

Math 122L - Brief Review of Prerequisites

1. If

(a) $f(x)$ is an even function, then $f(-x) = \underline{\hspace{2cm}}$

(b) $f(x)$ is an odd function, then $f(-x) = \underline{\hspace{2cm}}$

2. What is the **definition** of the derivative of f at x ?

3. Use the **definition** of the derivative to derive $f'(x)$ for $f(x) = \frac{1}{\sqrt{1+x}}$.

4. Find the derivative of each of the following functions (you need not use the definition here):

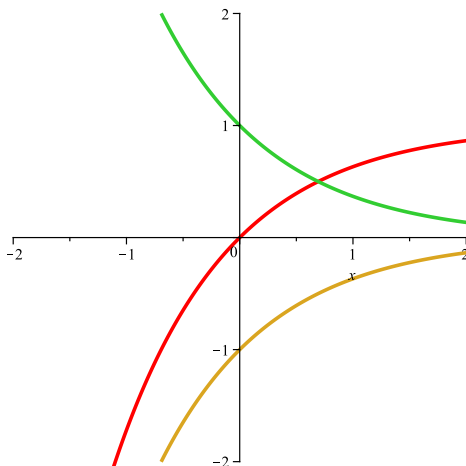
(a) $f(x) = x \sin(e^x)$

(b) $f(x) = \frac{2^x}{1+x^3}$

(c) $f(x) = \arctan(x \cos(x))$

(d) $f(x) = e^{\arcsin(x^2)}$

5. On the graph below, identify which graph is f , f' and f'' . Explain how you know.



6. Find the line tangent to the function $f(x) = xe^{kx}$ at $x = 0$. Assuming that $k > 0$, does the linear approximation underestimate or overestimate xe^{kx} near 0? Explain your answer carefully.

7. The table below gives the values of the functions $f(x)$ and $g(x)$ at specified values of x .

x	1	2	3	4	5
f(x)	0	3	6	8	2
g(x)	1	4	5	2	0

(a) Using the table, estimate the value of the derivative of $f(g(x))$ at $x = 2$.

(b) Using the table, estimate the value of the derivative of $g(f(x))$ at $x = 2$.

8. Suppose $P(t)$ is the monthly payment, in dollars, on a mortgage which will take t years to pay off. What are the units of $P'(t)$? Is $P'(t)$ positive or negative? Explain.

9. Let

$$f(x) = \begin{cases} c^x + x & \text{if } x < 1 \\ x^c + 2 & \text{if } x \geq 1 \end{cases}$$

Answer the following without using a graphing calculator.

(a) Define what it means for a function, $g(x)$, to be continuous at the point $x = a$.

(b) What value(s) of c make $f(x)$ continuous?

(c) Define what it means for a function, $g(x)$, to be differentiable at the point $x = a$.

(d) For this value(s) of c that you found in part (b), is $f(x)$ differentiable? Why or why not?

10. The position of a particle (in centimeters) at time t (in seconds) is $s(t) = \frac{1}{3}t^3 - 5t^2 + 24t$.

(a) When is the particle at rest?

(b) When is the particle moving to the right?

(c) When is the particle speeding up?

(d) Find the total distance traveled by the particle over the interval $0 \leq t \leq 10$.

11. Find the following limits, or state that they do not exist, noting that a , b , c , and d are constants greater than 1. Make sure to justify your answers (not with a calculator).

(a) $\lim_{x \rightarrow \infty} \frac{4a^{-x} + 2b}{3c + d^{-2x}}$

(b) $\lim_{x \rightarrow c^-} \frac{|x - c|}{2x - 2c}$

(c) $\lim_{x \rightarrow -a} \frac{x^2 - a^2}{(x)(x + a)}$