9.30.

Expected Value.

90: 1   60-70: 4
70-80: 2   50-60: 3
40-50: 5

Alt Midterm: $\boxed{\begin{array}{c} 58.18 \\ 13.0 \end{array}}$ [40, 50]

Very important to do your HW!

How to understand $Y = (\sqrt{x} \cdot 10)$
say 45 and 70.

sample space, event, independence, random variable

probability:

Complement rule: $P(A^c) = 1 - P(A)$

Addition rule: If two they are disjoint:

$$P(A \cup B) = P(A) + P(B)$$

$$(P(A) + P(B) = P(A \cup B) + P(A \cap B))$$

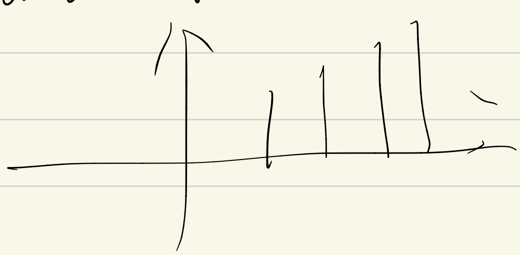
Today:

mass density function:

$|P(X = c)|$ is a function of c

a random variable

(draw a graph)



Recall last week we talked about: each flip is independent?

Flip a coin. (Flip until we get tail) stop

X is a random variable: (the # of tosses till tail)

$X = 1 \quad (\frac{1}{2})$

$X = 2 \quad (\frac{1}{2})^2$

$X = 3 \quad (\frac{1}{2})^3$

$\vdots$

$X = 4: \quad (\frac{1}{2})^4$

$\vdots$

Question: What's the density function?

$f(n) = P(X = n) = (\frac{1}{2})^n \Longrightarrow$ the n function

Another question: What's the average (weighted)?

◎ a simple Example: $X=0$ with $P=\frac{1}{2}$ $\left(P(X=0)=\frac{1}{2}\right)$

$X=1$ with $P(X=1)=\frac{1}{2}$.

$\therefore \quad 0\cdot\frac{1}{2}+1\cdot\frac{1}{2}=\frac{1}{2}$. $\Rightarrow$ weight average:

or $\sum_{i=1}^{n} P(X=a_i)\, a_i$ = weight average = Expectation

How about the last example?

$$E(X)=\sum_{n=1}^{\infty} P(X=n)\cdot n=\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n\cdot n$$

we meet the infinity sum ok, by series

Ex: consider roll a die. Roll it until you get a 6. then the time is a random variable

(1) $P(X=1)=\frac{1}{6}$ (3) $E(X)=?\ \sum n\left(\frac{5}{6}\right)^{n-1}\frac{1}{6}$

(2) $P(X=n)=\left(\frac{5}{6}\right)^{n-1}\frac{1}{6}$

Expectation: $\sum_{i=1}^{\infty\,(\text{could be infinity})} x_i\, P(X=x_i)$

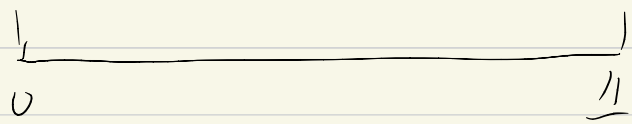
Outline: we see random variable, take finite value / countably many values over pt, are called <u>discrete random variable</u>

Also, it can be continuous:

Suppose we want to measure the heights of an adult,

the life expectancy ... : The random variable X can take on

any value in the interval



Like for discrete random variable, we have mass function $P(X=c)$

For continuous function, we have probability density function f for a continuous random variable

$$f(c) = P(X=c)$$

If want to calculate $P(c \le X \le b)$, take the density to the single and to add up $f(c)$ for every c in the interval $[a,b]$

$$P(a \le X \le b) = \int_a^b f(x)\,dx$$

Also:

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Also we know that: