9.02.2019.

Riem Sum (Σ)

(Summation notation Σ)

① Review. $\sum_{i=1}^{n} f(x_i)\cdot\Delta x_i$

1. LHS
2. RHS $= \sum_{i=1}^{n} f(x_i)\cdot\Delta x_i$
3. Midpt Sum
4. trapezoid Rule

$$c\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} c a_i$$

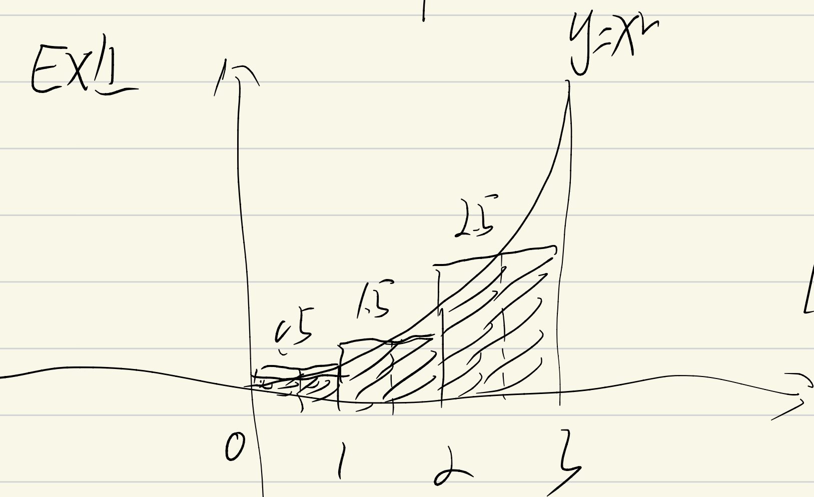
$$\sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i = \sum_{i=1}^{n} (a_i + b_i)$$

$$\sum_{i=1}^{n} k = ? \Rightarrow nk$$

$$\sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)\cdot\Delta x_i$$

$$\sum_{i=1}^{n} \frac{f(x_{i-1})+f(x_i)}{2}\cdot\Delta x_i$$

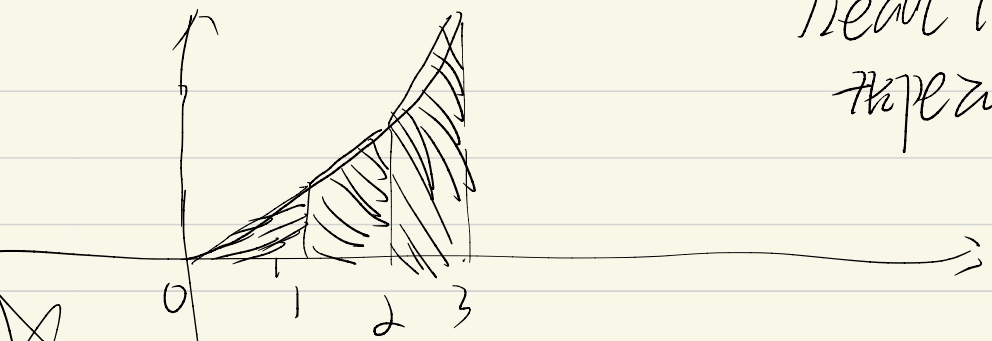
EX1.



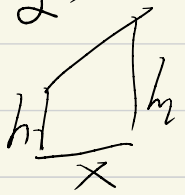
Midpt Sum for this case

$$\sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)(x_i - x_{i-1}) = f(0.5) + f(1.5) + f(2.5).$$

trapezoid sum



Recall the formula of area of trapezoid: $\frac{(h_1+h_2)}{2}\cdot\Delta x = \frac{f(x_i)+f(x_{i+1})}{2}\Delta x_i$



$$\sum_{i=0}^{n-1}\frac{(f(x_i)+f(x_{i+1}))}{2}\cdot\Delta x_i = \sum_{i=0}^{n-1}\frac{f(x_i)}{2}\cdot\Delta x_i + \sum_{i=0}^{n-1}\frac{f(x_{i+1})}{2}\cdot\Delta x_i$$

$$= \frac{1}{2}LHS + \frac{1}{2}RHS.$$

Over estimate? Underestimate? LHS, RHS, MPS, TRS

always Increasing: LHS ↓ (under)
RHS ↑ (over)

$f(x_i)$ / $f(x_i)>0$ $f(x)$ @ $x\in(x_i, x_{i+1})$

always decreasing: LHS ↑
RHS ↓ (Exactly)

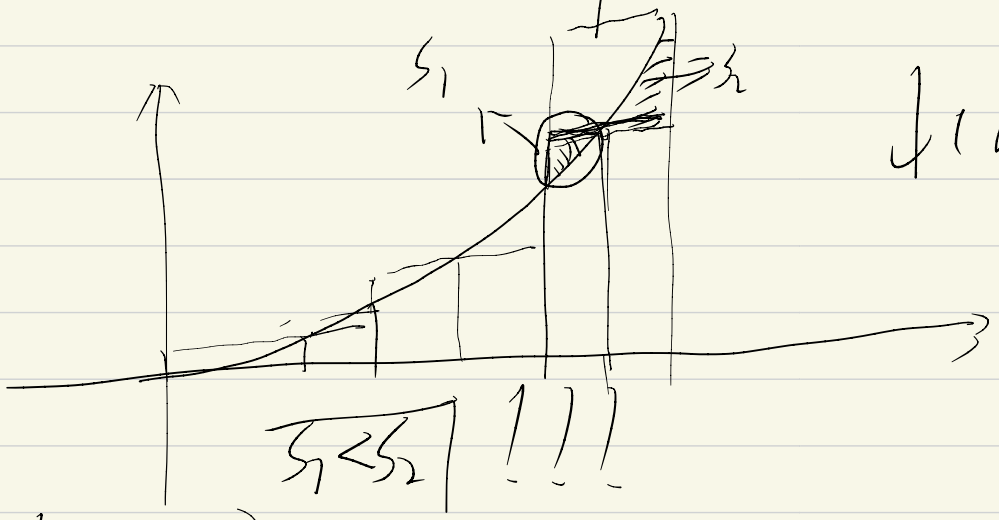
How about the MPS TRS? (In best case, =)

convex function: $f''(x)\geq 0$
always under the decay

$\frac{1}{2}$)

Trapezoid: $\int$ is the trapezoid area under graph

Midpt



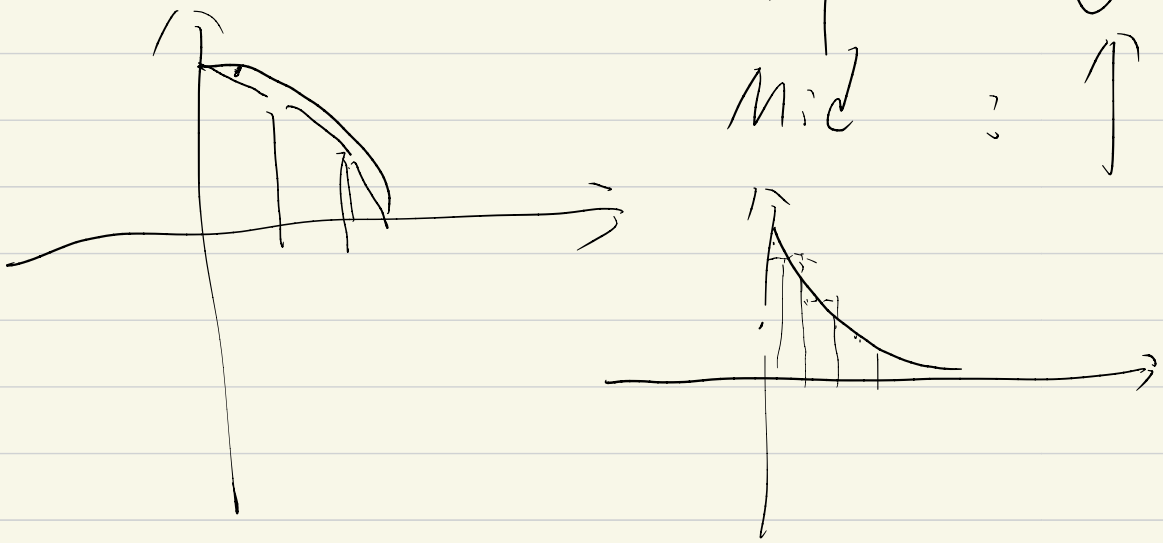
↓ (under)

$S_1 < S_2$!!!

How to $f''(x) \leq 0$?

Trapezoid: ↓

Mid: ↑



$\int_{[0,\pi]}$

$y = x^3$ $y = \sin x$ on $[0, \frac{\pi}{2}]$

$y = x^{\frac{1}{2}}$ $(0,2]$ $y = \ln(x)$

check the following Ex: (Which is HW, Bottom need to textbook)