1. If
   (a) \( f(x) \) is an even function, then \( f(-x) = f(x) \)
   (b) \( f(x) \) is an odd function, then \( f(-x) = -f(x) \)

2. What is the **definition** of the derivative of \( f \) at \( x \)?

   **Solution:**
   \[
   f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
   \]

3. Use the **definition** of the derivative to to derive \( f'(x) \) for \( f(x) = \frac{1}{\sqrt{1+x}} \).

   **Solution:**
   \[
   f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{1+x+h}} - \frac{1}{\sqrt{1+x}}}{h}
   \]
   \[
   = \lim_{h \to 0} \frac{1}{\sqrt{1+x+h}} \cdot \frac{1}{\sqrt{1+x}} \cdot \frac{1}{h} \cdot \frac{1}{\sqrt{1+x+h}} + \frac{1}{\sqrt{1+x}}
   \]
   \[
   = \lim_{h \to 0} \frac{\frac{1}{\sqrt{1+x+h}} - \frac{1}{\sqrt{1+x}}}{h}
   \]
   \[
   = \lim_{h \to 0} \frac{1}{h} \cdot \frac{1}{\sqrt{1+x}-\sqrt{1+x+h}} \cdot \frac{1}{\sqrt{1+x}+\sqrt{1+x+h}}
   \]
   \[
   = \lim_{h \to 0} \frac{-h}{\frac{1}{\sqrt{1+x+h}} + \frac{1}{\sqrt{1+x}}}
   \]
   \[
   = -\frac{1}{2} \cdot \frac{1}{(1+x)^{3/2}}
   \]

4. Find the derivative of each of the following functions (you need not use the defintion here):
   (a) \( f(x) = x \sin(e^x) \)

      **Solution:**
      \[
      f'(x) = (1) \sin(e^x) + x \left( \cos(e^x)e^x \right)
      \]

   (b) \( f(x) = \frac{2x}{1+x^3} \)

      **Solution:**
      \[
      f'(x) = \frac{2x \ln(2)(1+x^3) - 2x^2(3x^2)}{(1+x^3)^2}
      \]

   (c) \( f(x) = \arctan(x \cos(x)) \)

      **Solution:**
      \[
      f'(x) = \frac{1}{1+(x \cos(x))^2} \cdot \left( \left(1 \cos(x)+x(-\sin(x)) \right) \right)
      \]

   (d) \( f(x) = e^{\arcsin(x^2)} \)

      **Solution:**
      \[
      f'(x) = e^{\arcsin(x^2)} \cdot \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x
      \]

5. On the graph below, identify which graph is \( f \), \( f' \) and \( f'' \). Explain how you know.
Solution: Labeling from top to bottom on the left-hand side of the graph:

- $f'(x)$
- $f(x)$
- $f''(x)$

6. Find the line tangent to the function $f(x) = xe^{kx}$ at $x = 0$. Assuming that $k > 0$, does the linear approximation underestimate or overestimate $xe^{kx}$ near 0? Explain your answer carefully.

**Solution:** $f'(0) = 1$ which means the tangent line has slope 1. The tangent line must pass through $(0, f(0)) = (0, 0)$. Thus, the tangent line is $T(x) = x$.

$f''(0) = 2k > 0$, meaning that the graph of $f(x)$ is concave up. Thus, $T(x)$ **underestimates** $f(x)$ near 0.

7. The table below gives the values of the functions $f(x)$ and $g(x)$ at specified values of $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>$g(x)$</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) Using the table, estimate the value of the derivative of $f(g(x))$ at $x = 2$.

**Solution:** $f'(g(2))g'(2) = f'(4)g'(2) \approx \left( \frac{f(5) - f(3)}{2} \right) \left( \frac{g(3) - g(1)}{2} \right) = (-2)(2) = -4$

(b) Using the table, estimate the value of the derivative of $g(f(x))$ at $x = 2$.

**Solution:** $g'(f(2))f'(2) = g'(3)f'(2) \approx \left( \frac{g(4) - g(2)}{2} \right) \left( \frac{f(3) - f(1)}{2} \right) = (-1)(3) = -3$

8. Suppose $P(t)$ is the monthly payment, in dollars, on a mortgage which will take $t$ years to pay off. What are the units of $P'(t)$? Is $P'(t)$ positive or negative? Explain.

**Solution:**

- units: **$\text{\$ per year}$**
- sign: $P'(t)$ is **negative** because as you increase the number of years needed to pay off the mortgage, the monthly payment decreases.
9. Let
\[ f(x) = \begin{cases} 
  c^x + x & \text{if } x < 1 \\
  x^c + 2 & \text{if } x \geq 1 
\end{cases} \]

Answer the following without using a graphing calculator.

(a) Define what it means for a function, \( g(x) \), to be continuous at the point \( x = a \).
\[ \text{Solution: } \lim_{x \to a^-} f(x) = f(a) = \lim_{x \to a^+} f(x) \]

(b) What value(s) of \( c \) make \( f(x) \) continuous? Explain.
\[ \text{Solution: } \lim_{x \to 1^-} f(x) = c + 1 \text{ and } \lim_{x \to 1^+} f(x) = 3. \] In order for \( f(x) \) to be continuous at \( x = 1 \), we need \( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \implies c = 2 \). Note that for this value of \( c \), \( f(x) \) is also continuous for \( x \neq 1 \).

(c) Define what it means for a function, \( g(x) \), to be differentiable at the point \( x = a \).
\[ \text{Solution: } \lim_{h \to 0^-} \frac{g(a + h) - g(a)}{h} = g'(a) = \lim_{x \to a^+} \frac{g(a + h) - g(a)}{h}, \text{ where } g'(a) \text{ is finite.} \]

(d) For this value(s) of \( c \), is \( f(x) \) differentiable? Explain.
\[ \text{Solution: } \lim_{h \to 0^-} \frac{f(x + h) - f(x)}{h} = \frac{d}{dx} (2^x + x) = 2^x \ln(2) + 1 \text{ and } \lim_{h \to 0^+} \frac{f(x + h) - f(x)}{h} = \frac{d}{dx} (x^2 + 2) = 2x. \]
Because these are not equal, \( f(x) \) is not differentiable.

10. The position of a particle (in centimeters) at time \( t \) (in seconds) is \( s(t) = \frac{1}{3}t^3 - 5t^2 + 24t \).

(a) When is the particle at rest?
\[ \text{Solution: } v(t) = s'(t) = t^2 - 10t + 24 = (t - 4)(t - 6) \implies v(t) = 0 \text{ at } t = 4 \text{ and } t = 6 \text{ seconds.} \]

(b) When is the particle moving to the right?
\[ \text{Solution: } v(t) > 0 \iff (t - 4)(t - 6) > 0 \iff 0 \leq t < 4 \text{ and } 6 < t. \]

(c) When is the particle speeding up?
\[ \text{Solution: } a(t) = v'(t) = 2t - 10 \iff a(t) > 0 \text{ when } t > 5. \text{ So } s(t) \text{ is concave down on } 0 \leq t \leq 5 \text{ and concave up on } t > 5. \text{ The particle is speeding up (to the left) on } 4 < t < 5 \text{ and to the right on } t > 6. \]

(d) Find the total distance traveled by the particle over the interval \( 0 \leq t \leq 10 \).
\[ \text{Solution: } \text{Here we need to account for the distance the particle travels to the left on } 4 < t < 6. \text{ The total distance traveled is } \]
\[ s(4) + |s(6) - s(4)| + (s(10) - s(6)) = \frac{112}{3} + \frac{4}{3} + \frac{112}{3} = 76. \]

11. Find the following limits, or state that they do not exist, noting that \( a, b, c, \) and \( d \) are constants greater than 1. Make sure to justify your answers (not with a calculator).

(a) \[ \lim_{x \to \infty} \frac{4a^{-x} + 2b}{3c + d^{-2x}} = \frac{2b}{3c} \]

(b) \[ \lim_{x \to c} \frac{|x - c|}{2x - 2c} = \frac{-1}{2} \]
(c) \( \lim_{{x \to -a}} \frac{x^2 - a^2}{(x)(x + a)} = \lim_{{x \to -a}} \frac{(x - a)(x + a)}{(x)(x + a)} = \lim_{{x \to -a}} \frac{x - a}{x} = 2 \)