Problem Set #3

1. Suppose function f passes through the following points:

x	0	2	4	6	8	10	12
$f(x)$	2	-1	7	2	5	8	5

   (a) Approximate $\int_0^{12} f(x)dx$ using the **smallest** Riemann Sum with 3 rectangles of equal width.

   (b) Approximate $\int_0^{8} (f(x))^2 dx$ using the **Trapezoid Rule** with 2 rectangles of equal width.

   (c) If by using a LHS, we could approximate $\int_1^5 f(x)dx$ by $\sum_{k=0}^{6} f(1+\frac{4k}{7})\cdot\frac{4}{7}$. If we instead want to approximate $\int_6^{10} f(x)dx$ with the same number of rectangles, how should we adjust the Riemann Sum?

2. Evaluate the following integrals. (First write down the definite integral and then use FTC or area to compute the integral.)

   (a) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(4 - 3\left(\frac{2i}{n}\right)^2 + 6\left(\frac{2i}{n}\right)^5\right)\cdot\frac{2}{n}$

   (b) $\lim_{n\to\infty} \sum_{i=0}^{n-1} \sqrt{4-\left(\frac{4i}{n}-2\right)^2}\cdot\frac{4}{n}$

   (c) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{-1}{n\cdot(1+\left(\frac{k}{n}\right)^2)}$

3. Evaluate the following:

   (a) $\frac{d}{dx}\int_0^x e^{-t^2}\,dt$

   (b) $\frac{d}{dx}\int_1^{x^2} \ln(t+t^2)\,dt$

   (c) $\frac{d}{dx}\int_0^x \frac{\sin t+1}{\cos^3 t}\,dt - \int_0^x \frac{d}{dt}\left(\frac{\sin t+1}{\cos^3 t}\right)dt$

4. Use u-substitution to find the following integrals.

   (a) $\int_4^9 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

   (b) $\int_0^1 \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}dx$

   (c) If f is continuous and even, and $\int_{-9}^{9} f(x)dx = 8$, find $\int_0^3 xf(x^2)dx$.

5. Find the following definite integrals and antiderivatives:

   (a) $\int_1^3 \frac{13x+7}{(3x-1)(2x+5)}\mathrm{d}x$;

   (b) $\int \frac{2x+1}{\sqrt{1-x^2}}\mathrm{d}x$;

   (c) $\int \frac{1}{(1+x^2)^2}\mathrm{d}x$.

6. Define the dirichlet function(see wiki or google it, which is a quite important example in math) on [0,1]:$f(x)=0$ when x is irrational number,while $f(x)=1$ when x is rational number.

   (a) compute the RHS(3),LHS(3) and the MPS(3)

(b) compute the RHS(n),LHS(n) and the MPS(n)

(c) Recall in last problem set/lab we define the minimum sum,now compute Min(3) and then compute Min(n).

(d) (Extra Creadit)Does there exist the definite integral $\int_0^1 f(x)dx$? Explain why.

hint: in each interval[a,b], there will always be irrational number inside it.