10.13.

Comparison Tests

Difficult to handle: $\frac{2+\sin n}{n}$

① Not geometric series

② can't use test for divergence $\Rightarrow$ actually converge!

③ can't use integral test directly $\Rightarrow$ $\frac{2+\sin x}{x}$ not monotone

The Comparison Test. Suppose that $\sum a_n$ and $\sum b_n$ with positive terms, Then:

1. $\sum b_n$ is convergent and $a_n \le b_n$ for all n, then $\sum a_n$ is convergent.
2. If $\sum a_n$ is divergent and $a_n \ge b_n$ for all n, then $\sum b_n$ is also divergent.

This case: $\frac{2+\sin n}{n} \geq \frac{1}{n} \Rightarrow$ divergent (by integral test)

Remark: a lot of time, we will use the comparison test when our series is comparable to one of these series.

1. p-series ($\sum \frac{1}{n^p}$ converge and diverge if $p \le 1$)
2. a geometric series (converge if $|r|<1$ and diverges if $|r| \ge 1$)

Example: $\sum \frac{5}{2n^2+4n+3}$

$$\Rightarrow \le \frac{5}{2n^2}$$

Example 2: $\sum \frac{\sin^2(n)}{n^3+2} \le \frac{1}{n^3}$

In order to use comparison test, the terms of our series must be smaller than those of a convergent series or larger than those of a divergent series

$$\underline{\sum \frac{1}{2^n+1}} \Rightarrow \text{convergent}$$

But sometimes, we might want:

$$\frac{1}{2^n-1} > \frac{1}{2^n}$$

But we have:

The Limit Comparison Test: Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms, and $\lim \frac{a_n}{b_n} = c$, > 0, c is finite and positive, then either both converge or both diverge.

Ex $\sum \frac{1}{2^{n-1}}$ converges

Ex3: $\sum c_n$ converges, does $\sum \ln(1+c_n)$ converges?

$c_n \geq 0$

$c_n \to 0$ $\boxed{\frac{c_n}{\ln(1+c_n)} \to 1}$ $\Rightarrow$ Yes.

Ex4: $\sum \frac{n}{\ln(n)}$, conv or div? $\Rightarrow$

① $\frac{n}{\ln n} \to \infty$, div.

(2) $\frac{n}{\ln(x)} \geq \frac{1}{x}$, div