Week 3. [First talk about the counter example

Fundamental Theorem of Calculus (下) 09.09

Last week,

$$\int_a^b f(x)\,dx = F(x)$$ when $F(x)$ is differentiable and $F'(x)$ is continuous

(上)

This week: $\left[\frac{d}{dx}\left(\int_a^x f(t)\,dt\right)\right] = ?$ when $f(t)$ is

(Actually prove, we prove at midterm

① Extreme Value Theorem: If f is continuous on a closed interval $[a,b]$, then f attains an absolute maximum value $f(c)$ and absolute minimum value

Let $g(x) = \int_a^x f(t)\,dt$

$$\frac{g(x+h) - g(x)}{h} = \frac{\int_x^{x+h} f(t)\,dt}{h}$$

$f(t)$ is continuous, by the Extreme Value Theorem, there exist a constant m, M,
such that $f(u)=m$, $f(v)=M$, ①

$m \le f(t) \le M$ for t in the interval $[x, x+h]$

$$\therefore \quad \underline{m \cdot h \le \int_x^{x+h} f(t)\,dt \le M \cdot h} \quad ②$$

$$\therefore \quad m \le \frac{\int_x^{x+h} f(t)\,dt}{h} \le M \quad ③$$

as $h \to 0$, $[x, x+h] \to x$,

and $m \to f(x)$, $(\uparrow)$ $M \to f(x)$ $(\downarrow)$ (why?)

Therefore $\frac{d}{dx}[g(x)] = \lim \frac{\int_x^{x+h} f(t)\,dt}{h} = \underline{f(x)}$

Fundamental Theorem of Calculus (I)

$$\frac{d}{dx}\left(\int_c^x f(t)\,dt\right) = f(x)$$

($f(t)$ evaluate at x.)

Ex: ① $g(x)=\int_1^x \sqrt{1+t^2}\,dt$

$g'(x)=\sqrt{1+x^2}$

② : $\left(\frac{d}{dx}\int_1^{x^4}\sec t\,dt\right)$? $\Rightarrow$ chain rule!

$\sec(x^4)\cdot 4x^3$

③ derivative of $g(y)=\int_2^y \sqrt{x^2+4x}\,dx$

④ $f(x)=\int_2^x (t^4+2)\,dt$

⑤ $h(t)=\int_1^{3} 3x^4\,dx$

⑥ : $G(x)=\int_x^1 \cos(\sqrt{t})\,dt$

⑦ : $h'(x)=(\cos(x))^6$, and $h(0)=10$

Find the st ct of $\int^{hx}$ initial - hp final