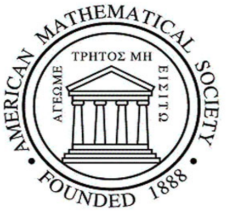
Triangle Area Graduate Mathematics Conference

Duke University, Department of Mathematics

25 February 2023

This conference is sponsored by the Duke University AMS and SIAM chapters



Schedule

9:00 - 9:30	Gross 353	**Breakfast and Welcome**		
9:30 - 10:30	Gross 330	**Plenary Talk: Dr. Veronica Ciocanel**, Duke University *Protein journey inside cells: how can mathematical modeling and data analysis help?*		
10:30 - 11:00	Gross 353	**Coffee Break**		
		Gross 304B	Gross 318	Gross 324
Session 1	11:00 - 11:30	**Tim Van Hoose** *Ruminations about Fiber: An Introduction to Nonlinear Schrodinger equations*	**Rena Chu** *A Tale of Two Sums (Part 1)*	**Jacob McErlean** *An Ensembling Algorithm of EDR Software Sensors via Vector Diffusion Map Synchronization*
	11:30 - 12:00	**Madelyne Brown** *Concentration of Eigenfunctions*	**Kyrie Johnson** *A Tale of Two Sums (Part 2)*	**Victor Gardner Pineda** *Enhanced Dissipation: The Dynamical perspective*
12:00 - 1:00	Gross 353	**Lunch**		
		Gross 304B	Gross 318	Gross 324
Session 2	1:00 - 1:30	**Zhongtian Hu** *Suppression of chemotactic blow up by active scalar*	**Kalani Thalagoda** *Computational Aspects of Bianchi Modular Forms*	**Andrew Shedlock** *Geodesically Equivalent Metrics and an Inverse Boundary Problem*
	1:30 - 2:00	**Spencer Whitehead** *The Search for More Fair Dice*	**Luke Conners** *Link Polynomials from Representation Theory*	**Yupeng Li** *Schubert Polynomials and Lorentzian Polynomials*
	2:00 - 2:30		**Ching-Lung Hsu** *Taxonomic Gibbs-type prior*	**Cole Butler** *Sexy Peacocks: The Mathematics of Sexual Selection in Birds*
2:30 - 3:30	Gross 330	**Teaching/DEI Fair**		

Plenary Talk

Protein journey inside cells: how can mathematical modeling and data analysis help?

Veronica Ciocanel - Duke University

9:30 - 10:30 Gross 330

The cellular cytoskeleton is made up of protein polymers (filaments) that are essential in proper cell and neuronal function as well as in developmental processes. These filaments represent the roads along which most proteins get transported to their cellular destinations. I will discuss several examples where questions about interactions between filament roads and protein cargo require the development of novel mathematical modeling, analysis, and simulation. These tools include partial differential equations, stochastic processes, and topological data analysis, and the applications include egg cell development and proper neuronal function.

Session Talks

Concentration of Eigenfunctions

Madelyne Brown - UNC Chapel Hill
11:30 - 12:00 Gross 304B

In this talk, we will discuss the growth of Laplace eigenfunctions on a compact, Riemannian manifold when restricted to a submanifold. We analyze the behavior of the restricted eigenfunctions by testing them against an arbitrary sequence of functions. Assuming the eigenfunctions and the test sequence have defect measures associated to them, we are able to further understand how the underlying geometry of the manifold affects the concentration of the eigenfunctions.

Sexy Peacocks: The Mathematics of Sexual Selection in Birds

Cole Butler - NC State University
2:00 - 2:30 Gross 324

A history of the mathematical and biological theory of sexual selection in birds is presented. Following this, I present a simple two-locus model of sexual selection alongside some interesting results.

A Tale of Two Sums (Part 1)

Rena Chu - Duke University
11:00 - 11:30 Gross 318

It is classical that the (generalised) Riemann Hypothesis (GRH) implies the (generalised) Lindelöf Hypothesis (GLH). As such, the overwhelming evidence for GRH likewise supports GLH, which describes a precise limit on the growth of Dirichlet L-functions on the critical line $\mathrm{Re}(s) = 1/2$. In this expository talk, we conduct a literature review of the beautiful duality between bounds of L-functions on the critical line and bounds on so-called short character sums with the ultimate goal of understanding number-theoretic consequences. There will be many pretty pictures, a couple of tables, and more than a few integrals/sums. We divide our discussion of this duality into two stand-alone parts, one for each direction.

Link Polynomials from Representation Theory

Luke Conners - UNC Chapel Hill
1:30 - 2:00 Gross 318

The category of finite-dimensional representations of (the quantized enveloping algebra of) a Lie algebra has a rigid algebraic structure, including a tensor product, an interesting braiding operation, and dual representations. In this talk, I'll describe how these algebraic structures can be encoded diagramatically in the form of an oriented tangle diagram. We'll also see how to obtain link polynomials, including the Jones polynomial, from this procedure. Some background in basic representation theory would be helpful but is not required.

Enhanced Dissipation: The Dynamical perspective

Victor Gardner Pineda - Duke University
11:30 - 12:00 Gross 324

One way of describing the dynamical phenomenona of diffusion and mixing in a bounded or periodic domain is to consider a particle following the dynamics of Brownian motion. The dynamics of this particle form a semigroup, and the generator of this semigroup is the heat equation. By analyzing the heat equation, we are able to determine the rate at which a fluid diffuses through the domain.
The PDE perspective has so far given us powerful insights on diffusion, mixing, and the connections between the two. However, when we add a deterministic flow to the dynamics, the interactions between the random diffusion and the deterministic mixing become difficult to understand from a PDE perspective. In this talk, I will explore the connections between diffusion and mixing through a dynamical perspective. I will begin by presenting the PDE perspective, then I will construct linear spaces of observables upon which the dynamics act. Through studying the evolution of these observables, we will be able to better understand the relationship between diffusion and mixing in dynamical systems derived from models in fluid mechanics.

Taxonomic Gibbs-type prior

Ching-Lung Hsu - Duke University
2:00 - 2:30 Gross 318

Gibbs-type priors are a popular Bayesian nonparametric tool for modeling exchangeable discrete data. Whereas its advance, few of these priors are tailored for the biological organisms which are classified into nested clusters with multiple layers in the biological taxonomy. Motivated by this application, this paper proposes a novel taxonomic Gibbs-type priors framework that accommodates any discrete data coming with such a taxonomic structure and provides finer control in modeling uncertainty on the marginal and conditionals of each layer. We describe its link to enriched processes, present a simple predictive scheme, and show that the resulting marginal of each layer still admits Gibbs-type. In addition, we identify the importance of an existing prior and demonstrate that suitable nonparametric mixtures of such priors are arbitrarily close to any Gibbs-type prior. As an exemplification, we construct an explicit taxonomic model and test it with the fungal data collected from Finland.

Suppression of chemotactic blow up by active scalar

Zhongtian Hu - Duke University
1:00 - 1:30 Gross 304B

Chemotactic blow up in the context of the Keller-Segel equation is an extensively studied phenomenon. In recent years, it has been shown that the presence of fluid advection can arrest singularity formation given that the fluid flow possesses mixing or diffusion enhancing properties and its amplitude is sufficiently strong - an effect that is conjectured to hold for more general classes of nonlinear PDE. In this talk, I will discuss some results on suppression of singularity formation in systems where Keller-Segel equation is coupled with fluid flow via buoyancy force. The talk is based on a joint work with Alexander Kiselev and Yao Yao.

Tale of Two Sums (Part 2)

Kyrie Johnson - Duke University
11:30 - 12:00 Gross 318

It is classical that the (generalised) Riemann Hypothesis (GRH) implies the (generalised) Lindelöf Hypothesis (GLH). As such, the overwhelming evidence for GRH likewise supports GLH, which describes a precise limit on the growth of Dirichlet L-functions on the critical line $\mathrm{Re}(s) = 1/2$. In this expository talk, we conduct a literature review of the beautiful duality between bounds of L-functions on the critical line and bounds on so-called short character sums with the ultimate goal of understanding number-theoretic consequences. There will be many pretty pictures, a couple of tables, and more than a few integrals/sums. We divide our discussion of this duality into two stand-alone parts, one for each direction.

Schubert Polynomials and Lorentzian Polynomials

Yupeng Li - Duke University
1:30 - 2:00 Gross 324

Lorentzian polynomials are homogeneous polynomials of degree n that satisfy particular signature condition after taking (n-2)th derivative. Lorentzian polynomials were used to prove the "strongest" version of Mason conjecture in matroids theory by Branden and Huh. In a recent work, Huh, Matherne, Meszaros and St. Dizier showed that Schur polynomials are Lorentzian polynomials and conjectured that serval polynomials in combinatorics are also Lorentzian. In particular, they conjectured that normalized Schubert and double Schubert polynomials are Lorentzian. In this talk, we will provide a potential solution to the conjecture by identifying normalized double Schubert polynomials with mixed volume of polytopes through mixed multiplicities and multidegree polynomials.

An Ensembling Algorithm of EDR Software Sensors via Vector Diffusion Map Synchronization

Jacob McErlean - Duke University
11:00 - 11:30 Gross 324

Several electrocardiogram-derived respiration (EDR) algorithms have been proposed to indirectly measure breathing activity from a single-channel ECG signal, but conclusively identifying a superior technique is challenging. We aimed to fuse the outputs of different EDR algorithms to create one EDR signal that is of higher quality. The approach was to apply the Hilbert transform to signals and construct a rank-one matrix of approximate phase deviations. The robustness property of VDM has recently been established, which states that the top eigenvector of this matrix provides a reliable global phase information under such noisy situation. Once phase shifts were estimated, signals could be aligned and averaged to obtain a better EDR estimate than individual estimates or a time-aligned ensemble.

Geodesically Equivalent Metrics and an Inverse Boundary Problem

Andrew Shedlock - NC State University
1:00 - 1:30 Gross 324

In Riemannian Geometry, all smooth distance minimizing curves are geodesics. As geodesics are determined the metric on a manifold, every geodesic contains some information about the underlying metric. Now geodesic equivalence between two manifolds is a condition by which every geodesic on our first manifold is simply a reparameterization of a geodesics on our second manifold. We discuss a smooth two parameter family of Riemannian metrics on the disc which are geodesically equivalent, but vary in their curvature and in their diameters. We then state some results on how geodesic equivalence is a condition that can be used to uniquely determine solutions to certain inverse problems.

Computational Aspects of Bianchi Modular Forms

Kalani Thalagoda - UNC Chapel Hill
1:00 - 1:30 Gross 318

Bianchi Modular forms are a generalization of classical modular forms to imaginary quadratic fields. We can use homology classes to compute certain parts of Bianchi modular forms. In this talk, I will talk about techniques we used to patch together information from homology to compute the entire Bianchi Modular form space. I will also share some fun examples we found by implementing these techniques for the field $Q(\sqrt{-17})$. The work presented in this talk is joint work with Dan Yasaki.

Ruminations about Fiber: An Introduction to Nonlinear Schrodinger equations

Tim Van Hoose - UNC Chapel Hill
11:00 - 11:30 Gross 304b

One prototypical model for light propagation in fiber optic cables is the cubic nonlinear Schrodinger equation (NLS). In this talk, we introduce (some of) the necessary tools to prove that solutions to this equation exist locally-in-time. We then discuss a modification to this equation, the so-called "dispersion-managed" NLS, which incorporates a time-periodic coefficient in the linear part of the equation, intended to model an anisotropy in the fiber optic cable which reduces noise as the signal propagates. We sketch the necessary modifications to the standard NLS theory to prove a similar local existence result for the dispersion-managed NLS.

The search for more fair dice

Spencer Whitehead - Duke University
1:30 - 2:00 Gross 304b

In three dimensions, there are 18 non-prismatic fair dice. In four dimensions, there are 64. The number in higher dimensions is not known–although there is a pretty good guess available. The study of polytopes of prescribed symmetry is in the 21st century a small but beautiful field mixing combinatorics and group theory with Euclidean geometry. In this talk I will give a gentle introduction to the theory of symmetric polytopes with a focus on uniform polytopes. I will describe Coxeter and Wythoff's calculus of kaleidoscopes and show how it can be used to generate a large number of examples of uniform polytopes in all dimensions. As time allows, I will present some original conjectures on how this calculus might be expanded to be complete.
As this talk is an introduction, no prior knowledge of polytopes will be assumed (although it may help to have seen a six-sided die prior to attending).