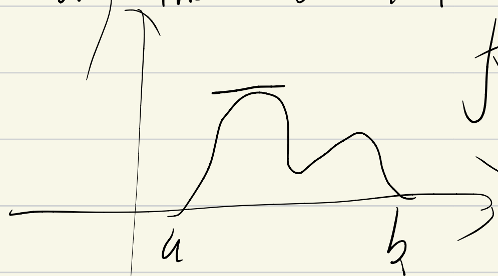
09.24 Mean Value Theorem and FTC II

The Rolle Theorem:



$f(a)=f(b) \Rightarrow f'(x)=0$ there exists some x.

$$F(x) = \int_a^x f(t)\,dt - \frac{\int_a^b f(x)\,dx\,(x-a)}{b-a}$$

$$F'(x) = f(x) - \frac{\int_a^b f(x)\,dx}{b-a} \Rightarrow F'(x^*) = 0$$

$$\Rightarrow f(x^*)(b-a) = \int_a^b f(x)\,dx$$

Mean Value Theorem

若 f is differentiable function on the interval $[a,b]$, there exists a number c between a and b such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

or equivalently: $f(b)-f(a) = f'(c)(b-a)$

The Fundamental Theorem of Calculus (Proof.)

(G is the antiderivative of H $\Longleftrightarrow G' = H$)

Complete the following definition:

$$\int_a^b f(x)\,dt = \lim \sum_{i=1}^{n} f(t_i)\,\Delta x \text{, where } t_i = \frac{i(b-a)}{n},\ \Delta t = \frac{b-a}{n}$$

If f is continuous, $F' = f$, rewrite your answer by F' instead of f.

$$\int_a^b f(t)\,dt = \lim \sum_{i=1}^{n} F'(t_i)\,\Delta t \quad , \quad \text{where } t_i = \frac{i(b-a)}{n},\ \Delta t = \frac{b-a}{n}$$

Note that Δt is small, $F'(t_i) \approx \frac{F(t_i) - F(t_{i-1})}{\Delta t}$ $\quad F'(t_i)\,\Delta t = F(t_i) - F(t_{i-1})$

$$\therefore \sum_{i=1}^{n} F'(t_i)\,\Delta t = \sum_{i=1}^{n} F(t_i) - F(t_{i-1})$$

$$= F(t_n) - F(t_0)$$

$$= F(b) - F(a)$$

When $\Delta t \to 0$, two things happen (1) $F'(t_i) = \frac{F(t_i) - F(t_{i-1})}{\Delta t}$ gets more accurate

Why?

(2) $\sum_{i=1}^{n} F'(\xi_i)\delta t \longrightarrow \int_a^b F'(t)\,dt$.

if f is continuous, and $F' = f$, then

$$\int_a^b f(t)\,dt = F(b) - F(a),$$

This is called Fundamental Theorem of Calculus

Caution: f shld be continuous on $[a, b]$!

Ex(1) Can we use FTC to $\int_{-1}^{1} \frac{1}{x^2}\,dx$? Why or why not

Application: The integral of f' is the net change.

Ex2: (1) $\int_{1}^{2} \frac{1}{x^2}\,dx$?

(2) $\int_{1}^{9} \frac{x-1}{\sqrt{x}}\,dx$?