9.03.219.

Definition of the Definite Integral.

(Quick connection: ① Nurbek (2 connects yesterday)

Idea: Rectangles were used to approximate the areas.

Question: S_n is equal? What's the difference?

LRS, RRS, MPS, TRS

Definition: (Continuous function (✓), with finite jump (✓), other? see in the text ...

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We saw in Section 5.1 that a limit of the form

$$\boxed{1} \quad \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x = \lim_{n\to\infty} [f(x_1^*)\,\Delta x + f(x_2^*)\,\Delta x + \cdots + f(x_n^*)\,\Delta x]$$

arises when we compute an area. We also saw that it arises when we try to find the distance traveled by an object. It turns out that this same type of limit occurs in a wide variety of situations even when f is not necessarily a positive function. In Chapter 6 we will see that limits of the form (1) also arise in finding lengths of curves, volumes of solids, centers of mass, force due to water pressure, and work, as well as other quantities. We therefore give this type of limit a special name and notation.

2 Definition of a Definite Integral If f is a function defined for $a \leqslant x \leqslant b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = (b - a)/n$. We let $x_0\,(= a), x_1, x_2, \ldots, x_n\,(= b)$ be the endpoints of these subintervals and we let $x_1^*, x_2^*, \ldots, x_n^*$ be any **sample points** in these subintervals, so x_i^* lies in the ith subinterval $[x_{i-1}, x_i]$. Then the **definite integral of f from a to b** is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

provided that this limit exists. If it does exist, we say that f is **integrable** on $[a, b]$.

Definition says: $\lim_{n\to\infty}$ LRS = RRS = MPS = TRS

= Max sum = Low sum !!!

(Impractical to be integrable.

Fundamental Theorem of Calculus: it's not clear if we can use it. Lesson will teach it.

Properties of Integral: (comes from the property of sum.)

(If they're possible, we can [illegible])

① $\int_a^b c\,dx = c(b-a)$

② $\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

③ $\int_a^b cf(x) = c\int_a^b f(x)\,dx$ C is constant

④ $\int_a^b (f(x)-g(x)) = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

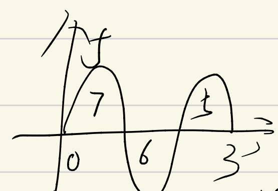
⑤ $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$ $a<c<b$

it even: $c>b$ or $c<a$

Good Example:

Key Idea

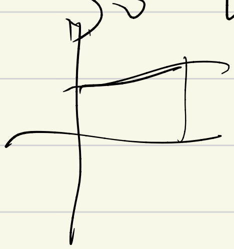
Limit of Riemann sum = Integral



what's $\int_0^3 f\,dx = ?$

$\boxed{7-6+5=6}$

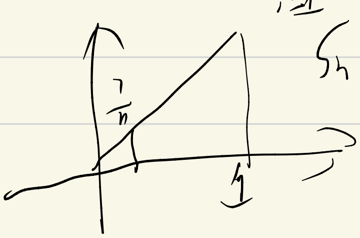
So to get Integral, we just compute the limit.



$\frac{1}{n}+\frac{1}{n}+\frac{1}{n} \dots \frac{1}{n}$

$= \sum_{i=1}^{n} \frac{1}{n}$

①

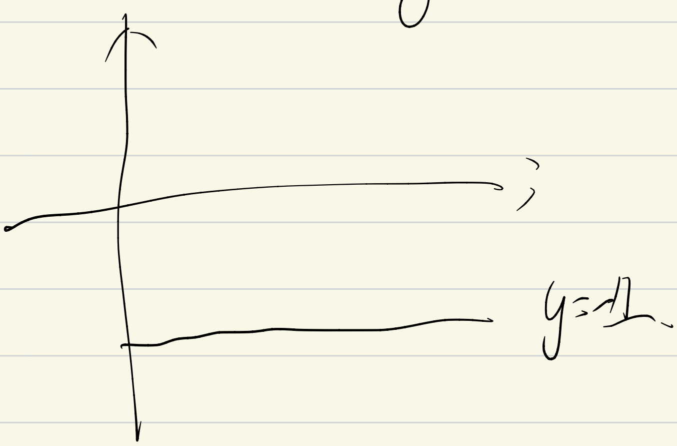


$S_n = \sum_{i=1}^{n} \left(\frac{i}{n}\right)\cdot\frac{1}{n}$

$= \sum_{i=1}^{n} \frac{i}{n^2}$

②

Also, can be negative (cases)



Counter-Question:

Express $\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\frac{1}{1+\left(\frac{i}{n}\right)^2}$ as a definite integral

($\int_0^1$ dx, $\left[f(x_i)\right]$)

Very Difficult to compute the limit $\Longrightarrow$ The Calculation of Definite Integral

Comparison: $f(x) \ge 0 \Rightarrow \int_a^b f(x)\,dx \ge 0$

$f(x) \ge g(x) \Rightarrow \int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$

$m \le f(x) \le M \Rightarrow m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$

[Intuitively true]

Ex1: Write $\lim_{n\to\infty} \sum_{i=1}^{n} (x_i^3 + x_i \sin x_i)\Delta x$ as integral on the interval $(0, \pi)$

Ex2: $\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{1+(i/n)^2}$

Ex3: express $\int_1^3 e^x\,dx$ as the limit of Riemann sums

Ex4: $\int_0^{10} f(x)\,dx = 17$, $\int_0^{8} f(x)\,dx = 12 \Rightarrow \int_8^{10} f(x)\,dx$

Ex5: Use the third comparison property to estimate $\int e^{-x^2}\,dx$