More Example: $\sum \frac{(-1)^n\sqrt{n}}{n+4}$

$\sum \frac{(-1)^n n}{n+5}$

10.11 Ratio Test.

Ratio test:

1. If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent and therefore convergent.

2. If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\lim\left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

3. If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$, the Ratio test is inconclusive; that is, no conclusion can be drawn about the convergence or divergence of $\sum a_n$.

Proof sketch: $a_n \sim L^n \Rightarrow$ behaves nearly like geometric series

You can prove it by using the comparison Thm. + Absolutely convergence.

Ex: $(-1)^n \frac{n^5}{3^n}$ $\Rightarrow$ absolutely converge

Ex2: $\frac{n^n}{n!}$? $\Rightarrow$ divergence

$$\left|\frac{c_{n+1}}{c_n}\right| = \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n} = \left(\frac{n+1}{n}\right)^n \rightarrow e.$$

Ex: Determine whether the series

$$\sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^n$$ is absolutely converge

(Ex: How to compute it?

$$s = \sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^n$$

$$\frac{1}{2}s = \sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^{n+1} = \sum_{n=2}^{\infty} (n-1)\left(\frac{1}{2}\right)^n$$

$$\frac{1}{2}s = 1\left(\frac{1}{2}\right) + \sum_{n=2}^{\infty} \left(\frac{1}{2}\right)^n$$

$$s = 2$$)

Notice: when $L = 1$

$$\sum \frac{1}{n^2} < \infty, \quad \sum \frac{1}{n} = \infty.$$

Ex: $\sum \frac{(n!)^2}{(2n)!}$

$$\Rightarrow \frac{C_{n+1}}{C_n} = \frac{((n+1)!)^2}{(2(n+1))!} \cdot \frac{(2n)!}{(n!)^2}$$

$$= \frac{(n+1)^2}{2n \cdot (2n-1) \cdots (2n+2)}$$

$k = 2$

Ex: $\sum \frac{3^k k!}{(3k)!}$

$$\Rightarrow \frac{3^{k+1}(k+1)!}{(3(k+1))!} \cdot \frac{(3k)!}{3^k k!} \Rightarrow \frac{\cancel{(k+1)}3}{3k+1 \, (3k+2) \cdot \cancel{3(k+1)}} \Rightarrow 0$$

Review: If AC $\Rightarrow$ Conditionally converge

Converge $\not\Rightarrow$ AC

$\sum -\frac{1}{n}$

10.22: Series practice (Lab)