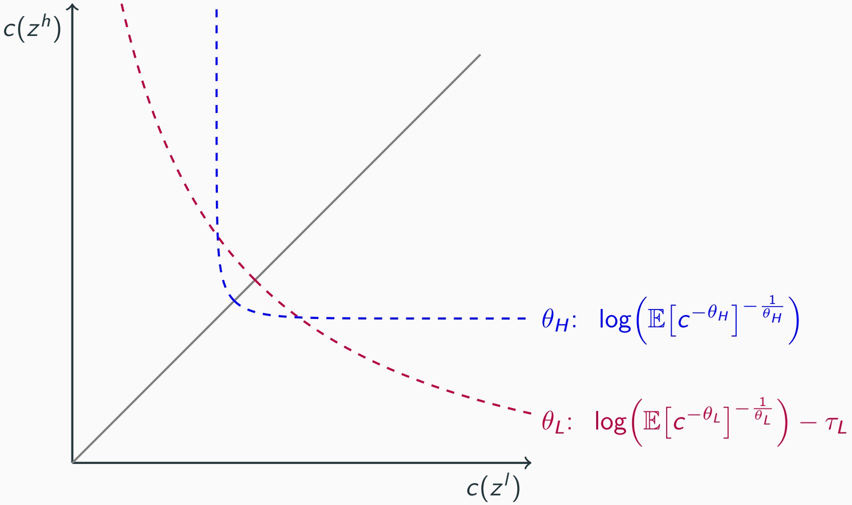


Illustration: Endogenous Heterogeneity in Equilibrium

Preferences over State-Contingent Consumption

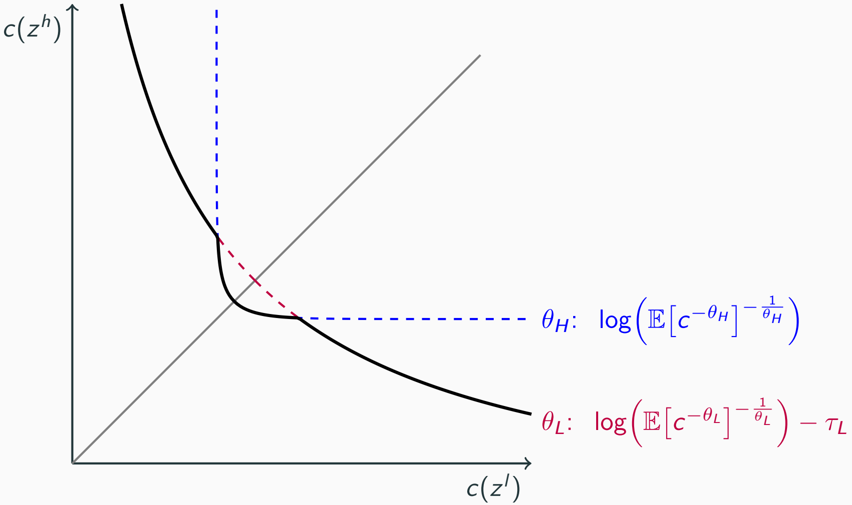
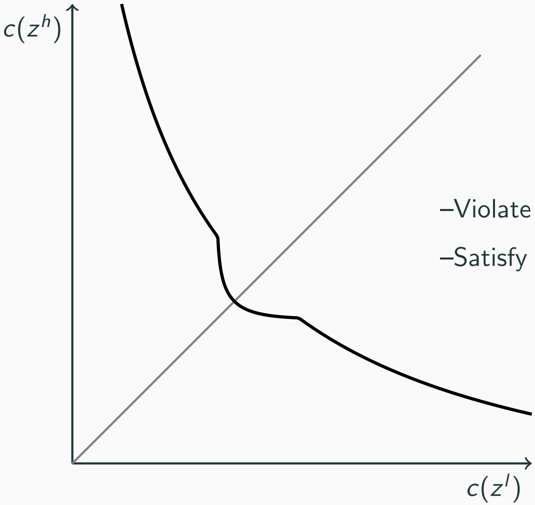


Illustration: Endogenous Heterogeneity in Equilibrium

Preferences over State-Contingent Consumption



–Violate Preference for Diversification

–Satisfy SOSD

Illustration: Endogenous Heterogeneity in Equilibrium

Equilibrium Case 1: All consumers type θ_L

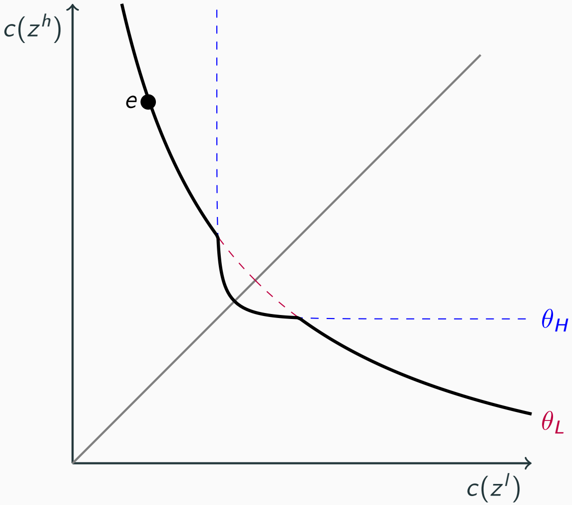


Illustration: Endogenous Heterogeneity in Equilibrium

Equilibrium Case 1: All consumers type θ_L

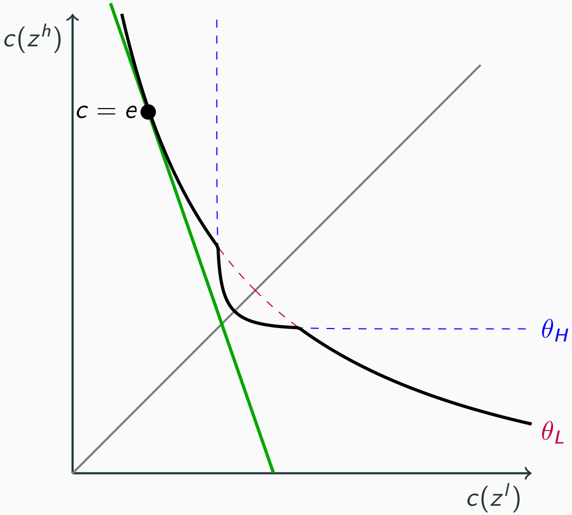


Illustration: Endogenous Heterogeneity in Equilibrium

Equilibrium Case 2: All consumers type θ_H

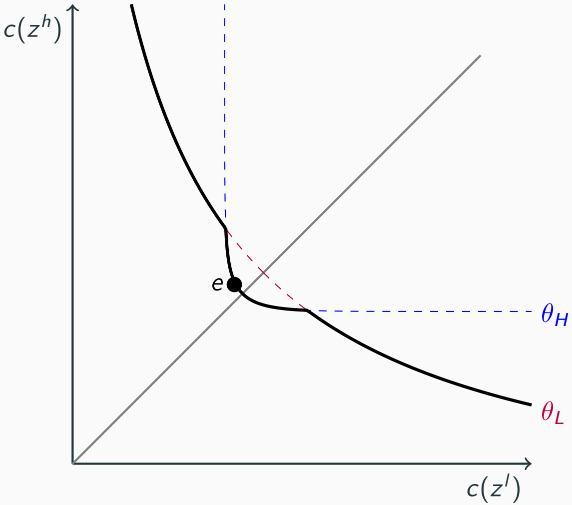


Illustration: Endogenous Heterogeneity in Equilibrium

Equilibrium Case 2: All consumers type θ_H

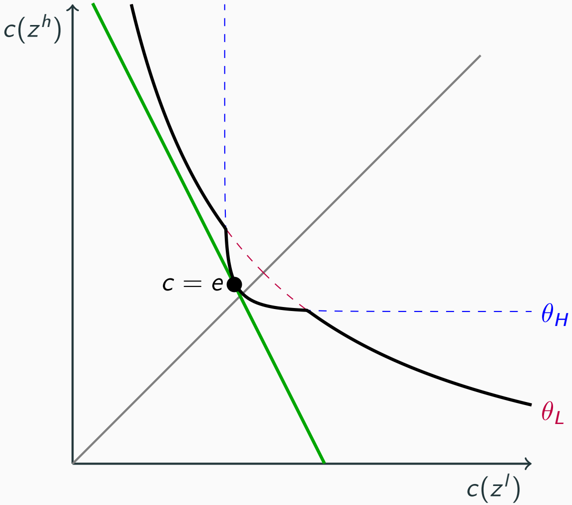


Illustration: Endogenous Heterogeneity in Equilibrium

Equilibrium Case 3: Heterogeneous types

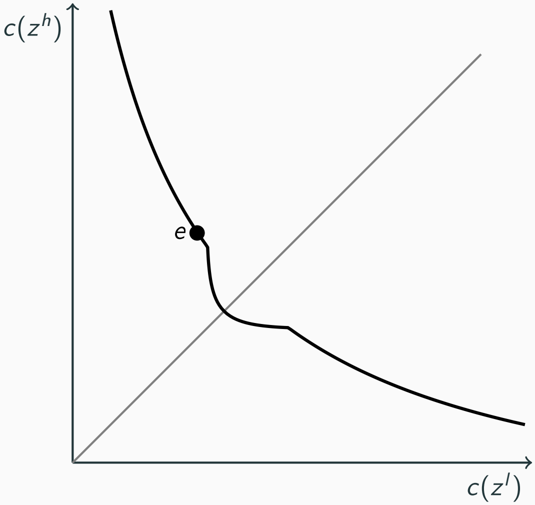


Illustration: Endogenous Heterogeneity in Equilibrium

Equilibrium Case 3: Heterogeneous types

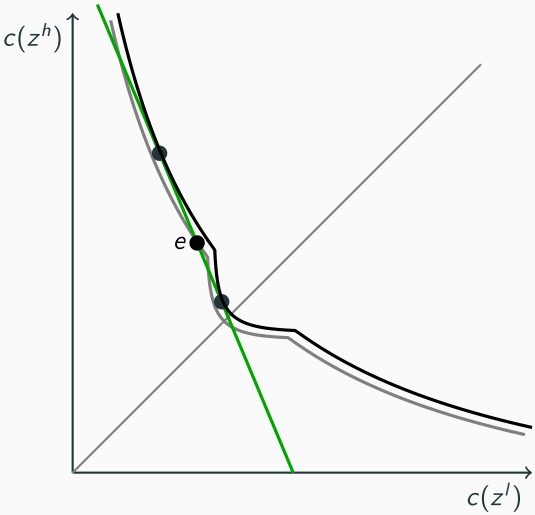
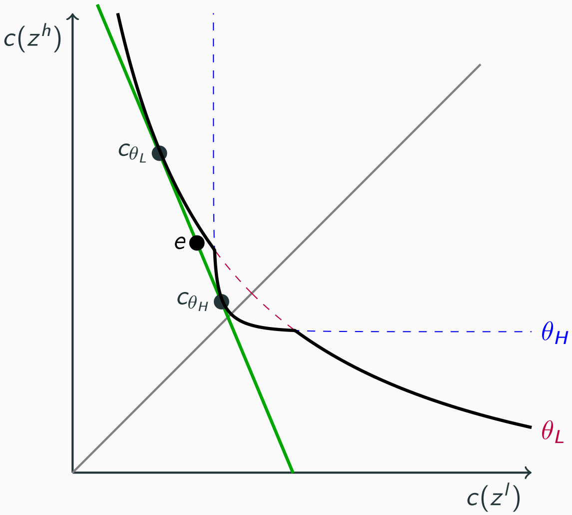


Illustration: Endogenous Heterogeneity in Equilibrium

Equilibrium Case 3: Heterogeneous types



Market-clearing: $e = \alpha c_{\theta_L} + (1-\alpha)c_{\theta_H}$, $\alpha =$ fraction type θ_L

Outline

Overview and Intuition

- Mixture Aversion axiom
- Parametric special case of the model used in application
- Attitudes toward risk on the margin

Applications

- Heterogeneous Stock Market Participation
- Rabin paradox

Decision-Theoretic Analysis

- Recursive framework
- Axiom and representation result

Related Literature and Additional Analysis

- Related literature in decision theory
- Related literature on nonparticipation and summary of numerical results

Utility specifications: EU, ORA, RDU

1. The first four specifications use the certainty equivalent defined by

$$\sup_{\theta \in \Theta} \left\{ \log\left(\mathbb{E}\left[w^{-\theta} \right]^{-\frac{1}{\theta}} \right) - \tau(\theta) \right\}.$$

   1.1 EZKP1 and EZKP2 assume $\Theta = \{\theta_H\}$, and gambles are therefore evaluated using expected-utility preferences with a CRRA of $\theta_H + 1$.

   1.2 ORA1 and ORA2 assume $\Theta = \{\theta_L, \theta_H\}$ where $\theta_L < \theta_H$ and $0 = \tau(\theta_H) < \tau(\theta_L) \equiv \tau_L$.

2. Specification RDU1 instead uses the following special case of rank-dependent utility:

$$\int \log(w)\, d(g \circ F)(w),$$

   where F is the cumulative distribution of wealth, and g is defined by

$$g(\alpha) = \begin{cases} (1+\theta_H)\alpha & \text{for } \alpha \le 1/2 \\ (1-\theta_H)\alpha + \theta_H & \text{for } \alpha > 1/2. \end{cases}$$

Rabin paradox

	Risk-Preference Model				
	EZKP1	EZKP2	ORA1	ORA2	RDU1
θ_H	3.00	25.00	25.00	145.00	0.20
θ_L	–	–	3.00	3.00	–
τ_L	–	–	0.02	0.02	–
Panel A: Binary 50-50 Gambles					
Loss	Gain that leads to indifference for wealth $300,000				
$100	100.13	100.87	100.87	105.12	150.06
$400	402.14	414.37	414.37	497.15	601.00
$1,000	1,013.51	1,094.95	1,094.95	2,023.89	1,506.27
$5,000	5,357.20	8,995.81	8,995.81	18,991.43	7,659.35
$10,000	11,539.60	∞	26,396.79	26,396.79	15,650.25
$20,000	27,302.60	∞	45,692.48	45,692.48	32,710.18

Rabin paradox

	Risk-Preference Model				
	EZKP1	EZKP2	ORA1	ORA2	RDU1
θ_H	3.00	25.00	25.00	145.00	0.20
θ_L	–	–	3.00	3.00	–
τ_L	–	–	0.02	0.02	–

Panel A: Binary 50-50 Gambles

Loss	Gain that leads to indifference for wealth $300,000				
$100	100.13	100.87	100.87	105.12	150.06
$400	402.14	414.37	414.37	497.15	601.00
$1,000	1,013.51	1,094.95	1,094.95	2,023.89	1,506.27
$5,000	5,357.20	8,995.81	8,995.81	18,991.43	7,659.35
$10,000	11,539.60	∞	26,396.79	26,396.79	15,650.25
$20,000	27,302.60	∞	45,692.48	45,692.48	32,710.18

Rabin paradox with background risk

	Risk-Preference Model				
	EZKP1	EZKP2	ORA1	ORA2	RDU1
θ_H	3.00	25.00	25.00	145.00	0.20
θ_L	–	–	3.00	3.00	–
τ_L	–	–	0.02	0.02	–

Panel B: Binary 50-50 Gambles with Background Risk					
Loss	Gain that leads to indifference for wealth \$300,000 $\pm$ \$7,000				
\$100	100.13	100.89	100.89	105.25	100.03
\$400	402.15	414.56	414.56	500.07	400.54
\$1,000	1,013.55	1,096.33	1,096.33	2,081.18	1,003.36
\$5,000	5,358.24	9,098.40	9,098.40	6,814.09	5,085.24
\$10,000	11,544.43	∞	22,003.17	13,123.15	11,990.94
\$20,000	27,329.83	∞	40,223.31	29,257.73	28,855.27

Rabin paradox with background risk

	Risk-Preference Model				
	EZKP1	EZKP2	ORA1	ORA2	RDU1
θ_H	3.00	25.00	25.00	145.00	0.20
θ_L	–	–	3.00	3.00	–
τ_L	–	–	0.02	0.02	–
Panel B: Binary 50-50 Gambles with Background Risk					
Loss	Gain that leads to indifference for wealth $300,000 $\pm$ $7,000				
$100	100.13	100.89	100.89	105.25	100.03
$400	402.15	414.56	414.56	500.07	400.54
$1,000	1,013.55	1,096.33	1,096.33	2,081.18	1,003.36
$5,000	5,358.24	9,098.40	9,098.40	6,814.09	5,085.24
$10,000	11,544.43	∞	22,003.17	13,123.15	11,990.94
$20,000	27,329.83	∞	40,223.31	29,257.73	28,855.27

Outline

Overview and Intuition

- Mixture Aversion axiom
- Parametric special case of the model used in application
- Attitudes toward risk on the margin

Applications

- Heterogeneous Stock Market Participation
- Rabin paradox

Decision-Theoretic Analysis

- Recursive framework
- Axiom and representation result

Related Literature and Additional Analysis

- Related literature in decision theory
- Related literature on nonparticipation and summary of numerical results

Outline

Overview and Intuition

- Mixture Aversion axiom
- Parametric special case of the model used in application
- Attitudes toward risk on the margin

Applications

- Heterogeneous Stock Market Participation
- Rabin paradox

Decision-Theoretic Analysis

- Recursive framework
- Axiom and representation result

Related Literature and Additional Analysis

- Related literature in decision theory
- Related literature on nonparticipation and summary of numerical results

Recursive Framework

Consumption space C (compact and connected metrizable)

- 1-period consumption lotteries: $\triangle(C)$
- 2-period temporal lotteries: $\triangle(C \times \triangle(C))$
- 3-period temporal lotteries: $\triangle(C \times \triangle(C \times \triangle(C)))$

Infinite-horizon temporal lotteries

$$D \approx C \times \triangle(D).$$

Mertens and Zamir (1985), Epstein and Zin (1989), Brandenburger and Dekel (1993), Gul and Pesendorfer (2004)

- Typical elements $(c, m) \in D$ where $c \in C$ and $m \in \triangle(D)$.
- Binary relation $\succsim$ on D.

Epstein-Zin Utility

Definition

A certainty equivalent is a continuous function $W : \triangle([a, b]) \to \mathbb{R}$ that satisfies $W(\delta_x) = x$ for all $x \in [a, b]$ and is FOSD monotone.

Epstein-Zin Utility

Definition

A certainty equivalent is a continuous function $W : \triangle([a, b]) \to \mathbb{R}$ that satisfies $W(\delta_x) = x$ for all $x \in [a, b]$ and is FOSD monotone.

For $V : D \to [a, b]$ and $m \in \triangle(D)$:

- $m \circ V^{-1}(E) = m(\{(\hat{c}, \hat{m}) \in D : V(\hat{c}, \hat{m}) \in E\})$.
- $m \circ V^{-1}$ is distribution of continuation values induced by m.

Epstein-Zin Utility

Definition

A certainty equivalent is a continuous function $W : \triangle([a, b]) \to \mathbb{R}$ that satisfies $W(\delta_x) = x$ for all $x \in [a, b]$ and is FOSD monotone.

For $V : D \to [a, b]$ and $m \in \triangle(D)$:

- $m \circ V^{-1}(E) = m(\{(\hat{c}, \hat{m}) \in D : V(\hat{c}, \hat{m}) \in E\})$.
- $m \circ V^{-1}$ is distribution of continuation values induced by m.

Definition

Epstein-Zin (EZ) value function $V : D \to \mathbb{R}$ satisfies:

$$V(c, m) = u(c) + \beta W(m \circ V^{-1}).$$

- $\beta \in (0, 1)$
- $u : C \to \mathbb{R}$ (continuous and nonconstant)
- $W : \triangle([a, b]) \to \mathbb{R}$ (certainty equivalent, $a = \min V$, $b = \max V$)

Outline

Overview and Intuition

- Mixture Aversion axiom
- Parametric special case of the model used in application
- Attitudes toward risk on the margin

Applications

- Heterogeneous Stock Market Participation
- Rabin paradox

Decision-Theoretic Analysis

- Recursive framework
- Axiom and representation result

Related Literature and Additional Analysis

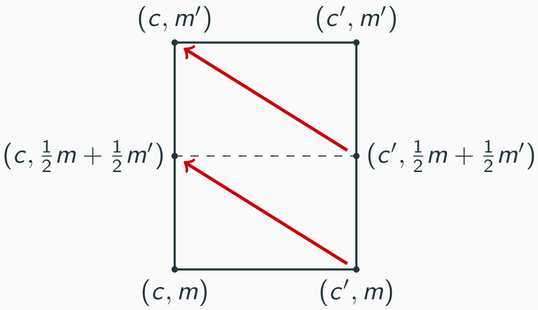
- Related literature in decision theory
- Related literature on nonparticipation and summary of numerical results

Mixture Aversion

Axiom (Mixture Aversion)

For any $c, c' \in C$ and $m, m' \in \triangle(D)$,

$$(c, \tfrac{1}{2}m + \tfrac{1}{2}m') \succsim (c', m) \implies (c, m') \succsim (c', \tfrac{1}{2}m + \tfrac{1}{2}m')$$



Return to Overview

ORA Representation

Definition

Optimal Risk Attitude (ORA) value function $V : D \to \mathbb{R}$ satisfies:

$$V(c, m) = u(c) + \beta \sup_{\phi \in \Phi} \int_D \phi\big(V(\hat{c}, \hat{m})\big)\, dm(\hat{c}, \hat{m})$$

- $\beta \in (0, 1)$
- $u : C \to \mathbb{R}$ (continuous and nonconstant)
- Φ is a collection of continuous and nondecreasing functions $\phi : V(D) \to \mathbb{R}$ such that

$$\sup_{\phi \in \Phi} \phi(x) = x, \quad \forall x \in V(D).$$

Can write

$$V(c, m) = u(c) + \beta \sup_{\phi \in \Phi} \mathbb{E}_m\big[\phi(V)\big]$$

Return to Overview

Representation Result

$$V(c,m) = u(c) + \beta \sup_{\phi\in\Phi} \int_D \phi\big(V(\hat{c},\hat{m})\big)\, dm(\hat{c},\hat{m})$$

$$\sup_{\phi\in\Phi} \phi(x) = x, \quad \forall x \in V(D).$$

Theorem

Suppose $\succsim$ has an Epstein-Zin representation (V, u, W, β). The following are equivalent:

1. *$\succsim$ satisfies Mixture Aversion.*
2. *The certainty equivalent W in the EZ representation of $\succsim$ is convex in probabilities.*
3. *$\succsim$ has an Optimal Risk Attitude representation (V, u, Φ, β).*

Proof Sketch

1. Mixture aversion $\implies W$ is convex.

2. Standard duality results can then be applied to show W takes the form

$$W(\mu) = \sup_{\phi \in \Phi} \int_a^b \phi(v)\, d\mu(v), \qquad \mu \in \triangle([a, b])$$

   (showing each ϕ is nondecreasing requires a new result)

3. Using the change of variables formula,

$$\begin{aligned} V(c, m) &= u(c) + \beta W(m \circ V^{-1}) \\ &= u(c) + \beta \sup_{\phi \in \Phi} \int_a^b \phi(v)\, d(m \circ V^{-1})(v) \\ &= u(c) + \beta \sup_{\phi \in \Phi} \int_D \phi(V(\hat{c}, \hat{m}))\, dm(\hat{c}, \hat{m}). \end{aligned}$$

Parametric Examples

$$W(\mu) = \sup_{\gamma} \int \phi(x|\gamma,\theta)\, d\mu(x) \qquad \text{(choosing ref point)}$$

$$W(\mu) = \sup_{\theta} \sup_{\gamma} \left\{ \int \phi(x|\gamma,\theta)\, d\mu(x) - \tau(\theta) \right\} \qquad \text{(choosing RP \& sensitivity)}$$

Example (Smooth Transformation)

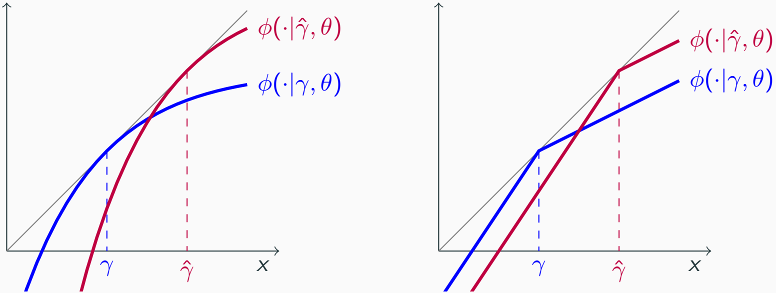
For $\gamma \in \mathbb{R}$ and $\theta > 0$,

$$\phi(x|\gamma,\theta) = \gamma + \frac{1}{\theta} - \frac{1}{\theta} \exp(-\theta(x-\gamma)).$$

Example (Kinked Transformation)

For $\gamma \in \mathbb{R}$ and $\theta \in [0,1]$,

$$\phi(x|\gamma,\theta) = \begin{cases} \gamma + (1-\theta)(x-\gamma) & \text{if } x \geq \gamma \\ \gamma + (1+\theta)(x-\gamma) & \text{if } x < \gamma. \end{cases}$$



Example (Smooth Transformation)

$$\phi(x|\gamma,\theta) = \gamma + \frac{1}{\theta} - \frac{1}{\theta}\exp(-\theta(x-\gamma)).$$

Example (Kinked Transformation)

$$\phi(x|\gamma,\theta) = \begin{cases} \gamma + (1-\theta)(x-\gamma) & \text{if } x \geq \gamma \\ \gamma + (1+\theta)(x-\gamma) & \text{if } x < \gamma. \end{cases}$$

Parametric Examples: EZKP Expected Utility $\subseteq$ ORA

Theorem

Suppose $h : \mathbb{R} \to \mathbb{R}$ is concave. For any value function $V : D \to \mathbb{R}$,

$$h^{-1}\big(\mathbb{E}_m\big[h(V)\big]\big) = \max_{\gamma \in \mathbb{R}} \mathbb{E}_m\big[\phi(V|\gamma)\big], \qquad \textit{where}$$

$$\phi(x|\gamma) = \gamma + \frac{h(x) - h(\gamma)}{h'(\gamma)}$$

Parametric Examples: EZKP Expected Utility $\subseteq$ ORA

Theorem

Suppose $h : \mathbb{R} \to \mathbb{R}$ is concave. For any value function $V : D \to \mathbb{R}$,

$$h^{-1}\big(\mathbb{E}_m\big[h(V)\big]\big) = \max_{\gamma\in\mathbb{R}} \mathbb{E}_m\big[\phi(V|\gamma)\big], \qquad \textit{where}$$

$$\phi(x|\gamma) = \gamma + \frac{h(x) - h(\gamma)}{h'(\gamma)}$$

Corollary

When $\phi(x|\gamma,\theta) = \gamma + \frac{1}{\theta} - \frac{1}{\theta}\exp(-\theta(x-\gamma))$,

$$-\frac{1}{\theta}\log\left(\mathbb{E}_m\big[\exp(-\theta V)\big]\right) = \sup_{\gamma\in\mathbb{R}} \mathbb{E}_m\big[\phi(V|\gamma,\theta)\big]$$

and

$$\sup_{\theta\in\Theta}\left\{-\frac{1}{\theta}\log\left(\mathbb{E}_m\big[\exp(-\theta V)\big]\right) - \tau(\theta)\right\} = \sup_{\theta\in\Theta}\sup_{\gamma\in\mathbb{R}} \mathbb{E}_m\big[\phi(V|\gamma,\theta) - \tau(\theta)\big]$$

Outline

Overview and Intuition

- Mixture Aversion axiom
- Parametric special case of the model used in application
- Attitudes toward risk on the margin

Applications

- Heterogeneous Stock Market Participation
- Rabin paradox

Decision-Theoretic Analysis

- Recursive framework
- Axiom and representation result

Related Literature and Additional Analysis

- Related literature in decision theory
- Related literature on nonparticipation and summary of numerical results

Outline

Overview and Intuition

- Mixture Aversion axiom
- Parametric special case of the model used in application
- Attitudes toward risk on the margin

Applications

- Heterogeneous Stock Market Participation
- Rabin paradox

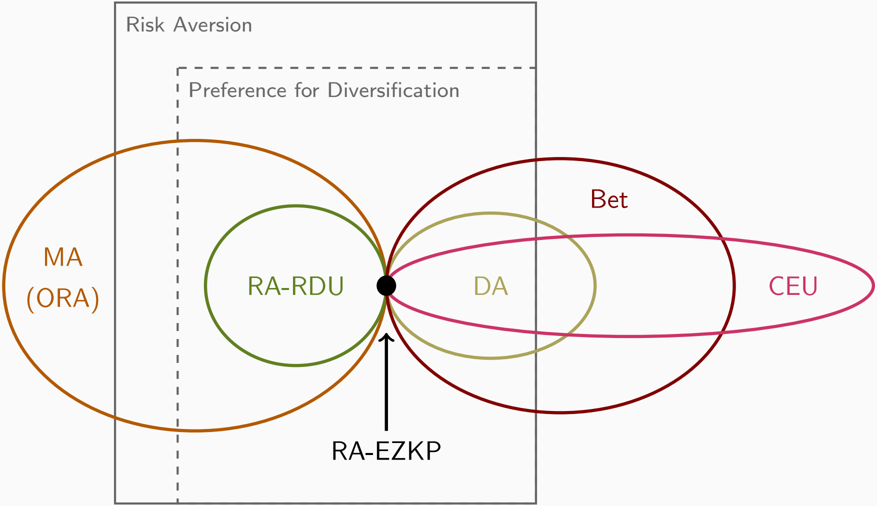
Decision-Theoretic Analysis

- Recursive framework
- Axiom and representation result

Related Literature and Additional Analysis

- Related literature in decision theory
- Related literature on nonparticipation and summary of numerical results

Related Non-EU Preferences



Key: mixture-averse preferences (MA), risk-averse Epstein-Zin-Kreps-Porteus expected utility (RA-EZKP), risk-averse rank-dependent utility (RA-RDU), betweenness (Bet), disappointment aversion (DA), cautious expected utility (CEU).

Outline

Overview and Intuition

- Mixture Aversion axiom
- Parametric special case of the model used in application
- Attitudes toward risk on the margin

Applications

- Heterogeneous Stock Market Participation
- Rabin paradox

Decision-Theoretic Analysis

- Recursive framework
- Axiom and representation result

Related Literature and Additional Analysis

- Related literature in decision theory
- Related literature on nonparticipation and summary of numerical results

Evidence and related literature

Two key facts:

Fact 1: Many households have limited or no participation in equity markets.

Fact 2: Although participation is positively correlated with wealth, a nontrivial fraction of wealthy households hold little or no public (or private) equity.

References: Mankiw and Zeldes (1991), Haliassos and Bertaut (1995), Heaton and Lucas (2000), Campbell (2006), Guiso and Sodini (2013)

Evidence and related literature

Some possible explanations:

Explanation 1: Expected utility with heterogeneity in risk aversion.

Explanation 2: Participation costs.

Explanation 3: First-order risk aversion or ambiguity aversion.

Evidence and related literature

Explanation 1: Expected utility with heterogeneity in risk aversion.

- Equity premium becomes less of a puzzle when attention is restricted to participants.
- But becomes more of a puzzle for those with little or no participation.

References: Mankiw and Zeldes (1991), Attanasio, Banks, and Tanner (2002), Brav, Constantinides, and Geczy (2002), Vissing-Jørgensen (2002), Vissing-Jørgensen and Attanasio (2003)

Evidence and related literature

Explanation 1: Expected utility with heterogeneity in risk aversion.

- Equity premium becomes less of a puzzle when attention is restricted to participants.
  (Shows up in calibration)
- But becomes more of a puzzle for those with little or no participation.

References: Mankiw and Zeldes (1991), Attanasio, Banks, and Tanner (2002), Brav, Constantinides, and Geczy (2002), Vissing-Jørgensen (2002), Vissing-Jørgensen and Attanasio (2003)

Evidence and related literature

EXPLANATION 2: Participation costs.

- Plausible values of entry and participation costs can rationalize the nonparticipation decisions of many households.
- But not very wealthy households.

References: Vissing-Jørgensen (2003), Gomes and Michaelides (2005), Briggs, Cesarini, Lindqvist, Östling (2015)

Evidence and related literature

EXPLANATION 2: Participation costs.

- Plausible values of entry and participation costs can rationalize the nonparticipation decisions of many households.
- But not very wealthy households.

(Complementary to ORA approach)

References: Vissing-Jørgensen (2003), Gomes and Michaelides (2005), Briggs, Cesarini, Lindqvist, Östling (2015)

Evidence and related literature

EXPLANATION 3: First-order risk aversion or ambiguity aversion.

Some minor differences from ORA approach:

- Sensitivity of first-order RA results to background risk.
- Ambiguous versus familiar environments.

A significant methodological difference:

- These approaches rely on heterogeneous preferences.
- Focus on partial equilibrium analysis (or single period GE analysis).

References: Ang, Bekaert, and Liu (2005), Barberis, Huang, and Thaler (2006), Dow and Werlang (1992), Epstein and Wang (1994), Epstein and Schneider (2007), Epstein and Miao (2003), Boyle, Garlappi, Uppal, and Wang (2012), Cao, Wang, and Zhang (2005), Chapman and Polkovnichenko (2009).

Evidence and related literature

Explanation 3: First-order risk aversion or ambiguity aversion.

Some minor differences from ORA approach:

- Sensitivity of first-order RA results to background risk.
- Ambiguous versus familiar environments.

A significant methodological difference:

- These approaches rely on heterogeneous preferences.
- Focus on partial equilibrium analysis (or single period GE analysis).

(ORA permits full GE analysis of asset prices and participation decisions.)

References: Ang, Bekaert, and Liu (2005), Barberis, Huang, and Thaler (2006), Dow and Werlang (1992), Epstein and Wang (1994), Epstein and Schneider (2007), Epstein and Miao (2003), Boyle, Garlappi, Uppal, and Wang (2012), Cao, Wang, and Zhang (2005), Chapman and Polkovnichenko (2009).

Infinite horizon: summary of calibration

The following calibration results come from Sarver (2017):

- Markov process (i.i.d.)
- Aggregate consumption growth to match MP (1985)
- Continuum of consumers
- Identical homogeneous preferences:

$$\mathcal{V}(w_t) = \max_{c_t, w_{t+1}} \Big\{(1-\beta)\log(c_t) + \beta\mathcal{R}(\mathcal{V}(w_{t+1}))\Big\},$$

$$\mathcal{R}(\mathcal{V}(w_{t+1})) = \max_{\theta\in\Theta}\left\{-\frac{1}{\theta}\log\big(\mathbb{E}\big[\exp(-\theta\mathcal{V}(w_{t+1}))\big]\big) - \tau(\theta)\right\}.$$

$$\Theta = \{\theta_L, \theta_H\}, \qquad \theta_H > \theta_L, \qquad \tau(\theta) = \begin{cases}\tau_H = 0 & \text{if } \theta = \theta_H \\ \tau_L > 0 & \text{if } \theta = \theta_L.\end{cases}$$

Value function has unique solution: $\mathcal{V}(w_t) = \Lambda + \log(w_t)$

- Utility from atemporal wealth gamble:

$$\mathcal{R}(\mathcal{V}(w_{t+1})) = \Lambda + \max_{\theta\in\Theta}\left\{\log\left(\mathbb{E}\left[w_{t+1}^{-\theta}\right]^{-\frac{1}{\theta}}\right) - \tau(\theta)\right\}.$$

Evaluating parameters using atemporal wealth gambles

	Risk-Preference Model				
	EZKP1	EZKP2	ORA1	ORA2	ORA3
θ_H	3.000	17.000	25.000	25.000	100.000
θ_L	–	–	3.000	4.000	3.000
τ_L	–	–	0.020	0.025	0.020

	Binary 50-50 Gambles				
Loss	Gain that leads to indifference for initial wealth $300,000				
$100	100.13	100.60	100.87	100.87	103.48
$400	402.14	409.84	414.37	414.37	462.37
$1,000	1,013.51	1,063.85	1,094.95	1,094.95	1,518.31
$2,000	2,054.80	2,273.08	2,420.72	2,420.72	9,254.96
$5,000	5,357.20	7,170.61	8,995.81	8,995.81	18,991.43
$10,000	11,539.60	27,901.22	26,396.79	32,281.88	26,396.79
$20,000	27,302.60	∞	45,692.48	58,228.29	45,692.48
$30,000	50,274.57	∞	75,052.03	110,405.61	75,052.03

Evaluating parameters using atemporal wealth gambles

	Risk-Preference Model				
	EZKP1	EZKP2	ORA1	ORA2	ORA3
θ_H	3.000	17.000	25.000	25.000	100.000
θ_L	–	–	3.000	4.000	3.000
τ_L	–	–	0.020	0.025	0.020

	Binary 50-50 Gambles				
Loss	Gain that leads to indifference for initial wealth $300,000				
$100	100.13	100.60	100.87	100.87	103.48
$400	402.14	409.84	414.37	414.37	462.37
$1,000	1,013.51	1,063.85	1,094.95	1,094.95	1,518.31
$2,000	2,054.80	2,273.08	2,420.72	2,420.72	9,254.96
$5,000	5,357.20	7,170.61	8,995.81	8,995.81	18,991.43
$10,000	11,539.60	27,901.22	26,396.79	32,281.88	26,396.79
$20,000	27,302.60	∞	45,692.48	58,228.29	45,692.48
$30,000	50,274.57	∞	75,052.03	110,405.61	75,052.03

	Risk-Preference Model				
	EZKP1	EZKP2	ORA1	ORA2	ORA3
θ_H	3.000	17.000	25.000	25.000	100.000
θ_L	–	–	3.000	4.000	3.000
τ_L	–	–	0.020	0.025	0.020
β^{-1}	1.010	1.010	1.010	1.010	1.010
Equilibrium Consumption Growth by Type					
$\mathbb{E}(\theta_H$ cons gr$)$	1.0180	1.0180	1.0102	1.0111	1.0053
$\sigma(\theta_H$ cons gr$)$	0.0360	0.0360	0.0175	0.0231	0.0042
$\mathbb{E}(\theta_L$ cons gr$)$	–	–	1.0527	1.0659	1.0475
$\sigma(\theta_L$ cons gr$)$	–	–	0.1182	0.1260	0.1101
% type θ_L	–	–	18.36	12.55	30.04
Asset Returns: σ(dividend growth) $= 0.10$					
R^f	1.0231	1.0077	1.0128	1.0088	1.0137
$\mathbb{E}(R)$	1.0374	1.0667	1.0567	1.0645	1.0550
$\sigma(R)$	0.1019	0.1048	0.1038	0.1046	0.1036
$\mathbb{E}(R) - R^f$	0.0143	0.0590	0.0439	0.0557	0.0413

	Risk-Preference Model				
	EZKP1	EZKP2	ORA1	ORA2	ORA3
θ_H	3.000	17.000	25.000	25.000	100.000
θ_L	–	–	3.000	4.000	3.000
τ_L	–	–	0.020	0.025	0.020
β^{-1}	1.010	1.010	1.010	1.010	1.010
	Equilibrium Consumption Growth by Type				
$\mathbb{E}(\theta_H$ cons gr)	1.0180	1.0180	1.0102	1.0111	1.0053
$\sigma(\theta_H$ cons gr)	0.0360	0.0360	0.0175	0.0231	0.0042
$\mathbb{E}(\theta_L$ cons gr)	–	–	1.0527	1.0659	1.0475
$\sigma(\theta_L$ cons gr)	–	–	0.1182	0.1260	0.1101
% type θ_L	–	–	18.36	12.55	30.04
	Asset Returns: σ(dividend growth) $= 0.10$				
R^f	1.0231	1.0077	1.0128	1.0088	1.0137
$\mathbb{E}(R)$	1.0374	1.0667	1.0567	1.0645	1.0550
$\sigma(R)$	0.1019	0.1048	0.1038	0.1046	0.1036
$\mathbb{E}(R) - R^f$	0.0143	0.0590	0.0439	0.0557	0.0413

	Risk-Preference Model				
	EZKP1	EZKP2	ORA1	ORA2	ORA3
θ_H	3.000	17.000	25.000	25.000	100.000
θ_L	–	–	3.000	4.000	3.000
τ_L	–	–	0.020	0.025	0.020
β^{-1}	1.010	1.010	1.010	1.010	1.010
Equilibrium Consumption Growth by Type					
$\mathbb{E}(\theta_H \text{ cons gr})$	1.0180	1.0180	1.0102	1.0111	1.0053
$\sigma(\theta_H \text{ cons gr})$	0.0360	0.0360	0.0175	0.0231	0.0042
$\mathbb{E}(\theta_L \text{ cons gr})$	–	–	1.0527	1.0659	1.0475
$\sigma(\theta_L \text{ cons gr})$	–	–	0.1182	0.1260	0.1101
% type θ_L	–	–	18.36	12.55	30.04
Asset Returns: $\sigma(\text{dividend growth}) = 0.10$					
R^f	1.0231	1.0077	1.0128	1.0088	1.0137
$\mathbb{E}(R)$	1.0374	1.0667	1.0567	1.0645	1.0550
$\sigma(R)$	0.1019	0.1048	0.1038	0.1046	0.1036
$\mathbb{E}(R) - R^f$	0.0143	0.0590	0.0439	0.0557	0.0413

	Risk-Preference Model				
	EZKP1	EZKP2	ORA1	ORA2	ORA3
θ_H	3.000	17.000	25.000	25.000	100.000
θ_L	–	–	3.000	4.000	3.000
τ_L	–	–	0.020	0.025	0.020
β^{-1}	1.010	1.010	1.010	1.010	1.010
	Equilibrium Consumption Growth by Type				
$\mathbb{E}(\theta_H$ cons gr)	1.0180	1.0180	1.0102	1.0111	1.0053
$\sigma(\theta_H$ cons gr)	0.0360	0.0360	0.0175	0.0231	0.0042
$\mathbb{E}(\theta_L$ cons gr)	–	–	1.0527	1.0659	1.0475
$\sigma(\theta_L$ cons gr)	–	–	0.1182	0.1260	0.1101
% type θ_L	–	–	18.36	12.55	30.04
	Asset Returns: σ(dividend growth) $= 0.10$				
R^f	1.0231	1.0077	1.0128	1.0088	1.0137
$\mathbb{E}(R)$	1.0374	1.0667	1.0567	1.0645	1.0550
$\sigma(R)$	0.1019	0.1048	0.1038	0.1046	0.1036
$\mathbb{E}(R) - R^f$	0.0143	0.0590	0.0439	0.0557	0.0413

Brandenburger, A. and E. Dekel (1993): "Hierarchies of Beliefs and Common Knowledge", *Journal of Economic Theory*, 59, 189–198 (page 43).

Campbell, J. Y. (2006): "Household Finance", *Journal of Finance*, 61, 1553–1604 (page 21).

Epstein, L. G. and S. E. Zin (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework", *Econometrica*, 57, 937–969 (pages 2, 43).

Guiso, L. and P. Sodini (2013): "Household Finance: An Emerging Field", in *Handbook of the Economics of Finance*, ed. by G. M. Constantinides, M. Harris, and R. Stulz, vol. 2, Amsterdam, The Netherlands: Elsevier (page 21).

Gul, F. and W. Pesendorfer (2004): "Self-Control and the Theory of Consumption", *Econometrica*, 72, 119–158 (page 43).

Haliassos, M. and C. C. Bertaut (1995): "Why Do So Few Hold Stocks?", *The Economic Journal*, 105, 1110–1129 (page 21).

Heaton, J. and D. Lucas (2000): “Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk”, *Journal of Finance*, 55, 1163–1198 (page 21).

Kreps, D. M. and E. L. Porteus (1978): “Temporal Resolution of Uncertainty and Dynamic Choice Theory”, *Econometrica*, 46, 185–200 (page 2).

Mankiw, N. G. and S. P. Zeldes (1991): “The Consumption of Stockholders and Nonstockholders”, *Journal of Financial Economics*, 29, 97–112 (page 21).

Mertens, J.-F. and S. Zamir (1985): “Formulation of Bayesian Analysis for Games with Incomplete Information”, *International Journal of Game Theory*, 14, 1–29 (page 43).

Sarver, T. (2017): “Risk Attitude Optimization and Heterogeneous Stock Market Participation”, working paper (page 68).

Sarver, T. (2018): “Dynamic Mixture-Averse Preferences”, *Econometrica*, 86, 1347–1382 (page 2).