ルMIL:

100-90: 2
80-89: 2
70-80: 3
60-70: 4
50-60: 5

Mean: 72.5
SD: 14.1

10.28.

Taylor Polynomials.

Definition of polynomials: (for one parameter/variable) or polynomial function.

$$f(x) = \sum_{k=1}^{n} a_n x^n + a_0 \quad \text{for } n \text{ (fixed index)}$$

a_n is constant

Motivation: approximate the non-polynomial functions by polynomials.

Start with those centered at x

Cit: $P_1(0) = f(0) \quad P_1'(0) = f'(0)$

Notice an advantage of polynomials: The nth derivatives are determined by a_n

The nth degree Taylor polynomials of a function at a defined

$$T_n(x) = \sum_{k=1}^{n} \frac{f^{(k)}(x)}{k!}(x-a)^k + f(a)$$

for $a=0 \Rightarrow \sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{n!} x^n + f(a)$

If we let n go to ∞, we get an infinite power series at a.

The Maclaurin series of f is the Taylor series $f(x)$ centered at $x=0$

In general, when is $f(x)$ the sum of it's Taylor series?

If we let

$$R_n(x) = f(x) - T_n(x)$$

so that: $$f(x) = T_n(x) + R_n(x)$$

$R_n(x)$ is called the remainder of Taylor series.

Then it follows that

$$\lim_{n\to\infty} T_n(x) = f(x).$$

Why we care about the Taylor polynom?

What is $T_n(a)$? $\Longrightarrow f(a)$

What is $T_n'(a)$? $\Longrightarrow f'(a)$

What is $T_n''(a)$? $\Longrightarrow f''(a)$

$T_n^{(k)}(a) \Longrightarrow f^{(k)}(a)$

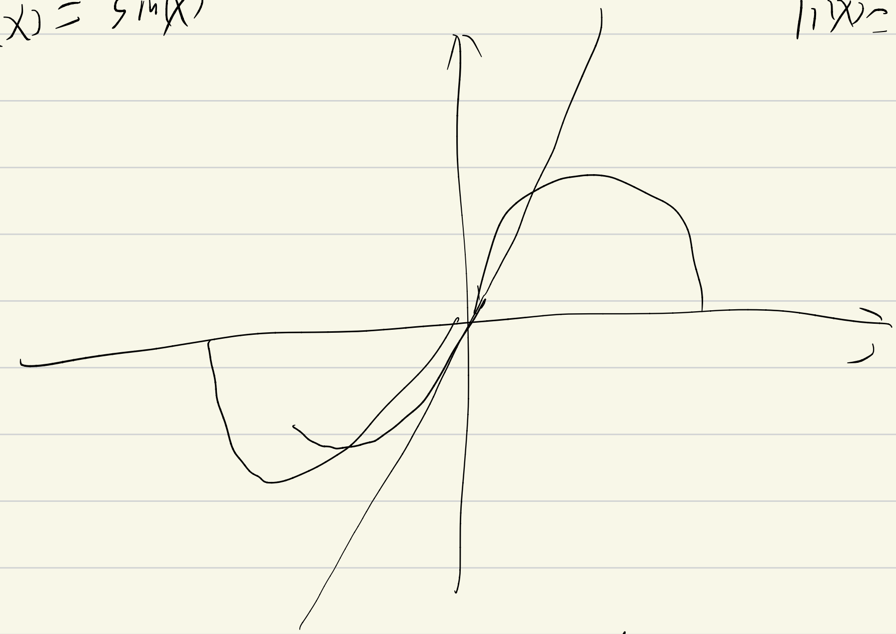
Ex: Find the Taylor polynomial centered at $X=0$ of degree 1 for the function

$f(x) = \sin(x)$

$T_1(x) = \sum_{k=1}^{1} \frac{f^{(k)}(0)}{k!}(x-0) + f(0)$

$f(0) = 0$

$f'(0) = \cos(0) = 1$



Find the taylor polynomial centered at $x=0$ of degree 3 for

$f(x) = \sin(x)$

Find the Taylor polynomial at $x=0$ of degree 5 for $f(x) = \sin(x)$

How about n?

$$\sin X = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

$$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n!}$$

$$e^x = \sum \frac{x^n}{n!}$$

Ex: Use 5th degree Taylor polynomial to estimate

$$\boxed{\int_0^{\frac{\pi}{2}} \sin(x)\cdot dx = ?}$$

Ex: Approximate $f(x) = \sqrt{x}$ with 3rd degree Taylor polynomial at $a=4$

Ex: $x\ln(x)$ with a 3rd degree Taylor polynomial centered at $c=1$.