11.12:

Introduction to differential equations.

Definition: An equation of the form $\frac{dy}{dx} = g(y, x)$ is called a differential equation: $y = f(x)$ is the solution to the differentiation equation $\frac{dy}{dx} = g(y, x)$ if when you substitute $f(x)$ for y in the expression $g(y, x)$, the result is $\frac{dy}{dx}$. (An equation that contains an unknown function and some of its derivatives)

Intuition: If we have seen how to describe the changing rate (derivative), how can we find the original function?

Ex3: Let t represent time, P denote the population.

$$\frac{dP}{dt} = k P \quad (\text{the changing rate proportional to population})$$

k is a constant.

(We can solve it in next lecture.)

Example 1: $\frac{dy}{dx} = 3x$

Antidifferentiate: $y = \frac{3}{2}x^2 + C$, we can directly integrate it.

Ex 2: $\frac{dy}{dx} = y$. , guess a solution $\Rightarrow Ce^x$, works:

We should know how to plug y, x into the equation to check whether it's a solution:

Ex3 cont: $P = Ce^{kt}$.

Ex4: $\frac{d^2y}{dx^2} = -y$.

Check: $y = C_1\sin(x) + C_2\cos(x)$.

In general, a differential equation can have an infinite number of solutions (up to a constant). But if we are given a point through which the solution function must pass (called an initial condition), there will be only one solution.

A differential equation with an initial condition is called initial value problem (IVP).

Intuition, given velocity and position, the process is determined.

Ex: Which of the following is the solution of $\frac{dy}{dt} = 2(y-1)$?

1. $1-2e^{2t}$ Yes.
2. $\cancel{1}+e^{2t}$ Yes.
3. $e^{2t}-2t$ No
4. $2+e^{2t}$ No.

Definition: Constant solution to a differential equation are called equilibrium solution.

$$\boxed{y(x) \equiv C}$$

Ex: 1. $\frac{dy}{dt} = ky$ has equilibrium solution $y=0$.

2. $\frac{dy}{dt} = y(2-y)(1+y)(\cos)$ has equilibria $y=0$, $y=2$, $y=-1$.

3. $\frac{dy}{dt} = y^2+1$ has no equilibrium point.