Practice With Differential Equations

1. For which values of c does the function $e^x + c$ satisfy the differential equation $y'(x) = y(x)$? Note: for very special differential equations one can add a constant to a solution and get another solution. However, this is *not* true for most differential equations.

2. The concentration of a particular drug in a patient's bloodstream declines at a rate proportional to the concentration with constant of proportionality $k = .2$ if time is measured in hours. If the patient's blood concentration is 10 mM shortly after the injection, what will the concentration be 12 hours later?

3. The concentration of another drug in a patient's bloodstream declines at a rate proportional to the concentration, but it is not known what the constant of proportionality is. However, the blood concentration is measured twice, once shortly after the injection when the concentration is 12 mM and two hours later when the concentration is 6.5 mM. Use this information to determine the constant of proportionality. Predict what the concentration will be 12 hours after the injection.

4. Suppose that the number of bacteria in a colony increases at a rate proportional to the number there with constant of proportionality $k = 2.3$ when time is measured in days. If we start with 20 bacteria, how many will we have after 8 days?

5. The number of bacteria in another colony increases at a rate proportional to the number there, but the constant of proportionality is not known. Suppose that we start with 30 bacteria and we count 342 after 2 days. Find the constant of proportionality and use it to predict how many bacteria there will be 10 days after the start of the experiment.

6. Solve the following initial-value problems by using the method of substitution. In each case, draw a rough graph of the solution and discuss what happens as $t \to \infty$.

   (a) $y'(t) = 2y(t) - 3, \qquad y(0) = 1.$

   (b) $y'(t) = -2y(t) + 3, \qquad y(0) = 10.$

7. An ingot of iron ore at 1000F is plunged into a water bath whose temperature is kept at 60F. The temperature of the ingot decreases according to Newton's law of cooling with constant of proportionality $k = 3$ when time is measured in minutes. Find a formula for the temperature of the ingot as a function of time. How long will it take until the temperature reaches 70F?

8. The temperature in a certain Duke dorm is a stuffy 76F at midnight when the power goes out. Outside the temperature holds steady at 35F. The students measure the temperature in the dorm at 1AM and find that it has decreased to 65F. Worried, they compute how low the temperature will fall by 7AM when Duke Energy Corporation has pledged that the heat will come back on. Explain what they did.

9. The population of a certain tropical island is currently 800 and, in the absence of immigration or emigration, increases at a rate proportional to the population itself (with proportionality constant $k = .03$ when time is measured in years). However, each year 35 people emigrate to the mainland. Find and solve an initial value problem for the population $N(t)$. Graph $N(t)$ and find the time when the population becomes zero.

10. An 800 gallon tank is filled with brine which contains 300 pounds of salt. Every minute two gallons of dilute brine containing .1 pound of salt per gallon are pumped in and two gallons of the (well-mixed) brine are pumped out. Find a differential equation and initial condition satisfied by $S(t)$, the salt in the tank at time t. Solve the initial-value problem to find $S(t)$. Draw a graph of $S(t)$ and describe its behavior as $t \to \infty$.

11. Let $B(t)$ denote the number of infectious bacteria in a patient's body. Suppose that, left unchecked, these bacteria would multiply at a rate proportional to the number there (with constant of proportionality equal to 1.6 if time is measured in hours). Suppose that the patient's immune system kills off 800 of these bacteria per hour. Suppose that at $t = 0$ the patient has B_0 bacteria in his system. For which values of B_0 will the immune system eventually conquer the infection?

12. In a certain polluted lake, the fish population is decreasing at a rate proportional to its size (with constant of proportionality equal to .04 when t is measured in months). However, environmentalists add 2000 fish per month to restock the lake. If the environmentalists continue this restocking for a long time, approximately how many fish will be in the lake?