Problem Set #2

1. Consider a small plane preparing to take off. Suppose that the plane needs to be travelling at least 28 m/s to be able to lift off. It takes 50 s to reach this speed. At various intervals, the speed is measured, given in the table below:

t	0	10	20	30	40	50
$v(t)$	0	4	10	16	21	28

   Suppose the runway is 700 m long.

   (a) Use a right hand sum to approximate the distance the plane covers on the ground before it takes off.

   (b) From your estimate in part (a), can you conclude that the runway is or is not long enough? Or do you have insufficient information?

   (c) Use trapezoid rule to approximate the distance the plane covers on the ground before it takes off.

   (d) From your estimate in part (b), can you conclude that the runway is or is not long enough? Or do you have insufficient information?

2. $\int_{-1}^{3}(3-2x)dx$

   (a) Evaluate this integral as a limit of Riemann sums. (Your answer should be a number.)

   (b) Evaluate the integral by interpreting it in terms of areas. Plot the function and indicate the corresponding area(s).

3. Consider the following sums:

$$S_n = \sum_{i=1}^{n} \frac{n}{i^2+n^2}, n = 1, 2, ...$$

   (a) Recall that the Right Hand Sum has the form RHS(n)=$\sum_{i=1}^{n} \Delta x \cdot f(a+i\Delta x)$, rewrite S_n as a RHS.

   (b) Use definition of integrals and FTC to find $\lim_{n\to+\infty} S_n$.

   (c) Denote the limit in problem (b) as S_∞. Compute S_1, S_2, S_3 and compare them with S_∞: S_1 __ S_∞, S_2 __ S_∞, S_3 __ S_∞.

   (d) According to the comparison in (c), what is the relationship between $S_n, n = 1, 2, ...$ and S_∞? Prove your claim by **one** of the following two ways:

      i. figures of Riemann sums and properties of Riemann sums;

      ii. MVT.

4. For this question, we will consider the maximum possible Riemann sum with n rectangles, which we will denote MAX(n), for the function x^2 on the interval $[-1, 1]$. It is important to recall that Riemann sums do not necessarily need to be either left-hand or right-hand sums.

   (a) Compute MAX(4)

   (b) Compute MAX(5)

   (c) Compute MAX($2n$) for any n (note: $2n$ is just an arbitrary even number)

5. Evaluate the following limits

   (a) $\lim_{x\to 0} \frac{5^x - 2^x}{x^5 - x}$

   (b) $\lim_{x\to\infty} x^{\frac{1}{x}}$

   (c) $\lim_{x\to 0} \frac{\sin(x^2)}{\sin^2 x}$