Monitoring and Evaluation by Financiers and Investor Activism

Dan Bernhardt
University of Illinois
1106 S. Prospect Ave
Champaign, IL 61820
(217) 355–4773
danber@illinois.edu

Soroush Ghazi
Duke University
213 Social Sciences Building
Durhan, NC 27708
(217) 417–6391
soroush.ghazi@duke.edu

Abstract

This paper considers a financier contemplating a venture capital investment in a firm whose true value is unknown. The financier makes information–gathering and investment decisions on an ongoing basis to decide whether to undertake the investment and, later, if he chooses to finance the firm, how to manage his investment. We characterize how the financier's information acquisition is affected by the liquidity of the market for his claims to the firm, and derive the implications for the pricing of the firm. We distinguish between two qualitatively different types of information acquisition: *evaluation* efforts made prior to a potential investment; and *monitoring* efforts of already–funded firms that impact upon the financier's decisions about whether to take an active position in the firm (e.g. replace management) and whether to change its financial stake. We investigate the effects of liquidity on share price, describing why the market responds more favorably to less liquid forms of finance, and explore the consequences for investor activism. Finally, we characterize the socially optimal levels of evaluation and monitoring in order to determine when a marginal increase in liquidity has welfare–enhancing effects on the financier's behavior.

1 Introduction

There is extensive empirical work documenting that financiers devote significant resources to investigating potential investments, monitoring ongoing investments, and taking active positions in the firm, often intervening extensively in the strategy of the firm (see *e.g.* Gompers and Lerner's (1999) exhaustive study of venture capitalists[1]). Surprisingly, there has been little research that distinguishes between a financier's investigation of a potential investment and his subsequent monitoring of the firm. In particular, it is not well understood how the intensity of a financier's investigation of a firm would influence his subsequent monitoring (and vice versa), nor how the nature of the financier's claims to the firm would affect a financier's incentives to monitor and investigate the firm, and to intervene where necessary in the firm's strategy. It is these issues that are at the heart of our paper.

This paper considers a financier contemplating an investment in a firm, who must make information–gathering and investment decisions on an ongoing basis. We distinguish between two qualitatively different types of information acquisition:

- *Evaluation* efforts made prior to a potential investment.
- *Monitoring* efforts of an ongoing entity in which the financier has made an investment.

The evaluation signal determines the financing decision; while the monitoring signal impacts upon the financier's decisions about whether to take an action that may increase the value of the firm (e.g. replace management), and whether to change his financial stake. The goal of the paper is to understand how the resources that the financier devotes to *evaluating* potential investments and *monitoring* already–funded firms, as well as his decision about whether to actively intervene are affected by the economic environment. In particular, we characterize how the financier's information acquisition decisions are affected both by his stake in the firm, and by the liquidity of the after–market for the financier's claims. We then derive the implications for the pricing of the firm. We compare the financier's evaluation and monitoring efforts with the socially optimal levels, and determine when a marginal increase in liquidity of the after–market has welfare–enhancing effects on the financier's actions.

While our primary focus is on determining how the financier's information acquisition decisions are affected by the economic environment, an important secondary goal is to derive the implications for the pricing of the firm. Empirically, it is well documented that the market responds more favorably to the announcement of less liquid forms of finance. For example, Hertzel and Smith (1993) find a significantly greater positive (7.8%) announcement effect

[1] "Venture investors typically concentrate in early stage companies and high–tech industries with a great deal of uncertainty...(where) firms have substantial intangible assets, which are difficult to value...market conditions are highly variable. (p.3)" "...informational asymmetries are significant and monitoring is valuable. (p.146)"

on a firm's publicly traded share price for private placement of restricted shares than for private placements in general. Similarly, Smith (1986) finds private debt conveys positive information, whereas public debt has either a negative or insignificant effect on share price.[2]

We consider the following economic environment. A firm's manager approaches a financier about providing capital to the firm, and in return he offers a claim to the firm's proceeds. Then the financier must decide how thoroughly to investigate the firm. The more the financier spends on this evaluation signal, the more accurate it is. If the financier makes the capital investment, then she can monitor the ongoing firm to obtain additional information. Again, the more spent on monitoring, the more reliable the signal. We show that the financier's choice of evaluation and monitoring effort imposes a tension on the share the manager offers to the financier. If monitoring reveals bad news, the financier has two alternatives:

1. The financier can intervene in the firm, and force the firm to take an action that raises the firm's value. For example, the financier might replace management or abandon the project and salvage some of the capital investment. However, intervention conveys bad news to the market, and the market price of the financier's claims to the firm reflects this information.

2. Alternatively, the financier can attempt to conceal the bad news and sell some of her stake in the firm before the news leaks out. Of course, trades by the financier also convey information to the market; the number of shares that the financier can sell without revealing bad news depends on the liquidity of the after–market. [3]

The liquidity of the market for the security affects (i) the financier's decision about whether to intervene in the firm's strategy, (ii) the financier's monitoring and evaluation investments, (iii) the manager's deal with the financier regarding the capital she provides and her subsequent claims to the firm, (iv) the share–price response to a financing decision, and (v) welfare.

We start our analysis with the two limiting cases where (i) monitoring becomes infinitely costly, in which case the manager induces the financier to evaluate efficiently by giving her the least possible share, which keeps her from excessive evaluation (*over-evaluation* problem), and (ii) evaluation becomes infinitely costly, where the manager makes the financier monitor efficiently by selling her the whole firm to keep her from monitoring insufficiently (*under-monitoring* problem). But this suggests when the financier both evaluates the firm before investment, and then monitors the firm after providing the fund, there exists a tension in

[2]See also Asquith and Mullins (1986), Masulis and Korwar (1986), Eckbo (1986), Mikkelson and Partch (1986), James (1987), Preece and Mullineaux (1994), Lummer and McConnell (1989), Slovin *et al.* (1992), Best and Zhang (1992), Billet *et al.* (1995), and Hadlock and James (2000).

[3]Similar strategic incentives would emerge if we considered venture capitalists who must choose whether to take firms public. Gompers and Lerner (1999) highlight evidence of grandstanding empirically.

the manager's decision regarding picking the optimal evaluation or monitoring, and of course depending on how deviation from the optimal evaluation and monitoring hurts the profit, he offers the financier a share in between.

If through monitoring the financier realizes that the firm she has funded has a negative net present value (NPV), but her claims to the firm are very liquid in the after–market, she has incentive to quietly sell her share and abandon the project. Otherwise, she prefers to intervene in the firm (or to liquidate the firm and take her share of the salvage value).

We show that the manager can make the financier increase her evaluation effort by offering her a bigger share of firm and consequently charging her more. However, the effect of this action on the financier's monitoring effort depends on the her optimal response to receiving a bad monitoring signal. In particular, when the financier intervenes in the firm, giving her a higher share also increases her monitoring effort. But when she sells her shares in the market, giving her a higher share causes her to monitor less. We show that in equilibrium, when the financier sells her share in the market following a bad monitoring signal, the manager makes the financier over-evaluate so that he reduces her over-monitoring. However, when she intervenes in the firm, the manager makes her over-evaluate to reduce her under-monitoring.

The other issue that we extensively explore is the effect of an increase in the after–market liquidity on the equilibrium outcome. If the manager does not respond to an increase in the after–market liquidity, the more the share that the financier can sell on the market without revealing her presence, (i) the higher the opportunity cost to the financier of intervening in the firm's strategy; (ii) the greater the value of an accurate monitoring signal on which the financier makes her trading decisions; and (iii) the smaller the costs of making a bad initial financing decision. In turn, the reduced costs of a bad financing decision cause the financier to evaluate the firm less thoroughly, which further raises the optimal level of monitoring. However, the manager can increase the share he offers and charge the financier more in order to induce her to increase her evaluation. We show that in equilibrium, when the financier sells her shares in the market, the manager's response to an increase in the after–market liquidity makes the financier increase both evaluation and monitoring effort, so that share–price responds more positively to more liquid finances. But there are times at which the manager's response to a more liquid finance is to give more shares to the financier just to provide incentive for the financier not to sell her shares in the market after discovering a bad investment decision, and instead intervene in the firm. In this case, the more liquid the after–market the higher the share that the manager has to give away. But this can't go on forever, and at some point the manager stops luring the financier and lets her dump her share on the market. Similar to the previous case, when the manager offers more share to the financier, the increase in her both evaluation and monitoring effort makes the share–price respond strongly positively to more liquid forms of finance, however, at the very point that the manager stops giving more share and let the financier dump her share on the market, we observe a sudden drop in the share–price.

We also show that the after–market liquidity has different effects when the manager has restrictions in setting his strategy. In particular, when the manager can only sell a fixed share to the financier, or when he has restriction in setting his price, as after-market liquidity increases, the restricted manager has not enough power to make the financier increase his evaluation along with monitoring. In this case, with lower evaluation from the financier's part, the share–price of a financed project responds negatively to more liquid after–markets.

Finally, show that the after-market liquidity yet has a different effect when the manager is shortsighted, and only cares about the current shareholders. In particular, shortsightedness makes the manager not care about evaluation as much as he should, and as a result, does not make the financier increase her evaluation effort as she wastefully increases her monitoring effort. In this case, similar to the previous case, a higher after-market liquidity causes a lower share–price response to a financing event.

Related Research. Almazan and Suarez (2003) also seek to explain why the announcement of bank loans produces significantly positive abnormal returns on firms' equity. They endogenize the choice of private versus public financing by a firm's management: private finance permits monitoring that precludes moral hazard by management, but management must be compensated for control rents they could receive with public finance. With public finance, the managerial compensation scheme can ameliorate these control rents, but it is more costly to provide the correct incentives to good firms. Hence, in equilibrium, good firms choose private finance.

The informed financier in our model faces tradeoffs similar to those of an informed institutional investor in Kahn and Winton (1998). The issues considered and modeling approaches are otherwise very different. Kahn and Winton consider an institutional investor *endowed* with bad news who weighs the costs and benefits from actively intervening. Intervention is costly, but intervention probabilistically raises the firm's payoff. Importantly, whether the institution intervenes is *not* public information; and *if* the institution intervenes, it *privately* observes whether its intervention was successful. The institution can trade on *both* pieces of private information. Thus, in Kahn and Winton, intervention *creates* private information on which the institution can trade. In contrast, in our model, intervention is *public* information and intervention *reveals* bad news to the market; the opportunity cost of intervention are the foregone expected trading profits. The papers also focus on very different issues. Kahn and Winton's model is one of a troubled established firm, where neither information acquisition nor financing are concerns, issues that are at the heart of our paper.

Maug (1998) also models how liquidity affects interventions in a troubled firm by a large shareholder. Here the intervention both raises the expected value of the firm *and* creates private information for the large shareholder about the outcome of the intervention. In

Maug, giving the large shareholder a greater stake causes it to internalize a greater portion of the intervention costs, but it is also presumed to reduce the amount of liquidity trade which, in turn, reduces the trading profits associated with intervention. In equilibrium, this latter effect never dominates: larger stakeholders are more likely to intervene.[4]

The next section presents the economic environment. Section 3 details equilibrium outcomes. The impact of the liquidity of the securities market for equilibrium decisions, actions and pricing are derived. Section 4 concludes. All proofs are in an appendix.

2 Model

2.1 The Firm

A firm has a project that needs one unit of capital to get started. Without loss of generality, we normalize the firm's residual assets to be zero. A financed project either succeeds and produces a cash flow of X, or it fails and produces nothing. A financed project has a positive NPV, *i.e.* $\rho X \geq 1$. The payoffs are received at the end of the game, and for simplicity, there is no discounting. It is common knowledge that the project is successful with the probability ρ. The firm's manager seeks to maximize the value of the shareholders. In order to get the project financed, at time zero, the firm's risk neutral manager offers s shares of the firm to a risk neutral financier and charges her p_0. The offer is take it or leave it.

2.2 The Financier

The financier has the capability to evaluate the project before she invests in it. However, evaluation is costly, and it generates a signal for the quality of the project that is not accurate. In particular, if the project is bad, and the financier spends $e\pi_E^2$ dollars on evaluation, with probability π_E she gets a (bad) evaluation signal that accurately shows the project is bad. With the residual probability $1 - \pi_E$ the financier receives a (good) evaluation signal which indicates either the project is good, or it's bad but she could not identify it.
After she has funded the project, the financier can monitor her investment. The monitoring technology is similar to that of evaluation. In particular, after spending $m\pi_M^2$, with probability π_M the financier receives a (bad) monitoring signal that accurately shows if the project is bad. With the residual probability, the financier can't distinguish between a good and bad project. After receiving a monitoring signal, the financier can either do nothing and let the project mature, or she can take an action. The two mutually exclusive actions that are available to the financier are one: liquidating the firm, and two: selling her shares in the

[4] See also the related model of Bolton and Von Thadden (1998), and discussion by Bhide (1993).

market. Liquidation generates a cash flow of $z < 1$, and hence the financier receives sz from liquidation.

The financier can not sell as many shares as she wants. In particular, the financier can sell only α shares, which represents the liquidity of the firm. Given that the price of each share is λ, the financier gains $\alpha\lambda < 1$ by selling her shares. If the financier sells her shares upon receiving a bad monitoring signal, the bad project matures and the current shareholders will receive nothing since a bad project generates no cash flow in the end. Indeed, by selling the worthless shares, the financier gains $\alpha\lambda$ at the future shareholders' expense.

2.3 Timing

At $t = 0$, the manager offers the financier s shares of the firm and asks p_0 in return. The financier evaluates the firm and observes the evaluation signal. Based on the evaluation signal, she decides whether to accept or reject the manager's offer. If she rejects, the game ends and every one receives zero. The news that the financier has accepted the offer or not is released at $t = 0.5$.

If she accepts, at $t = 1$ she monitors the firm, and receives a monitoring signal. Based on her monitoring signal, she can either do nothing and let the project yield at $t = 2$, or she can take action. If she decides to take action, she can either liquidate the firm, or sell her shares in the market. If she liquidates the firm, the game ends at $t = 1$, otherwise, the project yields at $t = 2$, and all shareholders receive their share of the cash flow.

3 Solution

In order to understand the model better, we start from the limiting cases. First, we analyze the case where the cost of monitoring m goes to infinity. This is of course equivalent to having no monitoring available to the financier. Next, we analyze the case where the evaluation becomes infinitely costly, *i.e.* e goes to infinity, which is equivalent to having the financier not evaluate the project. The advantage of focusing on evaluation and monitoring one at a time is that we can disentangle their effects completely, and see what we should expect when either effect dominates.

3.1 A Simple Model of Evaluation

3.1.1 Assumptions

In this model, all the assumptions of the general model are satisfied except that we assume the financier can not monitor her investment. As a result, after investing in the firm, she can not receive a signal, and hence she can not take further actions. Thus, it is useful if we state the adjusted timing of the game.

3.1.2 Timing

At $t = 0$ the manager offers the financier s shares and asks her p_0 dollars in return. At $t = 1$ the financier evaluates the firm and observes the evaluation signal. Moreover, based on the signal, she decides whether to accept the manager's (s, p_0) offer or not. If she rejects the offer, the project won't be executed, and everyone receives zero. If she accepts, at time $t = 2$ the project yields and everyone receives his/her share of the payoff.

3.1.3 Solution of the Evaluation Model

The manager's strategy is to offer a share and price (s, p_0) to the financier. The financier's strategy is to decide about her evaluation effort π_E, and then given the signal that she receives from evaluation, she decides whether to invest or not. Notice that if she invests she has to pay the cost p_0, and in return she gets a share s of the cash flow. Thus, if she invests in a bad project, she receives nothing, which contradicts her individual rationality condition. As a result, she invests only if she receives a positive evaluation signal. Now the question is how much the financier should spend on evaluation. Notice that the probability that the financier invests, which equals the probability of receiving a positive evaluation signal, is $\rho + (1 - \rho)(1 - \pi_E)$. Moreover, given that the financier has spend π_E on evaluation and received a positive signal, the project yields X with probability $\frac{\rho}{\rho+(1-\rho)(1-\pi_E)}$. Thus, the financier's expected profit from investment is

$$\big(\rho+(1-\rho)(1-\pi_E)\big)\big(s\frac{\rho}{\rho + (1-\rho)(1-\pi_E)}X - p_0\big) - e\pi_E^2 = s\rho X - \big(\rho+(1-\rho)(1-\pi_E)\big)p_0 - e\pi_E^2.$$

Hence, given (s, p_0), the financier maximizes this target function to obtain her optimal evaluation level. The first order condition (FOC) gives us the optimal evaluation $\pi_E = \frac{1-\rho}{2e}p_0$.

Knowing the financier's best response, *i.e.* her incentive compatibility (IC) condition, the manager sets (s, p_0) to maximize the firm's profit. Of course the manager can not charge as much as he wants, or give the financier as low a share as he wishes, because then the financier might not invest. In fact, the manager needs to make the financier weakly better off than not investing, *i.e.* satisfy her individual rationality (IR) constraint. Given that the financier's IC and IR conditions are satisfied, and the financier's optimal evaluation is π_E, the manager (the firm) receives p_0 with the probability that the financier invest, which is $\rho + (1 - \rho)(1 - \pi_E)$. He also has to pay one unit in order to invest. Finally, the probability that the firm succeeds given that the financier's evaluation will be π_E is $\frac{\rho}{\rho+(1+\rho)(1-\pi_E)}$, and hence, we can write the manager's target function as

$$\Big(\rho+(1-\rho)(1-\pi_E)\Big)\Big(p_0-1+(1-s)\frac{\rho}{\rho+(1+\rho)(1-\pi_E)}X\Big)$$
$$=\big(\rho+(1-\rho)(1-\pi_E)\big)(p_0-1)+(1-s)\rho X,$$

and hence, the manager's problem is

$$\max_{s,p_0}\ \Big(\rho+(1-\rho)(1-\pi_E)\Big)(p_0-1)+(1-s)\rho X \tag{1}$$
$$\text{s.t. the financier's IR and IC are satisfied.}$$

Because this model is not complicated, one can solve the manager's problem directly, however, it will be illustrative if we employ the method that we are going to use for the general model. Here, instead of focusing on the manager, we see what a social planner would do. Then, given that the social planner produces the optimal (social) profit, and the manager can acquire all this profit, we see if and how the manager can mimic the social planner. Another virtue of this method is that it enables us to compare the financier's evaluation with the socially optimal evaluation level.

3.1.4 Evaluation and Social Planner

In the simple evaluation game described above, it is the financier who has to pay the evaluation cost, and because she alone has to internalize the whole evaluation cost, her decision might be different than that of a (risk neutral) social planner. Similar to the financier, if the social planner makes a mistake and finances a bad project, the whole investment will be wasted. Thus, the social planner invests only upon receiving a positive evaluation signal.

Notice that given the social planner's evaluation level π_E, the probability that the social planner funds the project is equal to the probability that she sees a positive evaluation signal, which is $\rho+(1-\rho)(1-\pi_E)$, and the project is successful with the updated probability $\frac{\rho}{\rho+(1-\rho)(1-\pi_E)}$. Moreover, instead of p_0, the social planner has to pay one unit of capital.

Similar to the derivation of the financier target function, the social planner's problem can be expressed as

$$\max_{\pi_E}\ \rho X-\big[\rho+(1-\rho)(1-\pi_E)\big]-e\pi_E^2. \tag{2}$$
$$\text{s.t. the social planner's IR condition is satisfied.}$$

Hence, the financier's optimal evaluation is

$$\pi_E^*=\frac{1-\rho}{2e}, \tag{3}$$

comparing which to the financier's optimal evaluation $\frac{1-\rho}{2e}p_0$ confirms the possibility of *over-evaluation* since $p_0\geq 1$.

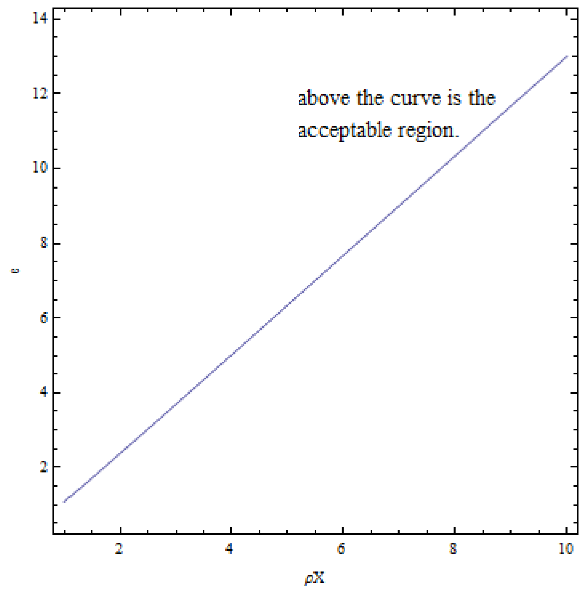

Figure 1: Above the curve (which behaves quite linearly for $\rho X \geq 1$) lies the acceptable region determined by the inequality in the lemma 1.

3.1.5 Manager and Financier

The manager already knows that if he charges any price p_0 above one unit, the financier will over-evaluate, and from the society's point of view, this is just a waste of resource. However, as long as the financier's IR condition is binding, *i.e.* the manager leaves no surplus for the financier, all the society's value belongs to the manager. The following lemma ensures us that if evaluation is not *too cheap*, the manager has no incentive to leave any surplus for the financier.

Lemma 1 *For any number of shares s, the manager wants to increase the price p_0 to extract all the surplus from the financier if and only if the following condition holds*

$$1 - \frac{1}{2e} \leq 2\sqrt{1 - 2\rho X \frac{1}{2e}}.$$

Proof. *See the appendix.*

As can be seen in graph 1 , the acceptable region is above a curve which quickly converges to a line as the NPV grows. Hence, for any NPV, we need the evaluation cost e not to be too small. The intuition is that when the evaluation cost is small, as the manager increases the price p_0, the financier affords to increase her evaluation so radically, that the decrease in her investment probability swamps the gain of a higher price. Moreover, since the magnitude of the price p_0 reflects the gain from investment, which in turn is positively related to NPV ρX, the more the NPV, the larger the evaluation cost we need. From this point on, we assume that the condition of the lemma 1 holds.

Equipped with lemma 1, we know that for any share s that the manager offers, he takes the whole (*i.e.* social) value of the project. The reader might find it useful to think as if the manager is paying the financier to evaluate the project for him. Of course this means that the manager has incentive to make the financier not waste any resource on over-evaluation. We need one more peace of information to conclude how the manager would accomplish this goal, which follows.

Corollary 1 *If the manager gives a higher share s to the financier, he charges her a higher price p_0, such that*

$$\frac{dp_0}{ds} = \frac{\rho X}{\rho + (1-\rho)(1-\pi_E)} > 0.$$

Notice that this relation immediately follows from the lemma 1 since the financier's IR condition binds, meaning her value function at her optimal decision is zero *i.e.*,

$$V^f = \max_{\pi_E} \; s\rho X - \big[\rho + (1-\rho)(1-\pi_E)\big]p_0 - e\pi_E^2 = 0,$$

and hence we have

$$\frac{dp_0}{ds} = -\frac{\frac{\partial V^f}{\partial s}}{\frac{\partial V^f}{\partial p_0}} = \frac{\rho X}{\rho + (1-\rho)(1-\pi_E)} > 0,$$

where the second equality follows from the envelope theorem.

But this readily means that the manager can make the financier evaluate efficiently by charging her $p_0 = 1$, and then taking all the surplus from her by giving her as low a share as he can $s = \underline{s}$, such that the financier still invests. Thus, in equilibrium the manager offers the financier $(\underline{s}, 1)$, and she evaluates the firm optimally, leaving all the social surplus to the manager.

In a summary, over-evaluation emerges when the financier is asked to pay a price higher than the cost of the project. And the intuition behind this solution is that the manager keeps the financier away from over-evaluation by asking $p_0 = 1$, and instead giving him the lowest possible share $\underline{\text{s}}$.

Our final note regarding the evaluation model is that it provides simple predictions for over-evaluation, which we present in the following lemma:

Lemma 2 *Over-evaluation is increasing in the number of shares s the manager offers, decreasing in the probability of success ρ, and increasing in $\frac{\rho X}{e}$, whose maximum is decreasing in ρX when the condition in lemma 1 holds. Using these rules, we see that the financier never evaluates more than 0.74: $\pi_E < 0.74$.*

Proof. *See the appendix.*

Notice that as ρX increases, we expect p_0 to increase which results in a higher evaluation if

e is fixed. However, if e satisfies lemma 1 with equality, *i.e.* it is the lowest acceptable cost at ρX, as ρX increases, the condition of lemma 1 implies that minimum e must increase as well, and this extra cost more than offsets the positive effect of ρX on evaluation.

3.2 A Simple Model of Monitoring

3.2.1 Assumptions

In this model, all the assumptions of the general model are satisfied except that we assume the financier can not evaluate the project before she invests in it. Of course if the financier invests, she can monitor her investment and based on the monitoring signal she can take further actions. Recall that if the financier liquidates the project, she receives sz, and the rest of the shareholders receive $(1-s)z$. If she sells her shares in the market, she receives $\alpha\lambda$ and the project is left to mature. It is useful if we restate the timing for the monitoring model.

3.2.2 Timing

At $t = 0$, the manager offers (s, p_0) to the financier. If she rejects, the game ends and everyone receives zero. If the financier accepts, at $t = 1$ the financier monitors her investment and receives a monitoring signal. Having seen the monitoring signal, at $t = 2$ the financier decides whether to sell her shares in the market, liquidate the firm or take no action. If she liquidates, the game ends and shareholders receive their share of the liquidation value. If the financier sells her shares in the market, or if she takes no action, at $t = 3$ the project yields and everyone receives his/her share.

3.2.3 Solution to the Monitoring Model

Similar to the evaluation model, the manager's strategy is to offer a share and price (s, p_0). The financier's strategy is either to accept or reject, and of course she accepts if and only if her IR condition is satisfied. Notice that if the financier accepts the manager's offer, she monitors her investment. If she discovers that the project is bad, it is in her best interest to interfere, since otherwise the project matures and yields nothing, but either dumping the shares on the market, or liquidating the project gives her a positive payoff. Thus, upon receiving a negative monitoring signal, she should decide on whether to dump her shares on the market, or to liquidate the project. Since the payoffs of liquidating and selling are sz and $\alpha\lambda$ respectively, the financier liquidates the project if and only if $s \geq \frac{\alpha\lambda}{z} := s_0$. From this point on, we assume that $\alpha\lambda \leq z$, so that $s_0 \in [0, 1]$.

Since there is no evaluation, the probability that the financier funds a good project is

ρ, and hence by accepting the manager's offer, she expects to receive $s\rho X$ from financing a good project. Moreover, the probability of financing a bad project is $1-\rho$, and given the monitoring cost of $m\pi_M^2$, the probability of finding a bad project through monitoring is $(1-\rho)\pi_M$. As a result, the financier expects to receive

$$V^f = \max_{\pi_M}\{s\rho X + (1-\rho)\pi_M \max\{\alpha\lambda, sz\} - (p_0 + m\pi_M^2)\}, \tag{4}$$

by accepting the manager's offer. Thus, her best response regarding the monitoring level is $\pi_M = (1-\rho)\frac{\max\{\alpha\lambda, sz\}}{2m}$, and she invests if and only if V^f is positive. Since we need the monitoring π_M not to exceed one, henceforth we require that $m \geq 0.5$, so that for any combination of other parameters we have $\pi_M \in [0,1]$.

Unlike the evaluation model, in order to write the manager's problem, we need to know more about the manager's attitude toward the future shareholders. First, suppose the manager only considers the current shareholders. Then, if the financier's IR condition is satisfied and she invests, the manager gains $p_0 - 1 + (1-s)\rho X$ from her investment, and also he expects to receive $(1-s)(1-\rho)\pi_M z$ in case the financier liquidates the project. Recall that the shareholders won't benefit if the financier sells her shares in the market. Hence the manager's problem can be expressed as

$$\max_{s,p_0}\ (p_0 - 1) + (1-s)\big(\rho X + (1-\rho)\pi_M z \mathbf{I}(s \geq s_0)\big), \tag{5}$$
$$\text{s.t. financier's IR and IC conditions are satisfied.}$$

However, when the financier sells her worthless shares in the market, some future shareholders are being swindled by her. If the manager takes the welfare of those future shareholders into account, he incurs an expected loss of $(1-\rho)\pi_M\alpha\lambda$, and thus the manager's problem becomes

$$\max_{s,p_0}\ (p_0 - 1) + (1-s)\big(\rho X + (1-\rho)\pi_M z \mathbf{I}(s \geq s_0)\big) - (1-\rho)\pi_M\alpha\lambda\mathbf{I}(s < s_0), \tag{6}$$
$$\text{s.t. financier's IR and IC conditions are satisfied.}$$

Recall that the financier's best monitoring response (*i.e.*) did not depend on the price p_0. As a result, as long as the financier IR condition is satisfied, the manager increases the price p_0. In other words, for any number of shares s, the manager's best price p_0 extracts all the surplus from the financier. It is easy to see (and is also shown in the next section) that when the manager cares about the future shareholder (*i.e.* the second case above), his target function equals a social planner's target function. The intuition is that a social planner tries to maximize the aggregate welfare (value) of the current shareholders, the financier, and the future shareholders. Thus, if the manager, who considers the current shareholders welfare and extract all the surplus from the financier, also considers the future shareholders, his total value equals the value of the social planner. In fact, we can show that in the monitoring model, similar to the evaluation model, the manager can achieve the socially optimal value by replicating the social planner's best response. So we analyze the social planner's problem to solve for the equilibrium of the monitoring model. Later on, we also show what happens if the manager does not care about the future shareholders.

3.2.4 Social Planner and Monitoring

Since the project has a positive NPV, a social planner always invests in it, and similar to the financier, the social planner won't take an action unless she observes a bad monitoring signal (otherwise her investment had no justification in the first place). Moreover, after observing a bad monitoring signal, the social planner takes action, otherwise her investment goes to waste. However, a social planner would never sell her shares in the market. The reason is that selling only transfers money from the future shareholders to the financier, which has no social value, but monitoring is costly. The social planner's problem is then

$$\max_{\pi_M} \ \rho X + (1-\rho)\pi_M z \mathbf{I}(s \geq s_0) - (1 + m\pi_M^2). \tag{7}$$

The reason for $\mathbf{I}(s \geq s_0)$ is that a social planner wants the financier not to monitor at all if she sells her shares in the market. This gives us the socially optimal value of monitoring

$$\pi_M^* = (1-\rho)\frac{z}{2m}, \tag{8}$$

when the financier liquidates, and of course no monitoring when she sells. Comparing (8) to the financier's best monitoring response $(1-\rho)\frac{max\{\alpha\lambda, sz\}}{2m}$ suggests that there is a potential under-monitoring problem when the financier liquidates ($s \geq s_0$), and also there is an over-monitoring problem when the financier sells in the market ($s < s_0$). Moreover, the financier's monitoring coincides with the socially optimal level of monitoring if and only if $s = 1$.

Next we show that when the manager cares about the future shareholders, his target function in (6) reduces to the social planner's target function in (7).

Lemma 3 *If the manager takes the future shareholders into account, the manager's target function equals the target function of a social planner.*

Proof. *See the appendix.*

3.2.5 Manager and Financier

Thus far, we know that the manager leaves no surplus for the financier. Hence, it is best for him if the financier's monitoring level coincides with the social planner's. In particular, for any share more than (or equal to) s_0 but less than one, the financier will under-monitor, and if the manager gives the financier any share less than s_0, she wastes resources on monitoring which has no value for him. So the equilibrium of the monitoring model is that the manager offers $s = 1$, and then he extracts all the surplus from the financier via increasing p_0 as much as he can, and the financier invests, monitors (social) optimally, and liquidates the project if she sees a bad monitoring signal.

The intuition of the result is that the financier has to internalize all the monitoring cost, however as long as she does not own the firm, she gains only a partial benefit from her monitoring effort. Thus, she has a tendency to under-monitor, which the manager corrects for by selling off all the firm. We also want to emphasize that although we needed the manager to care about the future shareholders so that we could use the auxiliary social planner's problem, in this simple model the result would remain the same even if the manager wouldn't care about the future shareholders. However, as we will show later, in the general model this distinction makes a difference.

Notice that the solution to under-monitoring, which is to offer the whole firm ($s = 1$), is in sharp contrast with the solution to over-evaluation, which is to offer the least possible share ($s = \underline{s}$). This tension between evaluation and monitoring creates a dynamism that we explore in the next section.

3.3 Solution to General Model

So far, we showed that evaluation makes the manager give as little share as possible to the financier, where as monitoring makes him want to sell the whole firm. Therefore, in a general model with both evaluation and monitoring, we expect to see a tension between those: the more important the evaluation, the smaller the s, and the more important the monitoring, the larger the s. However, when evaluation is not much costlier than monitoring, we expect to see evaluation playing the dominant role, for evaluation can save the financier the whole investment where as monitoring can only save her (a fraction of) the salvage value.

We start from the financier's and manager's problem, and then compare the financier's best responses to the social planner's best responses, so that we know the potential problems regarding over/under evaluation/monitoring that the manager faces, and how he can achieve the best outcome. Notice that the manager tries to make the financier emulate the social planner only if he can extract all the surplus from her. We present a sufficient condition that guarantees it, and after analyzing the manager's choice, we explore the comparative statics regarding an increase in liquidity.

Similar to the monitoring model, if the financier finds out that she has funded a bad project, she either liquidates the project when $s \geq s_0$, or she sells her shares in the market when $s < s_0$. This yields her $\max\{sz, \alpha\lambda\} \leq 1$, and since for her investment, she has to pay the evaluation and monitoring costs along with $p_0 \geq 1$, she does not invest in a project that she knows is bad. This immediately means that similar to the evaluation model, she rejects the manager's offer if she receives a negative evaluation signal. Moreover, she accepts his offer if she receives a positive evaluation signal, and also she expects to see a non-negative profit from her investment. Given that her profit is non-negative, the probability that the financier invests, which equals the probability of receiving a positive evaluation signal, is the sum of the probability that the project is good, and the probability that the project is bad but the financier does not receive a bad evaluation signal: $\big(\rho + (1-\rho)(1-\pi_E)\big)$. Moreover, given the evaluation π_E, the updated probability that the project is good equals $\frac{\rho}{\rho+(1-\rho)(1-\pi_E)}$, and hence given that the financier's evaluation and monitoring levels are π_E and π_M respectively, the financier's expected profit from investment is

$$\big(\rho + (1-\rho)(1-\pi_E)\big)\Bigg(s\frac{\rho}{\rho+(1-\rho)(1-\pi_E)}X + \big(1 - \frac{\rho}{\rho+(1-\rho)(1-\pi_E)}\big)\pi_M \max\{\alpha\lambda, sz\}$$
$$- (p_0 + m\pi_M^2)\Bigg) - e\pi_E^2 =$$
$$s\rho X + (1-\rho)(1-\pi_E)\pi_M \max\{\alpha\lambda, sz\} - \big(\rho + (1-\rho)(1-\pi_E)\big)(p_0 + m\pi_M^2) - e\pi_E^2, \tag{9}$$

which gives the financier the highest profit V^f at

$$\text{If the financier sells } (s < s_0) \begin{cases} \pi_E = \left(\frac{1-\rho}{2e}\right)(p_0 + m\pi_M^2 - \pi_M\alpha\lambda) \\ \pi_M = \frac{(1-\rho)(1-\pi_E)}{\rho+(1-\rho)(1-\pi_E)}\frac{\alpha\lambda}{2m}. \end{cases} \tag{10}$$

$$\text{If the financier liquidates } (s \geq s_0) \begin{cases} \pi_E = \left(\frac{1-\rho}{2e}\right)(p_0 + m\pi_M^2 - \pi_M sz) \\ \pi_M = \frac{(1-\rho)(1-\pi_E)}{\rho+(1-\rho)(1-\pi_E)}\frac{sz}{2m}. \end{cases} \tag{11}$$

Hence, the financier accepts the manager's offer if and only if she receives a positive evaluation signal, and at her best evaluation and monitoring responses she expects a non-negative profit (IR condition).

As for the manager, he wants to maximize the current and future shareholders' profit, in the sense of (6). Later on, we analyse the case where the manager does not care about the future shareholders. After the financier invests $p_0 \geq 1$ dollars, the manager has to pay one dollar to initiate the project, and the rest goes to the current shareholders. Also, when a good project yields, or a bad project gets liquidated, the shareholders take their shares. Thus, given that the financier invests with probability $\big(\rho + (1-\rho)(1-\pi_E)\big)$, the manager's problem can be expressed as the following:

$$\max_{s,p_0} \Big(\rho + (1-\rho)(1-\pi_E)\Big)(p_0 - 1)$$
$$+ (1-s)\Big(\rho X + (1-\rho)(1-\pi_E)\pi_M \mathbf{I}(s \geq s_0)z\Big) - (1-\rho)(1-\pi_E)\pi_M \mathbf{I}(s < s_0)\alpha\lambda.$$
$$\text{s.t. the financier invests, and his FOCs are satisfied.} \tag{12}$$

In the following section we represent the social planner's problem, so that in later sections we can compare the financier's responses with the socially optimal ones.

3.3.1 Social Planner

Here we present the social planner's problem for comparison. We explore what a social planner would want the financier to do. The social planner's target function is quite similar to the financier's target function in (9), except that she internalizes the real cost of the project (*i.e.* one) instead of p_0, and also she internalizes the value of all shareholders (*i.e.* $s = 1$). Finally, she does not value selling. In particular, if she is forced to sell her shares in the market, she prefers not to monitor at all. Of course this is only relevant when the manager offers the financier $s < s_0$. Thus, the social planner's problem can be represented as

$$\max_{\pi_E,\pi_M} \rho X + (1-\rho)(1-\pi_E)\pi_M z\mathbf{I}(s \geq s_0) - \big(\rho + (1-\rho)(1-\pi_E)\big)(1 + m\pi_M^2) - e\pi_E^2. \quad (13)$$

Hence, the social planner's best response is

$$\text{If the financier liquidates } (s \geq s_0) \begin{cases} \pi_E^* = \left(\frac{1-\rho}{2e}\right)(1 + m{\pi_M^*}^2 - \pi_M^* z) \\ \pi_M^* = \frac{(1-\rho)(1-\pi_E^*)}{\rho+(1-\rho)(1-\pi_E^*)}\frac{z}{2m}. \end{cases} \quad (14)$$

And if the financier sells, the social planner would want her not to monitor $\pi_M^* = 0$, and evaluate at $\pi_E^* = \frac{1-\rho}{2e}$.

3.3.2 Manager and Financier

Similar to previous sections, one crucial fact is that the manager has the opportunity to leave no surplus for the financier. Indeed, we can show that when the marginal cost of evaluation e is not too low, the manager makes the financier's individual rationality constraint bind, and hence the following lemma holds.

Lemma 4 *When the marginal cost of evaluation e is not too low, the manager leaves no surplus for the financier. In particular*

(i) at a given share s, the manager increases the price p_0 as high as he can.

(ii) at a given price p_0, the manager decreases share s as low as he can.

This makes the financier indifferent between investing and not investing.

Proof. *See the appendix.*

Although this is an existence result, simulation shows that for $m = 0.5$ – the lowest m that we allow – at a given ρX the acceptable e lies just above the curve in figure 1. In fact the two curves are so close that they can not be told apart by eye. We assume that henceforward, parameters e and ρX are such that lemma 4 holds. Lemma 4 part (i) enables us to define an optimal $p_0(s)$ for any s that the manager would choose. As we expect, the manager asks the financier a higher price $p_0(s)$ as he offers her a higher share s. In fact using lemma 4 we can show that:

Corollary 2 *We have the following relation between the optimal price $p_0(s)$ and the number of shares s:*

$$\frac{\partial p_0}{\partial s} = \frac{\rho X + (1-\rho)(1-\pi_E)\pi_M \mathbf{I}(s \geq s_0) z}{\rho + (1-\rho)(1-\pi_E)}.$$

Proof. *See the appendix.*

For simplicity, after this point whenever we say the manager's best price, or simply the manager's price p_0, we mean $p_0(s)$.

Corollary 2 shows that the manager's optimal price p_0 always increases as he offers more shares. But as soon as he starts offering enough share to financier such that she starts liquidating bad projects (*i.e.* s hits s_0), the manager suddenly starts to charge more than before, or equivalently $\frac{\partial p_0}{\partial s}$ suddenly increases. This is simply due to the fact that for shares larger than s_0, the amount that the financier makes from finding a bad project sz also increases with her share of the firm. And since the manager gets all the surplus, as s increases he can increase the price p_0 even more than when the financier sells bad projects.

Corollary 2 also enables us to draw the manager's optimal price p_0 at each share s in a figure like figure 2 (left). Notice that after the financier starts to liquidates $s > s_0$, the blue curve rises faster. In the figure, the manager's best choice (s, p_0) is a typical point in which the financier sells. Recall that the manager has incentive to give a small share to the financier when the role of evaluation is relatively more important, in which case the financier is likely to sell when received a bad monitoring signal.

The right panel is a copy of the left panel in which liquidity has increased. Notice that the part of the curve in which the financier sells shifts up, as α increases. In order for the manager to leave no surplus for the financier, at a fixed s he increases p_0, and at a fixed p_0 he decreases s, as predicted by the lemma 4.

The fact that the financier funds the project affects its stock price. We define the price of a funded project P_F to be the share price of such a project in a competitive stock market. Recall that at $t = 0.5$, after the financier has evaluated the project and accepted the offer, the (updated) probability that the project is good is $\frac{\rho}{\rho+(1-\rho)(1-\pi_E)}$. And hence the expected probability of finding a bad project through monitoring is $\left(1 - \frac{\rho}{\rho+(1-\rho)(1-\pi_E)}\right)\pi_M$. Finally, a good project yields X, and a bad project if liquidated yields z. Thus, the price of a funded project is

$$P_F = \frac{\rho X + (1-\rho)(1-\pi_E)\pi_M \mathbf{I}(s \geq s_0) z}{\rho + (1-\rho)(1-\pi_E)}, \quad (15)$$

which equals $\frac{\partial p_0}{\partial s}$ from the corollary 2.

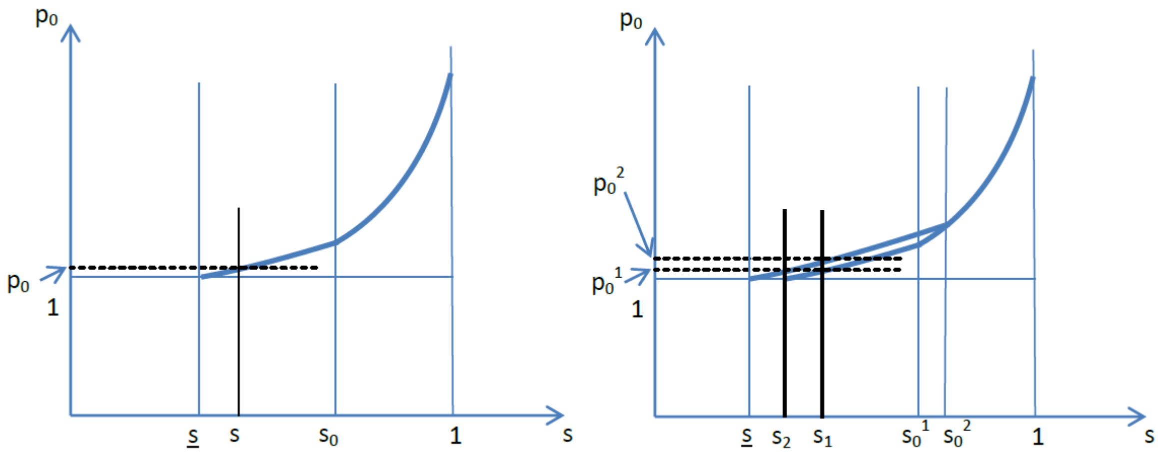

Figure 2: **(left)** The manager's optimal price $p_0(s)$ is the thick blue curve. The lowest share that the manager can offer is $\underline{s}$, and after s_0 the financier starts to liquidate bad projects, also (s, p_0) represents a typical solution for the model.
(right) Aa a result of an increase in liquidity $\alpha_2 > \alpha_1$, the manager has to give more shares to the financier for her to liquidate $s_0^2 > s_0^1$. Notice that when s_1 is fixed, p_0^1 increases to p_0^2, and when p_0^1 is fixed, s_1 decreases to s_2.

We are interested to see how the liquidity α affects the price of a funded project. Of course if the financier decides to liquidate the project upon receiving a bad monitoring signal ($s \geq s_0$), liquidity plays no role in the model, and hence it has no effect on the price of a funded project P_F, or any other variable for that matter. However, if the financier sells the firm, changing liquidity will change both the manager's and the financier's decisions, and hence affecting the price of a funded project as well.

One prediction of the model is that as the liquidity changes, the sign of $\frac{\partial P_F}{\partial \alpha}$ depends on how "free" the manager is in setting the price p_0 and the share s he offers.

Proposition 1 *Given that the manager sets either s or p_0, and takes the other one as given, we have the following results:*

(i) As long as the financier liquidates upon receiving a bad monitoring signal (with an interior solution $s > s_0$), a change in liquidity α, has no effect on the behavior of the financier and the manager.

(ii) As long as the financier sells upon receiving a bad monitoring signal (or she is at the verge of liquidating $s = s_0$), an increase in liquidity α will increase the monitoring, and decrease the evaluation, which in turn decreases the price of a funded project P_F.

Proof. *See the appendix.*

This becomes important, for instance, if the manager can not offer more than a certain number of shares, or if he can not set the price as high as he wants. In order to provide intuition on this result, we define direct and indirect effects, their role, and why/when one dominates.

Notice that the price of a funded project when the financier sells her shares upon receiving a bad monitoring signal reduces from (15) to

$$P_F = \frac{\rho X}{\rho + (1-\rho)(1-\pi_E)}, \tag{16}$$

which reflects the fact that the market only values the financier's evaluation effort. When the liquidity of the firm increases, upon receiving a bad monitoring signal, the financier can dump more shares on the market and hence she makes more money from selling. It increases the value of monitoring, and because the financier can now recover more from a bad investment, her incentive for evaluation falls. But the less the evaluation, the more the probability of a bad investment, which again increases the value of monitoring, and this goes on. As a result, the financier puts more effort on monitoring and less effort on evaluation, and the lower the evaluation, the lower the price of a funded project (P_F).

In the above discussion, we assumed a passive role for the manager as if he does not change his choices when the liquidity changes. The effects in which we only consider the financier's decision, and treat the manager's choices as fixed are defined as the *direct* effects. But the manager's move also affects the financier's response, which in turn affects the price of a funded project. These types of effects are called the *indirect* effects. Another way to see this is to write $\pi_E = \pi_E\big(p_0(\alpha), \alpha\big)$, and then

$$\frac{\partial P_F}{\partial \alpha} = \frac{\partial P_F}{\partial \pi_E}\Big(\frac{\partial \pi_E}{\partial \alpha} + \frac{\partial \pi_E}{\partial p_0}\frac{\partial p_0}{\partial \alpha}\Big),$$

where the first partial in the parenthesis denotes the direct effect, and the second one denotes the indirect effect.

So far, we have argued that an increase in liquidity has a negative direct effect on the price of a funded project. Next, suppose the manager can only set the price p_0, and not the share s. Obviously, the manager sees that the financier's value is increasing as liquidity goes up. By lemma (4), we know that he charges the financier more to make her indifferent between investing and not investing. But an increase in the initial payment p_0 makes a bad investment more costly to the financier, which in turn drives her to evaluate the project more. So this indirect effect pushes up the price of a funded project P_F. Briefly said, an increase in liquidity has a negative direct effect, and a positive indirect effect on the price of a funded project. We show that when the manager can set only one of the price or share, the (negative) direct effect dominates the (positive) indirect effect, and the price of a funded project drops:

$$\frac{\partial \overset{-}{P_F}}{\partial \alpha} = \frac{\partial \overset{+}{P_F}}{\partial \pi_E} \Big(\frac{\partial \overset{-}{\pi_E}}{\partial \alpha} + \frac{\partial \overset{+}{\pi_E}}{\partial p_0} \frac{\partial \overset{+}{p_0}}{\partial \alpha} \Big). \tag{17}$$

It is worthwhile to observe that as the manager gains more authority, the liquidity's effect changes substantially. In particular, when the manager can choose both s and p_0, the indirect effect of an increase in liquidity is stronger and as a results, the price of funded project increases. At the first glance, this result might look surprising, however, the intuition behind it makes it clearer.

Notice that evaluation is good in general, but monitoring is only good when the financier liquidates, and when she sells, it is a complete waste of resources. Now recall that as the liquidity increases, the financier (when sells) starts to gain more from monitoring and less from evaluation, which makes her substitute resources from evaluation to monitoring, and hence wasting them. Also remember that when s was fixed, the manager would increase p_0 to extract the newly generated surplus. This extra payment would make the financier more cautious about her investment, inducing her to evaluate more and hence monitor less. Previously, this would weaken the direct effect, but was not strong enough to reverse it. However, if the manager increases s as well, he can charge the financier even further. This is the source of the *extra* indirect effect that we mentioned above, which forces the financier to evaluate still more and monitor less. We show that this larger indirect effect now dominates the direct effect. Briefly said, in response to an increase in liquidity, the manager increases both the price p_0 and share s, in order not to let the financier waste money by diverting resources from evaluation to monitoring. We can also write $\pi_E = \pi_E\big(p_0(s(\alpha), \alpha), \alpha\big)$, and based on the above discussion,

$$\frac{\partial \overset{+}{P_F}}{\partial \alpha} = \frac{\partial \overset{+}{P_F}}{\partial \pi_E} \bigg(\frac{\partial \overset{-}{\pi_E}}{\partial \alpha} + \frac{\partial \overset{+}{\pi_E}}{\partial p_0} \Big(\frac{\partial \overset{+}{p_0}}{\partial \alpha} + \frac{\partial \overset{+}{p_0}}{\partial s} \frac{\partial \overset{+}{s}}{\partial \alpha} \Big) \bigg). \tag{18}$$

Notice that the only difference between (18), (17) is the presence of $\frac{\partial p_0}{\partial s} \frac{\partial s}{\partial \alpha}$, which as we just argued is positive, and moreover makes the RHS positive.

We need two more lemmas to prove our claim about the positive effect of liquidity on the price of a funded project. First, as once stated earlier, because the manager takes the value of present and future shareholders into account, and he leaves no surplus for the financier, his target function is the same as the target function of a social planner in (13), with the exception that the social planner maximizes her target function by choosing the socially optimal values of evaluation and monitoring, but in contrast, the manager chooses p_0, which pins down s (see figure 2 for illustration), and then the financier chooses (her) optimal evaluation and monitoring level. This is formally stated as

Lemma 5 *When the manager takes both the current and the future shareholders into account, his target function is the same as the social planner's target function:*

$$V^m = \max_{p_0} \; \rho X + (1-\rho)(1-\pi_E)\pi_M \mathbf{I}(s \geq s_0) z - \Big(\rho + (1-\rho)(1-\pi_E)\Big)(1+m\pi_M^2) - e\pi_E^2.$$

The only difference is that the manager maximizes this target function over p_0, and then the financier decides about π_E and π_M accordingly.

Proof. *See the appendix.*

In order to proceed further, we need the following lemma about the optimal price p_0 that the manager sets. We show that

Lemma 6 *When the financier sells upon receiving a bad monitoring signal ($s < s_0$), the optimal price p_0 for the manager is*

$$p_0 = 1 + \pi_M \alpha\lambda \Big[1 + \frac{\rho}{\rho + (1-\rho)(1-\pi_E)}\Big]. \tag{19}$$

Proof. *See the appendix.*

At this point we are able to state our proposition about if/when evaluation and monitoring are substitutes, and how liquidity affects the price of a funded project.

Proposition 2 *The followings hold:*

(i) When the financier sells upon receiving a bad monitoring signal ($s < s_0$) we have

1. $\frac{\partial \pi_E}{\partial p_0} > 0$.
2. $\frac{\partial \pi_M}{\partial p_0} < 0$.
3. $\frac{\partial \pi_E}{\partial \alpha}$, $\frac{\partial \pi_M}{\partial \alpha}$, $\frac{\partial P_F}{\partial \alpha}$ *are all positive.*

(ii) If the financier liquidates upon receiving a bad monitoring signal ($s > s_0$) we have

1. $\frac{\partial \pi_E}{\partial p_0} > 0$.
2. $\frac{\partial \pi_M}{\partial p_0} > 0$.
3. $\frac{\partial \pi_E}{\partial \alpha}$, $\frac{\partial \pi_M}{\partial \alpha}$, $\frac{\partial P_F}{\partial \alpha}$ *are all zero.*

Proof. *See the appendix.*

Notice that the effect of an increase in price p_0 on monitoring π_M is negative when the financier sells, but it is positive when the financier liquidates. To see the intuition, recall that an increase in p_0 makes the financier evaluate more because she incurs a larger loss in case of a bad investment. When she sells, because the selling value $\alpha\lambda$ is fixed, the financier

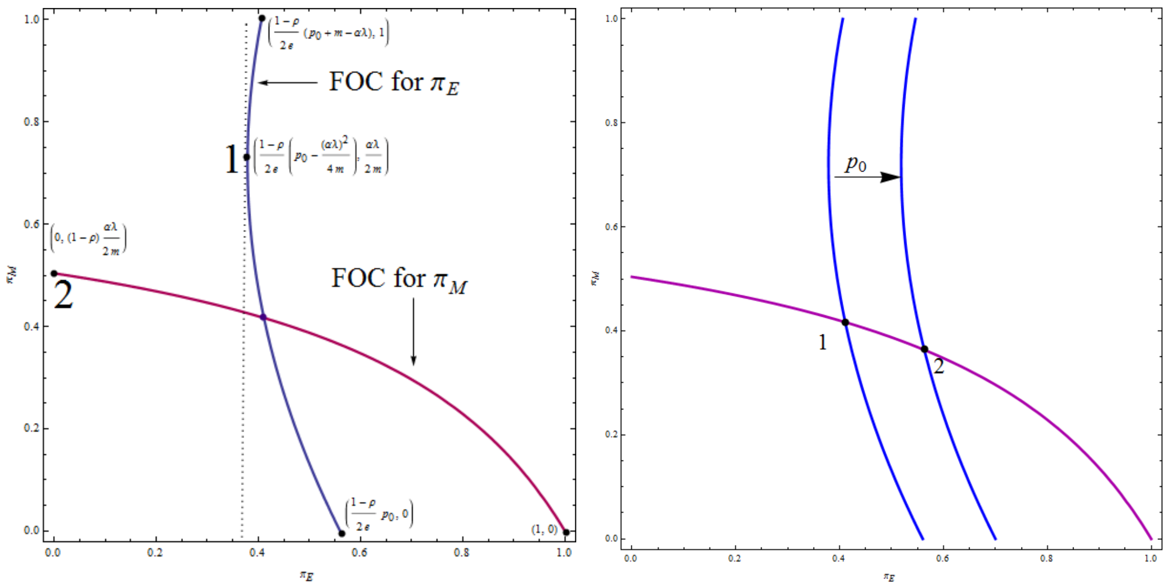

Figure 3: **(left)** The financier's FOC curves when she sells.
(right) As p_0 increases, the FOC for π_E shifts to the right (the solution moves from the point 1 to 2), increasing π_E and decreasing π_M.

substitutes monitoring with evaluation. However, when she liquidates, an increase in p_0 which is followed by an increase in s, would increase the liquidation value sz as well, which also gives the financier more incentive to monitor. As the proposition suggests, the added incentive for monitoring dominates the substitution effect.

It is easy to visualize the relations stated in proposition 2 if we draw the financier's FOCs in the (π_E, π_M) space. For instance, consider the case when the financier sells upon receiving a bad monitoring signal. Then her FOCs in (11) can be drawn as in figure 3 (left). By comparing the points 1 and 2 in the figure 3 (left), it is clear that the curve for the FOC of π_E is always downward sloping at the point of intersection. Note that an increase in the price that the financier has to pay p_0 only shifts the FOC curve for π_E to the right. The reason is that although the manager increases both p_0 and s together ($\frac{\partial p_0}{\partial s} = P_F$ by lemma 2), only p_0 enters into the financier FOCs. The result of an increase in p_0 is moving from the point 1 in the figure 3 (right panel) to the point 2, and hence an increase in evaluation, and a decrease in monitoring, as predicted in the proposition 2.

In contrast, when the financier liquidates, the liquidation value sz enters into her FOCs (the left panel of the figure 4). Thus, as p_0 increases, we should also track the resulting increase in s. One can see from the left panel that as s increases, the point 2 (left panel) migrates up, and the point 1 (left panel) migrates to up and left. Both of these effects (increase in p_0 and s) are depicted in the figure 4 (right). From the proposition 2, the result is an increase in both π_E and π_M as depicted in the figure 4 (right).

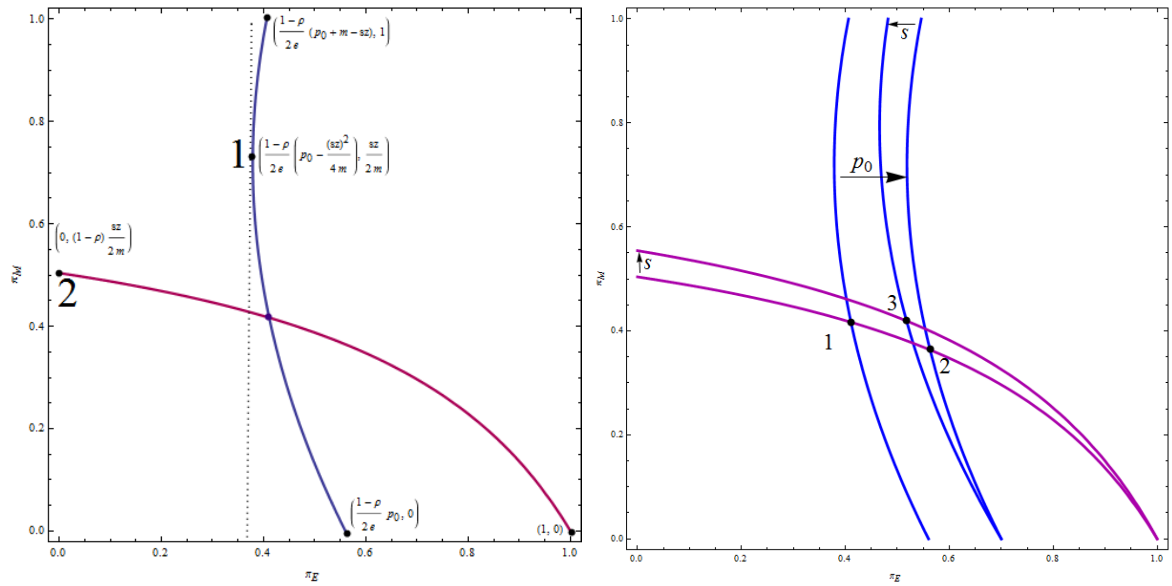

Figure 4: **(left)** The financier's FOC curves when she liquidates.
(right) As p_0 increases, the FOC for π_E shifts to the right (the solution moves from the point 1 to 2). Moreover, the increase in s changes both curves (almost) as if they rotate around their intersection with the π_E axis (the solution moves from the point 2 to the point 3), resulting in an increase in both π_E and π_M.

The merits of drawing the FOCs are beyond visualization of proposition 2. Using this tool, we can compare the financier's decision with that of the social planner. We start from the case where the financier liquidates a discovered bad project. Notice that in this case, the financier's FOCs in (11) is similar to that of a social planner in (14), except that for the social planner $p_0 = 1$ and $s = 1$. In contrast, the manager offers $s \leq 1$, and $p_0 \geq 1$, such that both of them don't hold with equality (see the left panel of figure 2). In order to compare the decisions of the social planner and the financier, we track the changes to the social planner's solution as we decrease s and increase p_0. Following the changes is easiest using a graph like figure 5, where we start from drawing the social planner's FOCs, and decrease s to observe the direction of changes. Notice that as the solution moves from the point 1 to 2, the financier starts to over-evaluate and under-monitor. Then we increase p_0 and again observe the direction of changes, which is further over-evaluation and under-monitoring. As a result, if s decreases, p_0 increases, or there is a combination of those, a liquidating financier over-evaluates and under-monitors. Notice that it coincided with the intuition, since the financier pays all the monitoring cost but she only gets a share of the benefit she under-monitors. Also, because the cost of the project is one, when the financier pays $p_0 > 1$, she over-evaluates. Even if she pays $p_0 = 1$, and the manager gives her $s < 1$ (which happens in equilibrium, see figure 2), she under-monitors and since evaluation and monitoring are substitute when

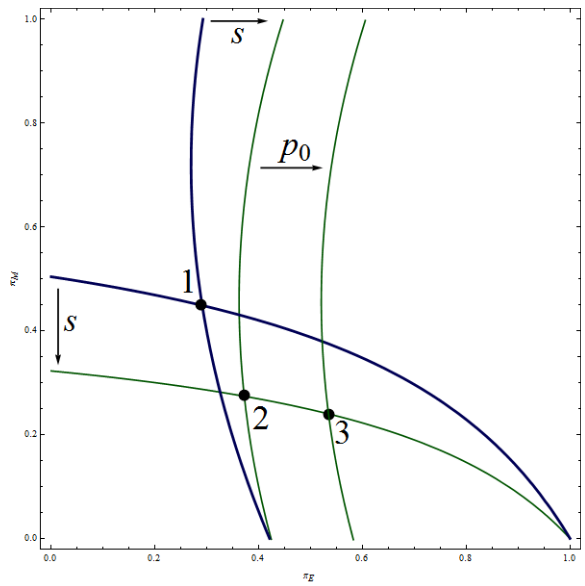

Figure 5: The blue (thick) curves are the FOCs of the social planner. As s decreases, the solution moves from the point 1 to 2. As p_0 increases, the solution moves from the point 2 to 3. Point 3 is a typical solution for the financier's FOC.

p_0 is fixed (at one), she over-evaluates. Notice that in the figure 2 (left), any acceptable point for the manager (which leaves no surplus for the financier) can be reached via first decreasing s at $p_0 = 1$, and then increasing p_0 to its optimal (highest) level $p_0(s)$. Thus, for every candidate of equilibrium in the model we have over-evaluation and under-monitoring when the financier liquidates the project.

In comparison, when the financier sells, all the monitoring effort is wasted. Hence the optimal monitoring is no monitoring at all, and the financier always over-monitors. Also using lemma 6, over-evaluation is immediate since we can replace p_0 from the lemma 6 into the financier's FOC for π_E to get

$$\pi_E = \Big(\frac{1-\rho}{2e}\Big)(p_0 + m\pi_M^2 - \pi_M\alpha\lambda) = \Big(\frac{1-\rho}{2e}\Big)\Big[1 + m\pi_M^2 + \frac{\pi_M\alpha\lambda\rho}{\rho+(1-\rho)(1-\pi_E)}\Big] > \frac{1-\rho}{2e} = \pi_E^*.$$

We formally state the results in the following proposition.

Proposition 3 *The next two statements hold:*

(i) When the financier sells upon receiving a bad monitoring signal, she over-evaluates and over-monitors.

(ii) When she liquidates a discovered bad project, she over-evaluates and under-monitors.

Proof. *See the appendix.*

Propositions 2 and 3 enable us to analyze the whole system better. Recall that when there was only evaluation, the manager offered the financier the lowest share possible $\underline{s}$ at price $p_0 = 1$, so that she does not waste money on over-evaluation. We also mentioned that since evaluation is preventive of a bad investment, rather than partially recovering it like monitoring, we expect to see the financier focusing on evaluation when it is not excessively costlier than monitoring. This suggests a way of investigating the system: we can see if/when the manager is willing to increase the price p_0 above one, which is unavoidably followed by an increase in s (above $\underline{s}$).

Let us start with the case where the financier sells a discovered bad project. It is easy to see that if the manager offers $p_0 = 1$ and $s = \underline{s}$, the financier under-evaluates and over-monitors. Over-monitoring is clear since all the monitoring effort is socially wasted. Then $p_0 = 1$ and together with $s = \underline{s}$ and $\pi_M = 0$ give us the socially optimal evaluation, but $\pi_M > 0$, and evaluation and monitoring are substitute when (s, p_0) is fixed (at $(\underline{s}, 1)$). Hence, $p_0 = 1$ together with $s = \underline{s}$ and $\pi_M > 0$ results in under-evaluation. By proposition 2, an increase in price p_0 (from $p_0 = 1$) increases evaluation and decreases monitoring, and hence it decreases both under-evaluation and over-monitoring. It is clear that the manager would increase the price until he achieve π_E^*. However, after this point, an increase in price p_0 would cause over-evaluation (and still less over-monitoring). Since we know by the proposition 3 that at equilibrium the financier ends up over-evaluating, proposition 2 suggests that the manager always prefers if the financier does a little more over-evaluation, in return of a little less over-monitoring.

There are two points that are worth pointing out. First, this result is intuitive since from the manager's point of view, not only the current shareholders don't get any benefit from the monitoring effort, but also the future shareholders are incurring a loss due to the financier's informed trading. Hence, the manager prefers if the financier wastes resources a bit more on evaluation rather than on monitoring and he achieves it through increasing p_0. However, he won't increase the price too far, because at some point the rate at which the financier wastes resources on evaluation stops justifying the benefits of smaller over-monitoring.

Second, if the manager does not care for the future shareholder's, then the harms of over-monitoring are lessened and he should increase the price less than what he does now. In the next section we examine this case, and indeed we show that when the manager does not care about the future shareholders he won't increase the price at all, offering $(\underline{s}, 1)$.

Next, suppose the financier liquidates upon receiving a bad monitoring signal. For simplicity, suppose she liquidates at any $s \geq \underline{s}$. Note that it simply means $\underline{s}z \geq \alpha\lambda$, for instance because the shares are very illiquid *i.e.* α is low. Then, from the graph 5 and by comparing the points 1 and 2, we know that if the financier is being offered $(\underline{s}, 1)$ she over-evaluates and under-monitors. By proposition 2, an increase in p_0 increases both evaluation and monitoring, and hence the over-evaluation problem worsens at the cost of less under-monitoring.

Hence, we predict that the manager increases the price p_0 so that the marginal cost of more over-evaluation equals the marginal benefit from less under-monitoring.

In contrast to the case where the financier sells a bad project and monitoring hurts, monitoring is appealing when she liquidates. The reason is clear from the social planner's point of view: recall that for a social planner, selling, although costly, creates no value because it's just a redistribution of wealth. However, liquidating creates value because it partially recovers a bad investment (the expected social value of monitoring and liquidation is $(1-\rho)(1-\pi_E)\pi_M z$). But the manager is entitled to put himself in social planner's shoes because the manager leaves no surplus for the financier, and his target function is the same as the social planner's target function (since the manager cares about current and future shareholders). In fact, when the financier liquidates, the manager wants her to do more monitoring. Hence, the mechanism that makes the manager bear more over-evaluation cost and increase the price p_0 is similar when the financier sells, and when she liquidates, with the exception that when selling, the manager wants to decrease over-monitoring, but when liquidating, the manager wants to decrease under-monitoring.

So far we have seen that when the manager sets both the share and the price he offers to the financier (s, p_0), an increase in liquidity α results in an increases in the price of a funded project P_F, given that $s \neq s_0$. In the next section we show that there is a possibility of a sudden decrease in price when the manager offers $s = s_0$ and liquidity increases.

3.3.3 Financier Switches

So far, we have explored the cases in which the financier does not change his behavior of selling or liquidating as the firm's liquidity α increases. We have seen that when the financier liquidates ($s > s_0 = \frac{\alpha\lambda}{z}$), an increase in liquidity does not change the price of a funded project P_F. And when the financier sells ($s < s_0$), an increase in liquidity increases the price of a funded project. However, in this section we see that there is a possibility of a drop in P_F as α increases, if the financier switches his best response from liquidation to selling.

Imagine the case where the manager is better off if the financier liquidates, and specifically the manager has the corner solution $s = s_0$ (*i.e.* the evaluation effect dominates). It should be clear by now that when the marginal costs of evaluation and monitoring are close, the manager is more worried about the financier's over-evaluation than his under/over-monitoring, and hence he gives the financier a small share so that she won't waste a lot of resources on over-evaluation. Also recall that informed trading hurts the manager, so the manager might want to give the financier just enough shares ($s = s_0$) to induce her to liquidate the firm upon receiving a bad monitoring signal. But as liquidity α increases, the manager should give up more shares to financier, since $s_0 = \frac{\alpha\lambda}{z}$ increases. So the manager charges the financier more, since by lemma 2 s and p_0 move in the same direction. But charging more, makes the financier increase her over-evaluation. Recall when the financier liquidates, increasing s improves under-monitoring, however, as just explained, $s = s_0$ already means that under-monitoring does not hurt the manager as much as over-evaluation (over-evaluation was the reason the manager was offering s_0 in the first place, *i.e.* the lowest share possible for liquidation). Thus, there is (usually) a point at which the manager stops giving up more shares to the financier, and lets her start dumping her share on the market.

There is an important point about this special case which is stated in the following proposition.

Proposition 4 *If as liquidity α increases, the manager wants to keep the financier at the verge of liquidation $s = s_0$, then we have:*

(i) $\frac{\partial p_0}{\partial \alpha}$ *and* $\frac{\partial s}{\partial \alpha}$ *are both positive.*

(ii) $\frac{\partial \pi_E}{\partial \alpha}$ *and* $\frac{\partial \pi_M}{\partial \alpha}$ *are both positive.*

(iii) $\frac{\partial P_F}{\partial \alpha} > 0$.

Proof. *See the appendix.*

Thus, if the manager keeps offering $s_0 = \frac{\alpha\lambda}{z}$ as α is increasing, just to stop the financier from selling, the financier increases both evaluation and monitoring. But remember that the price of a funded project $P_F = \frac{\rho X + (1-\rho)(1-\pi_E)\pi_M z}{\rho + (1-\rho)(1-\pi_E)}$ is increasing in both evaluation and monitoring. Hence as liquidity α increases, of course the price of a funded project increases as well.

Now suppose the manager has reached a point where it is not profitable any more to give up more shares to the financier in order to keep her away from selling bad shares in the market. At this point, with an incremental increase in the firm's liquidity α, he would suddenly decrease the number of shares to a new $s < s_0$, to let her dump her shares on the market.

Notice that this decrease in the offered shares s is associated by a decrease in the price p_0. But by proposition 2, we know $\frac{\partial \pi_E}{\partial p_0} > 0$, and hence the sudden decrease in the offered price p_0 is followed by a sharp decrease in evaluation π_E, which in turn significantly decreases the price of a funded project $P_F = \frac{\rho X}{\rho + (1-\rho)(1-\pi_E)}$. Of course the disappearance of the cash flow from liquidation of a potential bad project also adds to the decrease in the price of a funded project P_F. Hence, we get the following corollary to the proposition 4.

Corollary 3 *If the manager stops selling more shares to the financier just to keep her liquidating, an increase in liquidity α makes an abrupt decrease in the price of a funded project P_F.*

Table 1: Result of increases in liquidity α, for $\lambda = 1.8$, $z = 0.35$, $e = 1.73$, $m = 0.5$, $\rho = 0.2$, $\rho X = 1.5$:

α	P_F	$\alpha\lambda$	s_0	s	p_0	π_E	π_M	V^m
0.114	1.890	0.205	0.586	0.618	1.037	0.235	0.163	0.617
0.118	1.890	0.212	0.607	0.618	1.037	0.235	0.163	0.617
0.122	1.898	0.220	0.627	0.627	1.056	0.239	0.165	0.616
0.142	1.989	0.256	0.730	0.730	1.256	0.283	0.190	0.612
0.162	2.092	0.292	0.833	0.833	1.466	0.330	0.213	0.599
0.182	2.209	0.328	0.936	0.936	1.686	0.378	0.234	0.577
0.186	2.236	0.335	0.957	0.957	1.732	0.389	0.238	0.571
0.190	1.863	0.342	0.977	0.646	1.109	0.244	0.257	0.566
0.194	1.865	0.349	0.998	0.648	1.114	0.245	0.262	0.564

Table 1 presents a simulation of the model as the liquidity α increases. We explain how the results are in line with propositions 2, 4.

Notice that in the beginning, for the first two rows the financier prefers to liquidate $s > s_0$, because low liquidity of the project α makes selling unattractive. As a result, the increase in liquidity α from the first row to the second row has no effect on any variable. However, as the liquidity increases the financier gets attracted to selling. But selling is hurtful for the manager, both because it returns no value to current shareholders, and because it hurts the future shareholders. As a result, after the third row, the manager starts giving more shares to the financier to prevent her from selling. But as explained above in detail, this makes the financier waste resources on over-evaluation. This becomes intolerable when the liquidity surpasses $\alpha = 0.190$ in the eighth row, and the manager stops wooing the financier to liquidate bad projects. Thus, from the eighth row till the end the financier continues to sell shares of a bad project in the market.

Notice that from $\alpha = 0.186$ to $\alpha = 0.190$, there is a sudden decrease in evaluation, which is followed by a sudden increase in monitoring because they are substitute when the financier sells. Also, as predicted by proposition 4, the price of a funded project P_F falls instantly.

Another point to note is that as we predicted, whenever the financier does not liquidate, monitoring effort increases with an increase in liquidity. Also, after the first two rows where the financier liquidates, an increase in liquidity decreases the manager's value V^m. The reason is that from the third row to the seventh row, the increase in over-evaluation is dissipating so much value that can't be replenished by the decrease in under-monitoring. This is not surprising because $s = s_0$ itself indicates that over-evaluation is playing the dominant role.

From the seventh row to the eighth row, the manager has to forgo the better technology of liquidation (by the financier) for the inferior technology of selling (by the financier). And from the eighth row to the end, the whole (increasing) monitoring effort is a waste, and over-evaluation is increasing as well.

In the next section we explore the possibility of having a manager who does not care about the terminal shareholders. We show that it imposes some changes to the results presented earlier.

3.4 Role of the Terminal Shareholders

In this section we analyze the case where the manager only cares about the current shareholders. Then, the manager does not internalize the loss of the shareholders that buy the financier's junk shares in the future, and we can rewrite the manager's problem as

$$\max_{s,p_0} \Big(\rho + (1-\rho)(1-\pi_E)\Big)(p_0 - 1) + (1-s)\Big(\rho X + (1-\rho)(1-\pi_E)\pi_M \mathbf{I}(s \geq s_0) z\Big).$$
$$\text{s.t. the financier invests, and his FOCs are satisfied.} \tag{20}$$

It is immediate that when the financier is not selling, the change in the manager's target function (that is due to selling to terminal shareholders) should not have any effect on the outcomes. Indeed, the comparison between (20) and (12) for $s \geq s_0$ confirms that. We also want to emphasize that this change in the manager's target function will not keep him from extracting all the surplus from the financier, and hence the lemma 4 holds.

As a result, we only need to focus on the case where the financier sells. Recall that when the financier sells as a result of monitoring, not only the current shareholders don't get any value, but also it hurts the terminal shareholders. Hence, the manager wants to decrease the level of monitoring. However, the only control variable the manager possesses is the price p_0 (recall that the number of shares s is determined from p_0 and lemma 4). But from the proposition 2, by increasing the price p_0 the manager can make the financier substitute monitoring with evaluation. Also, at $p_0 = 1$ the financier is under-evaluating (because only $p_0 = 1$ and no monitoring would result in the socially optimal evaluation, but here the monitoring is positive, and it's substitute with evaluation). However, as the reader recalls from the lemma 6, the manager prefers to increase the price p_0 so that he partially alleviates the negative effects of monitoring at the cost of over-evaluation.

Notice that without caring about the terminal shareholders, selling, and hence monitoring is not as hurtful to the manager as it was before. As a result, we predict that the manager's incentive to replace over-evaluation with over-monitoring drops. Another important issue to notice is that the manager and the social planner's target functions are different now. The fact the manager does not internalize the loss of the terminal shareholders is equivalent to having a social planner that sees benefit in dumping shares on the market. This fictitious social planner would instead solve the problem

$$\max_{\pi_E,\pi_M} \ \rho X + (1-\rho)(1-\pi_E)\pi_M\alpha\lambda - \Big(\rho + (1-\rho)(1-\pi_E)\Big)(1+m\pi_M^2) - e\pi_E^2,$$

and hence, her FOCs are

$$\begin{cases}\pi_E = \left(\frac{1-\rho}{2e}\right)(1+m\pi_M^2 - \pi_M\alpha\lambda) \\ \pi_M = \frac{(1-\rho)(1-\pi_E)}{\rho+(1-\rho)(1-\pi_E)}\frac{\alpha\lambda}{2m}.\end{cases} \tag{21}$$

By comparing this FOCs and (10), it is immediate that the manager can achieve this 'socially' optimal responses from the financier via setting $p_0 = 1$. And then his optimal choice of share is to retain as much share as he can. This result that the manager's new optimal evaluation is lower than before is equivalent to saying that 'over-evaluation' plays an even stronger role when the manager does not care about the terminal shareholders.

The fact that now evaluation is playing an stronger role, also has consequences for the response of the system to an increase in liquidity α. The mechanism through which the increase of liquidity α affects the agents' responses is the following. Since the manager offers the financier the least number of shares s to neutralize her 'over-evaluation', as the liquidity α increases, the manager affords to give her even a smaller share. But when the price is fixed (at $p0 = 1$), evaluation and monitoring are substitute, and hence the increase in liquidity α leads to an increase in monitoring and a decrease in evaluation. However, monitoring has no effect on the price of a funded project ($P_F = \frac{\rho X}{\rho+(1-\rho)(1-\pi_E)}$ when the financier sells), but the decrease in evaluation translates into a decrease in the price of a funded project P_F, and this concludes all the system variables. The following proposition formally summarizes this discussion.

Proposition 5 *If the manager does not care about the terminal shareholders, we have the following results:*

(i) If the financier liquidates upon receiving a bad monitoring signal, the manager's value function is the same as before, and the previous results still hold.

(ii) If the financier sells upon receiving a bad monitoring signal, we have

1. $p_0 = 1$, *and the manager offers the least number of shares he can.*
2. $\frac{\partial \pi_E}{\partial \alpha} < 0$.
3. $\frac{\partial \pi_M}{\partial \alpha} > 0$.
4. $\frac{\partial P_F}{\partial \alpha} < 0$.

Proof. *See the appendix.*

Finally, it is worthwhile to summarize the comparison of the financier's responses with the (real) social planner's, in the two cases where the manager cares and cares not about the terminal shareholders.

Corollary 4 *Let π_E^* and π_M^* denote the social planner's choice of evaluation and monitoring, and π_E^c,π_M^c denote the financier's best response when the manager cares about all shareholders (initial and terminal). Also, let π_E^{nc} and π_M^{nc} denote the financier's best response when the manager does not care about the terminal shareholder. Then, if the financier liquidates upon a bad monitoring signal, nothing changes and the financier over-evaluates and under-monitors. But if the financier sells upon a bad monitoring signal, we have:*

(i) $\pi_E^{nc} < \pi_E^* < \pi_E^c$.

(ii) $\pi_M^*(=0) < \pi_M^c < \pi_M^{nc}$.

Proof. *See the appendix.*

This result is very intuitive, since we would expect that a manager that doesn't care about the future shareholders, wouldn't try to preclude an informed selling via evaluation. And the rest follows by the substitution of evaluation and monitoring.

4 Conclusion

This paper considers a financier who must make two qualitatively different types of information acquisition decisions: *evaluation* efforts made prior to a potential investment; and *monitoring* efforts of already–funded firms that impact upon the financier's decisions about whether to take an active position in the firm (e.g. replace management) and whether to change its financial stake. We characterize how the financier's information acquisition is affected both by his stake in the firm and by the liquidity of the after–market for his claims to the firm. We then derive the implications for the pricing of the firm, detailing why the market responds more favorably to less liquid forms of finance. Finally, we compare the financier's choices of evaluation and monitoring expenditures with the socially optimal levels and characterize the optimal level of after–market liquidity.

Less liquid after–markets make it more important for the financier to identify bad projects, causing the financier to make more thorough evaluation expenditures. Less liquid after–markets also reduce the value of trading on bad monitoring news, raising relatively the value of efficiently intervening, thereby revealing the bad news to the public, thus precluding the possibility of trading on it. As a consequence, share prices respond more favorably to the announcement of less liquid forms of finance.

However, we also show that the financier typically over–evaluates from a social perspective because he internalizes only a portion of the benefits of financing, but incurs all of the capital financing costs. As a consequence, reducing the level of liquidity in the after–market need not have welfare–enhancing effects on the financier's actions. In particular, if the financier would always seek to trade on adverse private information after a bad monitoring signal, then a more liquid after–market, although it inefficiently raises monitoring expenditures, also efficiently reduces evaluation expenditures toward the optimal level.

References

Almazan, A. and J. Suarez, 2003, “Managerial Compensation and the Market Reaction to Bank Loans”, *The Review of Financial Studies*, **16(1)**, 237–261.

Asquith, P., and D. Mullins, 1986, “Equity Issues and Offering Dilution”, *Journal of Financial Economics*, **15**, 61–89.

Bhide, A., 1993, “The Hidden Costs of Stock Market Liquidity”, *Journal of Financial Economics*, **34(1)**, 31–51.

Billet, M., J. Flannery, and J. Garfinkel, 1995, “The Effect of Lender Identity on a Borrowing Firm’s Equity Return”, *Journal of Finance*, **50**, 699–718.

Bolton, P., and E. von Thadden, 1998, “Banks, Liquidity, and Corporate Control”, *Journal of Finance*, **53(1)**, 1–25..

Eckbo, B.E., 1986, “Valuation Effects of Corporate Debt Offerings”, *Journal of Financial Economics*, **15**, 119–151.

Gompers, P. and J. Lerner, 1999, “The Venture Capital Cycle”, *MIT Press*, Cambridge, MA.

Hadlock, C., and C. James, 2000, “Bank Lending and the Menu of Financing Options, mimeo.

James, C., 1987, “Some Evidence on the Uniqueness of Bank Loans”, *Journal of Financial Economics*, **19**, 217–235.

Kahn, C. and A. Winton, 1998, “Ownership Structure, Speculation and Shareholder Intervention”, *Journal of Finance*, **53(1)**, 99–129.

Lummer, S., and J. McConnel, 1989, “Further Evidence on the Bank Lending Process and the Capital-Market-Response to Bank Loan Agreements”, *Journal of Financial Economics*, 99–122.

Masulis, R., and A. Korwar, “Seasoned Equity Offerings”, *Journal of Financial Economics*, **15**, 91–118.

Maug, E., 1998, “Large Shareholders as Monitors: Is there a Trade-off between Liquidity and Control”, *Journal of Finance*, **53(1)**, 65–98.

Mikkelson, W., and M. Partch, 1986, “Valuation Effects of Security Offerings and the Issuance Process”, *Journal of Financial Economics*, **15**, 31–60.

Preece, D., and D. Mullineaux, 1994, “Monitoring by Financial Intermediaries: Banks vs. Non-banks”, *Journal of Financial Services Research*, **8**, 193–202.

Slovin, M., S. Johnson, and J. Glascock, 1992, “Firm Size and the Informational Content of Bank Loan Announcements”, *Journal of Banking and Finance*, **16**, 1057–1071.

Smith, C. Jr., 1986, "Investment Banking and the Capital Acquisition Process", *Journal of Financial Economics*, **15**, 3–29.

5 Appendix: Proofs

Proof of the lemma 1: Recall that when there is no monitoring, the financier's problem can be written as

$$V^f = \max_{\pi_E} s\rho X - \big[\rho + (1-\rho)(1-\pi_E)\big]p_0 - e\pi_E^2,$$

which gives us

$$\pi_E = \frac{1-\rho}{2e}p_0. \tag{22}$$

Also, if the financier IR and IC hold, the manager's target function is

$$V^m(p_0, s) = \big[\rho + (1-\rho)(1-\pi_E)\big](p_0 - 1) + (1-s)\rho X.$$

Taking its derivative with respect to p_0 and imposing $\frac{\partial}{\partial p_0}V^m(p_0, s) \geq 0$ gives us

$$\begin{aligned}\big[\rho + (1-\rho)(1-\pi_E)\big] - (1-\rho)\frac{\partial \pi_E}{\partial p_0}(p_0 - 1) &= 1 - (1-\rho)\pi_E - (1-\rho)\frac{\partial \pi_E}{\partial p_0}(p_0 - 1)\\ &= 1 - \frac{(1-\rho)^2}{2e}p_0 - \frac{(1-\rho)^2}{2e}(p_0 - 1)\\ &= 1 - 2\frac{(1-\rho)^2}{2e}p_0 + \frac{(1-\rho)^2}{2e} \geq 0.\end{aligned}$$

As a result,

$$p_0 \leq \frac{1 + \frac{(1-\rho)^2}{2e}}{2\frac{(1-\rho)^2}{2e}}. \tag{23}$$

Now we check if it is possible for the manager to increase the price p_0 this much before the value of the financier V^f hits zero. Using the financier's best evaluation response, we check what prices p_0 satisfy her IR, *i.e.* $V^f \geq 0$:

$$\begin{aligned}s\rho X - \big[\rho + (1-\rho)(1-\pi_E)\big]p_0 - e\pi_E^2 &= s\rho X - \big[1 - \frac{(1-\rho)^2}{2e}p_0\big]p_0 - e\frac{(1-\rho)^2}{4e^2}p_0^2,\\ &= s\rho X - p_0 + \frac{(1-\rho)^2}{4e}p_0^2 \geq 0,\end{aligned}$$

and hence

$$p_0 \leq \frac{1 - \sqrt{1 - 2s\rho X\frac{(1-\rho)^2}{2e}}}{\frac{(1-\rho)^2}{2e}}. \tag{24}$$

So far, we have shown that the manager wants to increase the price to the RHS of (23), but he can't increase it above the RHS of (24). Thus, the manager makes the financier break even if and only if the RHS of (24) is less than the RHS of (23).

$$
\begin{aligned}
\frac{1-\sqrt{1-2s\rho X\frac{(1-\rho)^2}{2e}}}{\frac{(1-\rho)^2}{2e}} &\leq \frac{1+\frac{(1-\rho)^2}{2e}}{2\frac{(1-\rho)^2}{2e}}\\
2-2\sqrt{1-2s\rho X\frac{(1-\rho)^2}{2e}} &\leq 1+\frac{(1-\rho)^2}{2e}\\
1-\frac{(1-\rho)^2}{2e} &\leq 2\sqrt{1-2s\rho X\frac{(1-\rho)^2}{2e}}. \qquad (25)
\end{aligned}
$$

Since this must hold for any number of shares s, and the worst s for inequality is $s = 1$, we replace s with one. Thus, we should have

$$
1-\frac{(1-\rho)^2}{2e}-2\sqrt{1-2\rho X\frac{(1-\rho)^2}{2e}} \leq 0. \qquad (26)
$$

Also, notice that we can treat ρ and ρX as two different variables (recall that X appears only in ρX, which must be larger than one because the project's NPV is positive). But this allows us to take derivative of the LHS of (26) with respect to ρ (*i.e.* ρX kept unchanged), and show that it is always negative.

$$
\begin{aligned}
\frac{\partial}{\partial\rho}\Big(1-\frac{(1-\rho)^2}{2e}-2\sqrt{1-2\rho X\frac{(1-\rho)^2}{2e}}\Big) &= \frac{1-\rho}{e}-\frac{2\rho X\frac{1-\rho}{e}}{\sqrt{1-2\rho X\frac{(1-\rho)^2}{2e}}}\\
&= \frac{1-\rho}{e}\Big(1-\frac{2\rho X}{\sqrt{1-2\rho X\frac{(1-\rho)^2}{2e}}}\Big) < 0,
\end{aligned}
$$

because $\rho X \geq 1$. But this ensures that (26) holds if and only if it holds for $\rho \to 0$, which is

$$
1-\frac{1}{2e}-2\sqrt{1-2\rho X\frac{1}{2e}} \leq 0,
$$

which is the same as the condition stated in the lemma 1. ■

Proof of the lemma 3 The result follows from substituting p_0 from the financier's binding IR condition (*i.e.* $V^f = 0$) into the manager's problem, since by (4),

$$
\begin{aligned}
p_0 &= s\rho X+(1-\rho)\pi_M\max\{\alpha\lambda, sz\}-m\pi_M^2\\
&= s\rho X+(1-\rho)\pi_M\big(\mathbf{I}(s\geq s_0)sz+\mathbf{I}(s<s_0)\alpha\lambda\big)-m\pi_M^2,
\end{aligned}
$$

substituting which in (6), gives us

$$
\begin{aligned}
&(p_0-1)+(1-s)\big(\rho X+(1-\rho)\pi_M z\mathbf{I}(s\geq s_0)\big)-(1-\rho)\pi_M\alpha\lambda\mathbf{I}(s<s_0) =\\
&\quad s\rho X+(1-\rho)\pi_M\big(\mathbf{I}(s\geq s_0)sz+\mathbf{I}(s<s_0)\alpha\lambda\big)-m\pi_M^2\\
&\qquad -1+(1-s)\big(\rho X+(1-\rho)\pi_M z\mathbf{I}(s\geq s_0)\big)-(1-\rho)\pi_M\alpha\lambda\mathbf{I}(s<s_0) =\\
&\quad \rho X+(1-\rho)\pi_M zI(s\geq s_0)-(1+m\pi_M^2),
\end{aligned}
$$

which equals the social planner's target function. ■

Proof of the lemma 2: The fact that over-evaluation is increasing in s results from the fact that the socially optimal s is $\underline{s}$. Note that in the evaluation model $\pi_E = \frac{1-\rho}{2e}p_0$, and $p_0(s)$ comes from (24), hence

$$\begin{aligned}\pi_E &= \frac{1-\rho}{2e}p_0\\ &= \frac{(1-\rho)}{2e}\frac{1-\sqrt{1-2s\rho X\frac{(1-\rho)^2}{2e}}}{\frac{(1-\rho)^2}{2e}}\\ &= \frac{1-\sqrt{1-s\frac{\rho X}{e}(1-\rho)^2}}{1-\rho},\end{aligned}$$

which is increasing in s, and its derivative with respect to ρ (holding ρX fixed) is

$$-\frac{1-\sqrt{1-s\frac{\rho X}{e}(1-\rho)^2}}{(1-\rho)^2\sqrt{1-s\frac{\rho X}{e}(1-\rho)^2}} < 0,$$

showing that π_E is also decreasing in ρ. Moreover, note that π_E is increasing in $\frac{\rho X}{e}$, but from lemma 1 we know that

$$1-\frac{1}{2e} \leq 2\sqrt{1-\frac{\rho X}{e}},$$

and hence the highest $\frac{\rho X}{e}$ happens when e is the smallest, which happens when ρX is the smallest *i.e.* $\rho X = 1$ (see figure 1). Calculating the lowest e for $\rho X = 1$ yields $e = 1.07735$, which results in the highest π_E to be 0.7320. ■

Proof of the lemma 4: (i) From (12) the manager's target function is

$$\Big(\rho+(1-\rho)(1-\pi_E)\Big)(p_0-1)+(1-s)\Big(\rho X+(1-\rho)(1-\pi_E)\pi_M\mathbf{I}(s\geq s_0)z\Big)\\ -(1-\rho)(1-\pi_E)\pi_M\mathbf{I}(s<s_0)\alpha\lambda,$$

whose partial derivative with respect to p_0 (given s) is

$$\begin{aligned}&\big(\rho+(1-\rho)(1-\pi_E)\big)-(1-\rho)\frac{\partial\pi_E}{\partial p_0}(p_0-1)\\ &\quad-(1-\rho)\mathbf{I}(s\geq s_0)(1-s)z\Big(\pi_M\frac{\partial\pi_E}{\partial p_0}-(1-\pi_E)\frac{\partial\pi_M}{\partial p_0}\Big)\\ &\quad+(1-\rho)\mathbf{I}(s<s_0)\alpha\lambda\Big(\pi_M\frac{\partial\pi_E}{\partial p_0}-(1-\pi_E)\frac{\partial\pi_M}{\partial p_0}\Big).\end{aligned}\tag{27}$$

Using (10) and (11), we calculate $\frac{\partial\pi_E}{\partial p_0}$ and $\frac{\partial\pi_M}{\partial p_0}$, where π_E and π_M are the financier's choice of evaluation and monitoring. For simplicity we define $\phi := \max(sz, \alpha\lambda)$, which is now constant because s is fixed. Differentiating the financier's first order condition gives us

$$\begin{cases} \frac{\partial \pi_E}{\partial p_0} &= \frac{1-\rho}{2e}\big(1+(2m\pi_M-\phi)\frac{\partial \pi_M}{\partial p_0}\big) \\ \frac{\partial \pi_M}{\partial p_0} &= \frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial p_0}\frac{\phi}{2m}, \end{cases} \tag{28}$$

where $\psi = \psi(\pi_E) := \frac{(1-\rho)(1-\pi_E)}{\rho+(1-\rho)(1-\pi_E)}$. Simplifying the last parenthesis in (27) we have:

$$\begin{aligned} \pi_M\frac{\partial \pi_E}{\partial p_0} - (1-\pi_E)\frac{\partial \pi_M}{\partial p_0} &= \psi\frac{\phi}{2m}\frac{\partial \pi_E}{\partial p_0} - (1-\pi_E)\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial p_0}\frac{\phi}{2m} \\ &= \frac{\partial \pi_E}{\partial p_0}\Big(\psi\frac{\phi}{2m} - (1-\pi_E)\frac{\partial \psi}{\partial \pi_E}\frac{\phi}{2m}\Big) \\ &= \frac{\partial \pi_E}{\partial p_0}\Big(\psi\frac{\phi}{2m} - (1-\pi_E)\frac{-\rho(1-\rho)}{\big(\rho+(1-\rho)(1-\pi_E)\big)^2}\frac{\phi}{2m}\Big) \\ &= \frac{\partial \pi_E}{\partial p_0}\Big(1+\frac{\rho}{\rho+(1-\rho)(1-\pi_E)}\Big)\psi\frac{\phi}{2m} \\ &= \frac{\partial \pi_E}{\partial p_0}\big(1+1-\psi\big)\psi\frac{\phi}{2m} \\ &= \frac{\partial \pi_E}{\partial p_0}\big(2-\psi\big)\pi_M, \end{aligned}$$

where throughout we only used the definitions of ψ and π_M. As a result, we can rewrite the condition that (27) is non-negative if and only if

$$\rho+(1-\rho)(1-\pi_E) \geq (1-\rho)\frac{\partial \pi_E}{\partial p_0}\Big(p_0 - 1 + \big(\mathbf{I}(s\geq s_0)(1-s)z - \mathbf{I}(s<s_0)\alpha\lambda\big)\pi_M(2-\psi)\Big).$$

Notice that for e large, the LHS converges to one, however, we show that the RHS converges to zero since $\frac{\partial \pi_E}{\partial p_0}$ converges to zero, and the parenthesis is always less than p_0+1. Notice that for large e, p_0 will be less than $\rho X+1$, because monitoring can only recover part of the one unit initial investment.

In order to derive $\frac{\partial \pi_E}{\partial p_0}$, we substitute $\frac{\partial \pi_M}{\partial p_0}$ from the second equation of (28) into the first equation to get

$$\frac{\partial \pi_E}{\partial p_0} = \frac{1-\rho}{2e}\Big(1+(2m\pi_M-\phi)\frac{\phi}{2m}\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial p_0}\Big),$$

and hence,

$$\frac{\partial \pi_E}{\partial p_0} = \frac{\frac{1-\rho}{2e}}{1-\frac{1-\rho}{2e}(\frac{2m}{\phi}\pi_M - 1)\frac{\partial \psi}{\partial \pi_E}\frac{\phi^2}{2m}}.$$

But from (10) or (11), we know that $\frac{2m}{\phi}\pi_M = \psi$, and thus we have

$$\begin{cases} \frac{\partial \pi_E}{\partial p_0} &= \frac{\frac{1-\rho}{2e}}{1+\frac{1-\rho}{2e}(1-\psi)\frac{\partial \psi}{\partial \pi_E}\frac{\phi^2}{2m}} \\ \frac{\partial \pi_M}{\partial p_0} &= \frac{\phi}{2m}\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial p_0}. \end{cases} \tag{29}$$

Later on, we show that $\frac{\partial \pi_E}{\partial p_0}$ is positive, but for now it's enough to show for e large, it goes to zero. Note that as e grows, π_E vanishes, and hence ψ goes to $1-\rho$, and $\frac{\partial \psi}{\partial p_0}$ goes to $-\rho(1-\rho)$. Also, $\frac{\phi^2}{2m}$ is always less than one, and as a result, $\frac{\partial \pi_E}{\partial p_0}$ is less than $\frac{1-\rho}{2e-\rho^2(1-\rho)^2}$, which of course goes to zero as e gets large. This proves that for e larger than a critical value the above inequality always hold, proving part (i). Although this result is only about the existence, simulation shows that for $m = 0.5$, this critical e is just slightly above the critical e for $m = \infty$ that we derived in lemma 1.
(ii) For the second part, it's enough to prove that if the manager gives the financier more share s of the firm, he charges her a (strictly) higher price.
From part one, given s, the manager increases p_0 to the point where the financier breaks even. Recall that the financier's value V^f is

$$V^f = \max_{\pi_E,\pi_M} s\rho X + (1-\rho)(1-\pi_E)\pi_M \max\{sz, \alpha\lambda\} - \big(\rho + (1-\rho)(1-\pi_E)\big)(p_0 + m\pi_M^2) - e\pi_E^2.$$

Thus, part (i) implies $V^f = 0$, which in turn enables us to write

$$\frac{\partial p_0}{\partial s} = -\frac{\frac{\partial V^f}{\partial s}}{\frac{\partial V^f}{\partial p_0}}.$$

But we can use the envelope theorem to calculate each partial in the RHS, without having to deal with the confounding effects $\frac{\partial \pi_E}{\partial s}$, $\frac{\partial \pi_E}{\partial p_0}$, $\frac{\partial \pi_M}{\partial s}$, and $\frac{\partial \pi_M}{\partial p_0}$. This gives us

$$\frac{\partial p_0}{\partial s} = \frac{\rho X + (1-\rho)(1-\pi_E)\pi_M \mathbf{I}(s \geq s_0) z}{\rho + (1-\rho)(1-\pi_E)}, \tag{30}$$

which is obviously positive. This shows that in the space of (s, p_0), the manager's best p_0 at each s is strictly increasing. Since the points from which the manager can choose are below this diagram (see figure 2, left), at each p_0 the manager's best s is the lowest s that the manager can offer. ■

Proof of the corollary 2: $\frac{\partial p_0}{\partial s}$ is derived in the proof of the lemma 4, equation (30). ■

Proof of the proposition 1: (i) Trivial when $(s > s_0)$.
(ii) Suppose s is fixed. Recall that $\frac{\partial \pi_E}{\partial p_0}$ and $\frac{\partial \pi_M}{\partial p_0}$ are derived in (29). In order to prove $\frac{\partial \pi_E}{\partial p_0} > 0$, first we show that the highest possible evaluation π_E in our model is less than 0.74, and the use this fact to show that the negative term in the denominator of $\frac{\partial \pi_E}{\partial p_0}$ is bounded below one. Recall from lemma 2 that it is true for $m = \infty$. Moreover, this occurred when $s = 1$, $\rho X = 1$, and e was the lowest possible e associated to $\rho X = 1$ by lemma 1 such that the manager leaves no surplus for the financier. At this point, we claim that when m decreases from ∞, at a fixed e (and s) monitoring increases (becomes positive) and evaluation decreases. Notice that if we fix both s and p_0, then this substitution between evaluation and monitoring is trivial. However, when m decreases, the financier has access

to a better monitoring technology, which gives her a higher profit margin, but this enables the manager to charge her more, and when p_0 increases, we expect to see higher evaluation and lower monitoring. In the terminology that we use in this paper, the direct effect of an increase in m is positive on π_E, but the indirect effect is negative. Here we show that the direct effect dominates.

Recall that the financier's FOC's are

$$\begin{cases} \pi_E = \left(\frac{1-\rho}{2e}\right)(p_0 + m\pi_M^2 - \pi_M\phi) \\ \pi_M = \frac{(1-\rho)(1-\pi_E)}{\rho+(1-\rho)(1-\pi_E)}\frac{\phi}{2m}, \end{cases} \tag{31}$$

where $\phi = \max\{sz, \alpha\lambda\}$. Also, if the manager leaves no surplus for the financier we have

$$s\rho X + (1-\rho)(1-\pi_E)\pi_M\phi - \big(\rho + (1-\rho)(1-\pi_E)\big)(p_0 + m\pi_M^2) - e\pi_E^2 = 0, \tag{32}$$

and hence

$$p_0 = \frac{s\rho X + (1-\rho)(1-\pi_E)\pi_M\phi - e\pi_E^2}{\rho + (1-\rho)(1-\pi_E)} - m\pi_M^2,$$

replacing which in the financier's FOC for π_E gives us

$$\begin{aligned} \pi_E &= \big(\frac{1-\rho}{2e}\big)\Big(\frac{s\rho X + (1-\rho)(1-\pi_E)\pi_M\phi - e\pi_E^2}{\rho + (1-\rho)(1-\pi_E)} - \pi_M\phi\Big) \\ &= \big(\frac{1-\rho}{2e}\big)\frac{s\rho X - e\pi_E^2 - \rho\pi_M\phi}{\rho + (1-\rho)(1-\pi_E)}, \end{aligned}$$

and after simplification we get

$$\pi_E = \big(\frac{1-\rho}{2e}\big)(s\rho X + e\pi_E^2 - \rho\pi_M\phi). \tag{33}$$

Thus, at a fixed s if take derivative with respect to m we have

$$\begin{cases} \frac{\partial\pi_E}{\partial m} = \big(\frac{1-\rho}{2e}\big)(2e\pi_E\frac{\partial\pi_E}{\partial m} - \rho\phi\frac{\partial\pi_M}{\partial m}) \\ \frac{\partial\pi_M}{\partial m} = -\psi\frac{\phi}{2m^2} + \frac{\phi}{2m}\frac{\partial\psi}{\partial\pi_E}\frac{\partial\pi_E}{\partial m}. \end{cases}$$

Now replacing $\frac{\partial\pi_M}{\partial m}$ from the second equation above in the first one gives us

$$\frac{\partial\pi_E}{\partial m} = \big(\frac{1-\rho}{2e}\big)\Big(2e\pi_E\frac{\partial\pi_E}{\partial m} - \rho\phi\big(-\psi\frac{\phi}{2m^2} + \frac{\phi}{2m}\frac{\partial\psi}{\partial\pi_E}\frac{\partial\pi_E}{\partial m}\big)\Big),$$

now taking the first sentence of the RHS to the LHS we have

$$\begin{aligned} \big(1-(1-\rho)\pi_E\big)\frac{\partial\pi_E}{\partial m} &= \big(\frac{1-\rho}{2e}\big)\Big(-\rho\phi\big(-\psi\frac{\phi}{2m^2} + \frac{\phi}{2m}\frac{\partial\psi}{\partial\pi_E}\frac{\partial\pi_E}{\partial m}\big)\Big) \\ \frac{\partial\pi_E}{\partial m} &= \frac{1-\rho}{2e}(-\phi)\frac{\rho}{\rho + (1-\rho)(1-\pi_E)}\big(-\psi\frac{\phi}{2m^2} + \frac{\phi}{2m}\frac{\partial\psi}{\partial\pi_E}\frac{\partial\pi_E}{\partial m}\big) \\ \frac{\partial\pi_E}{\partial m} &= \frac{1-\rho}{2e}(-\phi)(1-\psi)\big(-\psi\frac{\phi}{2m^2} + \frac{\phi}{2m}\frac{\partial\psi}{\partial\pi_E}\frac{\partial\pi_E}{\partial m}\big) \\ \frac{\partial\pi_E}{\partial m} &= \frac{\frac{1-\rho}{2e}\psi(1-\psi)\frac{\phi^2}{2m^2}}{1 + \frac{1-\rho}{2e}\frac{\phi^2}{2m}(1-\psi)\frac{\partial\psi}{\partial\pi_E}}. \end{aligned} \tag{34}$$

Next, recall that when $m = \infty$, by lemma 2 we have $\pi_E < 0.74$. But the function $(1-\psi)\frac{\partial \psi}{\partial \pi_E} = \frac{-\rho^2(1-\rho)}{[\rho+(1-\rho)(1-\pi_E)]^2}$ in the denominator takes the minimum of (almost) -0.57 on the rectangle $\{(\rho, \pi_E) : 0 \le \rho \le 1, 0 \le \pi_E \le 0.74\}$, which is more than enough for the denominator to be positive, since $\frac{1-\rho}{2e} = \pi_E^* < 1$, $m \ge 0.5$, and $\phi = \max\{\alpha\lambda, sz\} \le 1$.

Thus, at $m = \infty$, we start with a positive $\frac{\partial \pi_E}{\partial m}$, and as m gets smallest than ∞, π_E gets smaller (and hence remains smaller than 0.74), which guarantees the $\frac{\partial \pi_E}{\partial m}$ to remain positive for any $m \ge 0.5$. But more importantly, when m moves down from ∞ to 0.5, π_E starts to get smaller, and hence always remains less than 0.74 for any combination of the system parameters, which keeps the denominator positive all the time. This fact will be used several times in the future proofs.

Finally, notice that the denominator in (29) is the same as the denominator in $\frac{\partial \pi_E}{\partial m}$ above, and because $\pi_E < 0.74$ all the time, it is positive, proving that $\frac{\partial \pi_E}{\partial p_0} > 0$, which readily (see (29 again)) proves that $\frac{\partial \pi_M}{\partial p_0} < 0$, because $\frac{\partial \psi}{\partial \pi_E} < 0$.

Next, we show that when s is fixed, liquidity has a positive effect on monitoring and a negative effect on evaluation. Recall that we claimed in the paper that we have

$$\overset{-}{\frac{\partial P_F}{\partial \alpha}} = \overset{+}{\frac{\partial P_F}{\partial \pi_E}} \Big(\overset{-}{\frac{\partial \pi_E}{\partial \alpha}} + \overset{+}{\frac{\partial \pi_E}{\partial p_0}} \overset{+}{\frac{\partial p_0}{\partial \alpha}} \Big).$$

Here, we show that the sign of each partial is indeed as it has been claimed. From (32) we deduce that

$$\frac{\partial p_0}{\partial \alpha} = -\frac{\frac{\partial V^f}{\partial \alpha}}{\frac{\partial V^f}{\partial p_0}} = \frac{(1-\rho)(1-\pi_E)\pi_M\lambda}{\rho + (1-\rho)(1-\pi_E)} > 0,$$

where in the last equality we used the envelope theorem. Also, it has been shown above that $\frac{\partial \pi_E}{\partial p_0} > 0$. The fact that $\frac{\partial P_F}{\partial \pi_E} > 0$ is trivial (see (16)). In order to show $\frac{\partial \pi_E}{\partial \alpha} < 0$, we need to use the financier's FOC's assuming (s, p_0) are fixed. Recall that the financier's FOC's are

$$\begin{cases} \pi_E = \left(\frac{1-\rho}{2e}\right)(p_0 + m\pi_M^2 - \pi_M\phi) \\ \pi_M = \psi\frac{\phi}{2m}, \end{cases}$$

Also, notice that we can take the partial derivative with respect to ϕ, since the only way that α enters the financier's FOC's is through ϕ. Taking derivative of the financier's FOC's gives us

$$\begin{cases} \frac{\partial \pi_E}{\partial \phi} = \left(\frac{1-\rho}{2e}\right)\left((2m\pi_M - \phi)\frac{\partial \pi_M}{\partial \phi} - \pi_M\right)\right) \\ \frac{\partial \pi_M}{\partial \phi} = \frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \phi}\frac{\phi}{2m} + \psi\frac{1}{2m}. \end{cases}$$

Replacing $\frac{\partial \pi_M}{\partial \phi}$ from the second equation into the first one gives us

$$\begin{aligned}
\frac{\partial \pi_E}{\partial \phi} &= \Big(\frac{1-\rho}{2e}\Big)\big((2m\pi_M - \phi)(\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \phi}\frac{\phi}{2m} + \psi\frac{1}{2m}) - \pi_M\big) \\
&= \Big(\frac{1-\rho}{2e}\Big)\big(\phi(\psi - 1)(\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \phi}\frac{\phi}{2m} + \psi\frac{1}{2m}) - \pi_M\big) \\
&= \Big(\frac{1-\rho}{2e}\Big)\big(-(1-\psi)(\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \phi}\frac{\phi^2}{2m} + \psi\frac{\phi}{2m}) - \pi_M\big) \\
&= \Big(\frac{1-\rho}{2e}\Big)\big(-(1-\psi)(\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \phi}\frac{\phi^2}{2m} + \pi_M) - \pi_M\big) \\
&= \Big(\frac{1-\rho}{2e}\Big)\big(-(1-\psi)\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \phi}\frac{\phi^2}{2m} - (2-\psi)\pi_M\big) \\
&= \frac{-\frac{1-\rho}{2e}(2-\psi)\pi_M}{1 + \frac{1-\rho}{2e}\frac{\phi^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}} < 0,
\end{aligned}$$

since the numerator is negative (because $\psi < 1$), and as we showed above the denominator is positive. We can also readily show that $\frac{\partial \pi_M}{\partial \phi} > 0$, since $\frac{\partial \pi_M}{\partial \phi} = \frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \phi}\frac{\phi}{2m} + \psi\frac{1}{2m}$ and both $\frac{\partial \psi}{\partial \pi_E}$ and $\frac{\partial \pi_E}{\partial \phi}$ are negative. So as we claimed in the paper, when we abstract from the indirect effects, monitoring and evaluation are substitute. Finally, we show that when s is fixed, the total effect liquidity on evaluation (considering both the direct effect and the indirect effect) is negative. In order to show that $\frac{\partial \pi_E}{\partial \phi} < 0$ when s is fixed, we let the manager increase p_0 as α increases, so that he makes the financier break even. Then, as we showed above, replacing p_0 from the $V^f = 0$ condition into the financier's FOC's give us

$$\begin{cases} \pi_E = \big(\frac{1-\rho}{2e}\big)(s\rho X + e\pi_E^2 - \rho\pi_M\phi) \\ \pi_M = \psi\frac{\phi}{2m}, \end{cases}$$

and hence

$$\pi_E = (\frac{1-\rho}{2e})(s\rho X + e\pi_E^2 - \rho\psi\frac{\phi^2}{2m}),$$

which is all in terms of π_E. Taking derivative of this expression with respect to ϕ gives us

$$\begin{aligned}
\frac{\partial \pi_E}{\partial \phi} &= (\frac{1-\rho}{2e})\big(2e\pi_E\frac{\partial \pi_E}{\partial \phi} - \rho(\frac{\phi^2}{2m}\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \phi} + 2\psi\frac{\phi}{2m})\big) \\
\big(1 - (1-\rho)\pi_E\big)\frac{\partial \pi_E}{\partial \phi} &= (\frac{1-\rho}{2e})\big(-\rho(\frac{\phi^2}{2m}\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \phi} + 2\psi\frac{\phi}{2m})\big) \\
\frac{\partial \pi_E}{\partial \phi} &= (\frac{1-\rho}{2e})\big(-(1-\psi)(\frac{\phi^2}{2m}\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \phi} + 2\psi\frac{\phi}{2m})\big) \\
&= \frac{-\frac{1-\rho}{2e}2\psi(1-\psi)\frac{\phi}{2m}}{1 + \frac{1-\rho}{2e}\frac{\phi^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}} < 0.
\end{aligned}$$

Notice that it clearly implies when s is fixed and the financier changes p_0 to extract all the surplus from the financier we also have $\frac{\partial \pi_M}{\partial \phi} > 0$, as suggested by the lemma, since

$\frac{\partial \pi_M}{\partial \phi} = \frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \phi}\frac{\phi}{2m} + \psi\frac{1}{2m}$, and $\frac{\partial \pi_E}{\partial \phi}$ and $\frac{\partial \psi}{\partial \pi_E}$ are both negative. Finally, when π_E decreases, from (16) we know that P_F decreases as well.

Now suppose p_0 is fixed and the manager can only set s. From (31), it is clear that s does not inter into the financier's choice of evaluation and monitoring when she sells. Moreover, previously in the proof we showed that when p_0 is fixed (no indirect effect), evaluation and monitoring are substitutes, and when ϕ (which equals $\alpha\lambda$ when the financier sells) increases (*e.g.* because α has increased) monitoring increases and evaluation decreases. Hence, when p_0 is fixed, an increases in α decreases π_E which in turn decreases P_F.

Finally, if $s = s_0$, lemma 4 shows that an increase in liquidity makes the financier start selling when either p_0 or s is fixed. As a result, for s fixed, p_0 increases, and for P_0 fixed, s decreases. In either case, the exact reasoning that we used above works, which completes the proof. ∎

Proof of the lemma 5: If the manager leaves no surplus for the financier, from (9) we have

$$s\rho X + (1-\rho)(1-\pi_E)\pi_M\big(\mathbf{I}(s<s_0)\alpha\lambda + \mathbf{I}(s \geq s_0)sz\big) - \big[\rho + (1-\rho)(1-\pi_E)\big](p_0 + m\pi_M^2) - e\pi_E^2 = 0,$$

which we can solve for the manager's optimal number of shares s in terms of p_0:

$$s = \frac{\big[\rho + (1-\rho)(1-\pi_E)\big](p_0 + m\pi_M^2) + e\pi_E^2 - (1-\rho)(1-\pi_E)\pi_M\big(\mathbf{I}(s<s_0)\alpha\lambda + \mathbf{I}(s\geq s_0)sz\big)}{\rho X}.$$

Substituting this into the manager's value function in (12) gives us exactly the value function stated in the lemma (5). Finally, since we have replaced s using the constraint that the manager leaves no surplus for the financier, the only remaining variable of choice for the manager is p_0. ∎

Proof of the lemma 6: When the financier sells, using the lemma 5 we can write the manager's problem as

$$\max_{p_0} \; \rho X - \big[\rho + (1-\rho)(1-\pi_E)\big](1 + m\pi_M^2) - e\pi_E^2.$$

Note that given p_0 (which determines s as well), the financier's evaluation and monitoring are both determined by her FOCs in (10). In order to do the above maximization, we need to know $\frac{\partial \pi_E}{\partial p_0}$ and $\frac{\partial \pi_M}{\partial p_0}$. From (10) we know that $\frac{\partial \pi_M}{\partial p_0} = \frac{\alpha\lambda}{2m}\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial p_0}$, where $\psi = \psi(\pi_E) = \frac{(1-\rho)(1-\pi_E)}{\rho+(1-\rho)(1-\pi_E)}$. Then, the first order condition of the manager's value function gives us

$$(1-\rho)\frac{\partial \pi_E}{\partial p_0}(1 + m\pi_M^2) - \big[\rho + (1-\rho)(1-\pi_E)\big]2m\pi_M\frac{\alpha\lambda}{2m}\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial p_0} - 2e\pi_E\frac{\partial \pi_E}{\partial p_0} = 0,$$

and hence, dividing by $\frac{\partial \pi_E}{\partial p_0}$ we get

$$(1-\rho)(1 + m\pi_M^2) - \big[\rho + (1-\rho)(1-\pi_E)\big]\pi_M\alpha\lambda\frac{\partial \psi}{\partial \pi_E} - 2e\pi_E = 0.$$

However, from (10) we know that $2e\pi_E = (1-\rho)(p_0 + m\pi_M^2 - \pi_M\alpha\lambda)$. Hence, we have

$$(1-\rho)\big[1 - p_0 + \pi_M\alpha\lambda\big] - \big[\rho + (1-\rho)(1-\pi_E)\big]\pi_M\alpha\lambda\frac{\partial\psi}{\partial\pi_E} = 0.$$

Now if we substitute for $\frac{\partial\psi}{\partial\pi_E}$, we get

$$(1-\rho)\big[1 - p_0 + \pi_M\alpha\lambda\big] + \pi_M\alpha\lambda\frac{\rho(1-\rho)}{\rho + (1-\rho)(1-\pi_E)} = 0.$$

Cancelling $(1-\rho)$, and taking p_0 to the other side gives us the desired result. ∎

Proof of the proposition 2: (i) When the financier sells, s does not enter into her FOCs in (10). Thus, we can borrow our calculations from lemma 4 part (i) with $\phi = \alpha\lambda$ to reach at (29), and then in a similar way to the proof of the proposition 1 we conclude result of the first two parts of (i).

For the third part, we first show that $\frac{\partial\pi_E}{\partial\alpha} > 0$, which immediately proves that $\frac{\partial P_F}{\partial\alpha} > 0$, because when the financier sells, from (16), P_F is increasing in π_E. Then we conclude about $\frac{\partial\pi_M}{\partial\alpha}$.

Let us write π_E in (10) in terms of ψ:

$$\begin{aligned}
\pi_E &= \Big(\frac{1-\rho}{2e}\Big)\Big[p_0 + m\pi_M^2 - \pi_M\alpha\lambda\Big] \\
&= \Big(\frac{1-\rho}{2e}\Big)\Big[p_0 + m\psi^2\frac{(\alpha\lambda)^2}{4m^2} - \psi\frac{\alpha\lambda}{2m}\alpha\lambda\Big] \\
&= \Big(\frac{1-\rho}{2e}\Big)\Big[p_0 + \frac{(\alpha\lambda)^2}{4m}(\psi^2 - 2\psi)\Big].
\end{aligned}$$

Also, representing p_0 in (19) in terms of ψ gives us

$$p_0 = 1 + \frac{(\alpha\lambda)^2}{2m}(2\psi - \psi^2).$$

Substituting this into the expression of π_E gives us

$$\pi_E = \Big(\frac{1-\rho}{2e}\Big)\Big[1 + \frac{(\alpha\lambda)^2}{4m}(2\psi - \psi^2)\Big]. \tag{35}$$

Now taking the partial derivative of π_E with respect to α gives us

$$\frac{\partial\pi_E}{\partial\alpha} = \Big(\frac{1-\rho}{2e}\Big)\Big[2\frac{\alpha\lambda^2}{4m}\psi(2-\psi) + \frac{(\alpha\lambda)^2}{4m}(2-2\psi)\frac{\partial\psi}{\partial\pi_E}\frac{\partial\pi_E}{\partial\alpha}\Big].$$

Simplifying, we get

$$\frac{\partial\pi_E}{\partial\alpha}\Big[1 - \frac{(\alpha\lambda)^2}{2m}(1-\psi)\frac{\partial\psi}{\partial\pi_E}\Big] = \Big(\frac{1-\rho}{2e}\Big)\Big[\frac{\alpha\lambda^2}{2m}\psi(2-\psi)\Big] = (\frac{1-\rho}{2e})\pi_M\lambda(2-\psi).$$

Thus,

$$\frac{\partial \pi_E}{\partial \alpha} = \frac{\frac{1-\rho}{2e}\pi_M \lambda(2-\psi)}{1-\frac{1-\rho}{2e}\frac{(\alpha\lambda)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}}. \tag{36}$$

Because $\psi \in (0,1)$ and $\frac{\partial \psi}{\partial \pi_E} < 0$, both the numerator and denominator are positive, proving that $\frac{\partial \pi_E}{\partial \alpha} > 0$.

Next, recall that we have $\pi_M = \psi(\pi_E)\frac{\alpha\lambda}{2m}$, and hence

$$\begin{aligned}\frac{\partial \pi_M}{\partial \alpha} &= \frac{\alpha\lambda}{2m}\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \alpha} + \psi\frac{\lambda}{2m} \\ &= \frac{\lambda}{2m}\Big(\alpha\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \alpha} + \psi\Big).\end{aligned}$$

Substituting $\frac{\partial \pi_E}{\partial \alpha}$ from above yields:

$$\begin{aligned}\frac{\partial \pi_M}{\partial \alpha} &= \frac{\lambda}{2m}\Big(\frac{\frac{1-\rho}{2e}\pi_M \alpha\lambda(2-\psi)\frac{\partial \psi}{\partial \pi_E}}{1-\frac{1-\rho}{2e}\frac{(\alpha\lambda)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}} + \psi\Big) \\ &= \frac{\lambda}{2m}\Big(\frac{\frac{1-\rho}{2e}\pi_M \alpha\lambda(2-\psi)\frac{\partial \psi}{\partial \pi_E} + \psi - \psi\frac{1-\rho}{2e}\frac{(\alpha\lambda)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}}{1-\frac{1-\rho}{2e}\frac{(\alpha\lambda)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}}\Big) \\ &= \frac{\lambda}{2m}\Big(\frac{\frac{1-\rho}{2e}\pi_M \alpha\lambda(2-\psi)\frac{\partial \psi}{\partial \pi_E} + \psi - \frac{1-\rho}{2e}\pi_M\alpha\lambda(1-\psi)\frac{\partial \psi}{\partial \pi_E}}{1-\frac{1-\rho}{2e}\frac{(\alpha\lambda)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}}\Big) \\ &= \frac{\lambda}{2m}\Big(\frac{\psi + \frac{1-\rho}{2e}\pi_M \alpha\lambda\frac{\partial \psi}{\partial \pi_E}}{1-\frac{1-\rho}{2e}\frac{(\alpha\lambda)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}}\Big) \\ &= \frac{\lambda}{2m}\psi\Big(\frac{1 + \frac{1-\rho}{2e}\frac{(\alpha\lambda)^2}{2m}\lambda\frac{\partial \psi}{\partial \pi_E}}{1-\frac{1-\rho}{2e}\frac{(\alpha\lambda)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}}\Big).\end{aligned}$$

Notice that the denominator is positive, and so is the numerator because the minimum of function $\frac{\partial \psi}{\partial \pi_E}$ on the rectangle $\{(\rho, \pi_E) : 0 \le \rho \le 1, 0 \le \pi_E \le 0.74\}$ is -0.962, proving $\frac{\partial \pi_M}{\partial \alpha} > 0$ when the financier sells.

(ii) Notice that when the financier liquidates, α does not enter her FOCs in (11), neither does it enter into the price of a funded project P_F in (15). Hence, the last claim (that the partials of π_E, π_M and P_F with respect to α are zero) is trivial.

For $\frac{\partial \pi_E}{\partial p_0}$, we start with the financier's FOC for π_E, use $\pi_M = \psi\frac{sz}{2m}$ to get an equation solely in terms of π_E, and then take the partial derivative with respect to π_E:

$$\begin{aligned}\pi_E &= \Big(\frac{1-\rho}{2e}\Big)[p_0 + m\pi_M^2 - \pi_M s] \\ &= \Big(\frac{1-\rho}{2e}\Big)\big[p_0 + m\psi^2\frac{(sz)^2}{4m^2} - \psi\frac{(sz)^2}{2m}\big] \\ &= \Big(\frac{1-\rho}{2e}\Big)\big[p_0 + \frac{(sz)^2}{4m}(\psi^2 - 2\psi)\big].\end{aligned}$$

Taking the partial derivative with respect to p_0, and using lemma 2 and (15) to get $\frac{\partial s}{\partial p_0} = \frac{1}{P_F}$ yields us

$$\begin{aligned}\frac{\partial \pi_E}{\partial p_0} &= \Big(\frac{1-\rho}{2e}\Big)[1 + \frac{2sz^2}{4m}\frac{1}{P_F}(\psi^2 - 2\psi) + \frac{(sz)^2}{4m}(2\psi - 2)\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial p_0}] \\ &= \Big(\frac{1-\rho}{2e}\Big)[1 + z\frac{sz}{2m}\psi\frac{1}{P_F}(\psi - 2) + \frac{(sz)^2}{2m}(\psi - 1)\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial p_0}] \\ &= \Big(\frac{1-\rho}{2e}\Big)[1 + \frac{z\pi_M}{P_F}(\psi - 2) + \frac{(sz)^2}{2m}(\psi - 1)\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial p_0}].\end{aligned}$$

Solving for $\frac{\partial \pi_E}{\partial p_0}$ we have

$$\frac{\partial \pi_E}{\partial p_0} = \frac{\Big(\frac{1-\rho}{2e}\Big)\Big[1 - \frac{z\pi_M}{P_F}(2-\psi)\Big]}{1 + \Big(\frac{1-\rho}{2e}\Big)\frac{(sz)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}}. \tag{37}$$

Notice that the denominator is positive as we showed in the proof of the proposition 1. As for the numerator, it is positive if and only if $z\pi_M(2-\psi) < P_F$. Moreover, we have

$$P_F = \frac{\rho X + (1-\rho)(1-\pi_E)\pi_M z}{\rho + (1-\rho)(1-\pi_E)} = X(1-\psi) + z\pi_M\psi.$$

Hence, the numerator is positive if and only if

$$z\pi_M(2-\psi) < X(1-\psi) + z\pi_M\psi,$$

or equivalently if and only if,

$$2z\pi_M(1-\psi) < X(1-\psi) \;\Leftrightarrow\; 2z\pi_M < X \Leftrightarrow \frac{sz^2}{2m}\frac{2\rho(1-\rho)(1-\pi_E)}{\rho + (1-\rho)(1-\pi_E)} < \rho X.$$

But $\rho X \geq 1$, $m > 0.5$ by assumption, and $\frac{2\rho(1-\rho)(1-\pi_E)}{\rho+(1-\rho)(1-\pi_E)} \in [0, 0.5]$ for $\{(\rho, \pi_E) \in [0,1]^2\}$ which proves $\frac{\partial \pi_E}{\partial p_0} > 0$.

For $\frac{\partial \pi_M}{\partial p_0}$, we start from the financier's FOC for π_M and take the partial derivative with respect to p_0. Recall that $\pi_M = \psi\frac{sz}{2m}$, and also $\frac{\partial s}{\partial p_0} = \frac{1}{P_F}$.

$$\begin{aligned}\frac{\partial \pi_M}{\partial p_0} &= \frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial p_0}\frac{sz}{2m} + \psi\frac{z}{2m}\frac{\partial s}{\partial p_0} \\ &= \frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial p_0}\frac{sz}{2m} + \psi\frac{sz}{2m}\frac{1}{sP_F} \\ &= \Big[\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial p_0} + \frac{\psi}{sP_F}\Big]\frac{sz}{2m}.\end{aligned}$$

Now we replace $\frac{\partial \pi_E}{\partial p_0}$ from (37) and simplify to analyse the sign.

$$\begin{aligned}
\frac{\partial \pi_M}{\partial p_0} &= \left[\frac{\frac{\partial \psi}{\partial \pi_E}\left(\frac{1-\rho}{2e}\right)\left[1-\frac{z\pi_M}{P_F}(2-\psi)\right]}{1+\left(\frac{1-\rho}{2e}\right)\frac{(sz)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}} + \frac{\psi}{sP_F}\right]\frac{sz}{2m} \\
&= \left[\frac{\frac{\partial \psi}{\partial \pi_E}\left(\frac{1-\rho}{2e}\right)\left[1-\frac{z\pi_M}{P_F}(2-\psi)\right] + \frac{\psi}{sP_F} + \frac{\psi}{sP_F}\left(\frac{1-\rho}{2e}\right)\frac{(sz)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}}{1+\left(\frac{1-\rho}{2e}\right)\frac{(sz)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}}\right]\frac{sz}{2m} \\
&= \left[\frac{\frac{\partial \psi}{\partial \pi_E}\left(\frac{1-\rho}{2e}\right)\left[1-\frac{z\pi_M}{P_F}(2-\psi)\right] + \frac{\psi}{sP_F} + \frac{\partial \psi}{\partial \pi_E}\left(\frac{1-\rho}{2e}\right)\frac{z\pi_M}{P_F}(1-\psi)}{1+\left(\frac{1-\rho}{2e}\right)\frac{(sz)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}}\right]\frac{sz}{2m} \\
&= \left[\frac{\frac{\partial \psi}{\partial \pi_E}\left(\frac{1-\rho}{2e}\right)\left[1-\frac{z\pi_M}{P_F}\right] + \frac{\psi}{sP_F}}{1+\left(\frac{1-\rho}{2e}\right)\frac{(sz)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}}\right]\frac{sz}{2m} \\
&= \left[\frac{\frac{\partial \psi}{\partial \pi_E}\left(\frac{1-\rho}{2e}\right)\left[sP_F-\pi_M sz\right] + \psi}{1+\left(\frac{1-\rho}{2e}\right)\frac{(sz)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}}\right]\frac{1}{sP_F}\frac{sz}{2m},
\end{aligned} \tag{38}$$

whose sign depends on the numerator of the ratio within the big bracket, which contains two terms. The first term is negative (because $\frac{\partial \psi}{\partial \pi_E} < 0$), and the second term is of course positive. We can continue our simplification using lemma 4 ($V^f = 0$):

$$s[\rho X + (1-\rho)(1-\pi_E)\pi_M z] - \big(\rho + (1-\rho)(1-\pi_E)\big)(p_0 + m\pi_M^2) - e\pi_E^2 = 0.$$

Dividing by $\big(\rho + (1-\rho)(1-\pi_E)\big)$, and using (15) we get

$$\begin{aligned}
sP_F &= p_0 + m\pi_M^2 + \frac{e\pi_E^2}{\rho + (1-\rho)(1-\pi_E)} \\
sP_F - \pi_M sz &= p_0 + m\pi_M^2 - \pi_M sz + \frac{e\pi_E^2}{\rho + (1-\rho)(1-\pi_E)} \\
\left(\frac{1-\rho}{2e}\right)(sP_F - \pi_M sz) &= \left(\frac{1-\rho}{2e}\right)(p_0 + m\pi_M^2 - \pi_M sz) + \frac{(\frac{1-\rho}{2e})e\pi_E^2}{\rho + (1-\rho)(1-\pi_E)} \\
\left(\frac{1-\rho}{2e}\right)(sP_F - \pi_M sz) &= \pi_E + \frac{(1-\rho)\pi_E^2}{2\big(\rho + (1-\rho)(1-\pi_E)\big)},
\end{aligned}$$

where in the last equation we used the financier's FOC in (11). If we replace $(\frac{1-\rho}{2e})(sP_F - \pi_M sz)$ with the above value, there will remain only ρ and π_E in the bracket's numerator

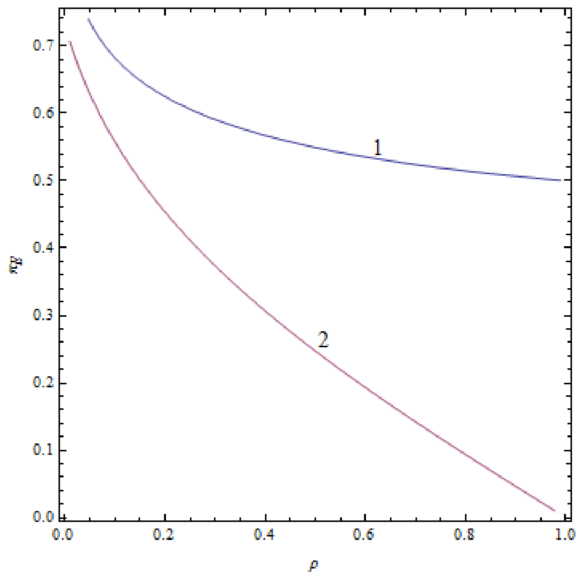

Figure 6: Below the curve number one lie the pairs (ρ, π_E) that satisfy the inequality (39). Points below the curve number two are the possible points that we can have in our model, *i.e.* they satisfy the inequality (40). As can be seen, the condition (39) is always satisfied in the model.

(since ψ also depends only on ρ and π_E). The numerator being positive is equivalent to

$$\begin{aligned} 0 &\leq \frac{\partial \psi}{\partial \pi_E}\left[\pi_E + \frac{(1-\rho)\pi_E^2}{2\big(\rho + (1-\rho)(1-\pi_E)\big)}\right] + \psi \\ -1 &\leq \frac{1}{\psi}\frac{\partial \psi}{\partial \pi_E}\left[\pi_E + \frac{(1-\rho)\pi_E^2}{2\big(\rho + (1-\rho)(1-\pi_E)\big)}\right] \\ 1 &\geq \frac{\rho}{(1-\pi_E)\big(\rho + (1-\rho)(1-\pi_E)\big)}\left[\pi_E + \frac{(1-\rho)\pi_E^2}{2\big(\rho + (1-\rho)(1-\pi_E)\big)}\right]. \end{aligned} \tag{39}$$

But recall from the proof of the proposition 1 that the largest π_E occurs when $m = \infty$, and at any given s, π_E decreases as m decreases. Moreover, we showed that for $m = \infty$ we have $\pi_E = \frac{1-\sqrt{1-s\frac{\rho X}{e}(1-\rho)^2}}{1-\rho}$, which is highest when $s = 1$, $\rho X = 1$, and e is the lowest possible such that the manager still extracts all the surplus from the financier, *i.e.* $e = 1.07735$. This allows us to calculate the highest possible π_E for any ρ which is

$$\pi_E \leq \frac{1 - \sqrt{1 - \frac{(1-\rho)^2}{1.07735}}}{1-\rho}. \tag{40}$$

Finally, by drawing the regions expressed in inequalities (39) and (40) in figure 6 , we can readily observe that any acceptable pair (ρ, π_E) in the model that satisfies (40) also satisfies (39) which completes the proof of $\frac{\partial \pi_M}{\partial p_0} > 0$ when the financier liquidates. ■

Proof of the proposition 3: (i) We begin with the socially optimal values of evaluation and monitoring (π_E^*, π_M^*), as our initial point for (π_E, π_M). Then, we show that if the financier changes one of his choices at each step, the result converges to the financier's best choice, which has the properties that we claimed. Recall that the social planner's best response, and the financier's best response come from the following system of equations:

$$\text{social planner}\begin{cases}\pi_M^* &= \psi(\pi_E^*)\frac{z}{2m}\\ \pi_E^* &= \left[\frac{1-\rho}{2e}\right](1+m{\pi_M^*}^2-\pi_M^* z),\end{cases}\quad \text{financier}\begin{cases}\pi_M &\overset{(1)}{=} \psi(\pi_E)\frac{sz}{2m}\\ \pi_E &\overset{(2)}{=} \left[\frac{1-\rho}{2e}\right](p_0+m\pi_M^2-\pi_M sz).\end{cases}$$

Now suppose we start from the social planner's solution, *i.e.* (π_E^*, π_M^*) as a solution to the financier's system. Then iterate the system to converge to a new solution for the financier's system. First, suppose we update π_E. Put π_M^* in equation (2) for π_E. Since $p_0 \geq 1$, and $sz \leq z$, π_E increases. Now because π_E increases, and ψ is decreasing in π_E, also $\frac{sz}{2m} \leq \frac{z}{2m}$, π_M in equation (1) decreases. Now because π_M goes down, and $m\pi_M^2 - \pi_M sz$ is decreasing in π_M, π_E goes up again. Next, because ψ is decreasing in π_E, π_M decreases, and this goes on until they converge to the new solution (provided that it converges). As it was shown, at each step π_E increases and π_M decreases, and thus in the final answer must have the property that the financier over-evaluates and under-monitors.

Part (ii) is proved in the text, just above the proposition. ■

Proof of the proposition 4: (i) The fact that s grows as α is growing simply follows from s being equal to $s_0 = \frac{\alpha\lambda}{z}$ in this special case. Moreover, by lemma 2 and (15) we have that $\frac{\partial p_0}{\partial s} = P_F$, hence $s = \frac{\alpha\lambda}{z}$ gives us $\frac{\partial p_0}{\partial \alpha} = \frac{\lambda}{z}P_F$.

(ii) We start from the financier's FOCs (11) when she liquidates:

$$\begin{cases}\pi_E = \left(\frac{1-\rho}{2e}\right)(p_0+m\pi_M^2-\pi_M sz)\\ \pi_M = \frac{(1-\rho)(1-\pi_E)}{\rho+(1-\rho)(1-\pi_E)}\frac{sz}{2m}.\end{cases}$$

Using $\psi = \frac{(1-\rho)(1-\pi_E)}{\rho+(1-\rho)(1-\pi_E)}$ and $sz = \alpha\lambda$, and then taking the partial derivative with respect to α gives us

$$\begin{cases}\frac{\partial \pi_E}{\partial \alpha} = \left(\frac{1-\rho}{2e}\right)(\frac{\lambda}{z}P_F + (2m\pi_M - sz)\frac{\partial \pi_M}{\partial \alpha} - \pi_M\lambda)\\ \frac{\partial \pi_M}{\partial \alpha} = \frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \alpha}\frac{\alpha\lambda}{2m} + \psi\frac{\lambda}{2m}.\end{cases}$$

Now replacing $\frac{\partial \pi_M}{\partial \alpha}$ from the second equation into the first one, and then solving it for $\frac{\partial \pi_E}{\partial \alpha}$, which is similar to the derivation of (37), gives us

$$\frac{\partial \pi_E}{\partial \alpha} = \frac{\frac{1-\rho}{2e}\frac{\lambda}{z}\Big[P_F - (2-\psi)z\pi_M\Big]}{1+\frac{1-\rho}{2e}\frac{(\alpha\lambda)^2}{2m}(1-\psi)\frac{\partial \psi}{\partial \pi_E}},$$

which is positive for the exact the same reasoning that follows (37).

Replacing $\frac{\partial \pi_E}{\partial \alpha}$ from above into the equation for $\frac{\partial \pi_M}{\partial \alpha}$ that came right before that, and simplifying the result along the same lines of derivation of (38), gives us

$$\frac{\partial \pi_M}{\partial \alpha} = \left[\frac{\frac{\partial \psi}{\partial \pi_E} \left(\frac{1-\rho}{2e} \right) \left[sP_F - \pi_M \alpha \lambda \right] + \psi}{1 + \left(\frac{1-\rho}{2e} \right) \frac{(\alpha\lambda)^2}{2m} (1-\psi) \frac{\partial \psi}{\partial \pi_E}} \right] \frac{\lambda}{2m}.$$

With the same argument that followed (38), we conclude that it has a positive sign.

(iii) Recall that $P_F = \frac{\rho X + (1-\rho)(1-\pi_E)\pi_M z}{\rho + (1-\rho)(1-\pi_E)}$ which could be rewritten in terms of ψ, the probability of funding a bad project given that the financier evaluated π_E, as

$$P_F = (1-\psi)X + \psi \pi_M z.$$

Differentiating this with respect to α gives us

$$\frac{\partial P_F}{\partial \alpha} = -\frac{\partial \psi}{\partial \pi_E} \frac{\partial \pi_E}{\partial \alpha} (X - \pi_M z) + \psi \frac{\partial \pi_M}{\partial \alpha} z,$$

which is positive since $\frac{\partial \psi}{\partial \pi_E} < 0$, $X > 1 > \pi_M z$, and both $\frac{\partial \pi_E}{\partial \alpha}$ and $\frac{\partial \pi_M}{\partial \alpha}$ are positive by the previous part. ■

Proof of the proposition 5: (i) This part is due to the two systems stemming from (12) and (20) being virtually the same when the financier sells.

(ii) 1. We start from differentiating the manager's value function in (20) with respect to p_0, given p_0 and s are related through lemma 4. The manager's target function is now:

$$\left[\rho + (1-\rho)(1-\pi_E) \right] (p_0 - 1) + (1-s)\rho X,$$

and keeping in mind that $\frac{\partial s}{\partial p_0} = \frac{1}{P_F}$, by lemma 2 and equation (16) we take its partial with respect to p_0:

$$-(1-\rho) \frac{\partial \pi_E}{\partial p_0} (p_0 - 1) + \left[\rho + (1-\rho)(1-\pi_E) \right] - \frac{\rho X}{P_F}.$$

Now by replacing $\frac{\rho X}{P_F}$ with $\rho + (1-\rho)(1-\pi_E)$, we get that

$$\frac{\partial V^m}{\partial p_0} = -(1-\rho) \frac{\partial \pi_E}{\partial p_0} (p_0 - 1) \leq 0,$$

proving that the manager sets the price at $p_0 = 1$. This is of course associated to the least number of shares s for which the financier still invests.

(ii) 2. We start from the financier's FOCs in (10):

$$\begin{cases} \pi_E = \left(\frac{1-\rho}{2e} \right) \left[1 + m\pi_M^2 - \alpha\lambda\pi_M \right] \\ \pi_M = \psi \frac{\alpha\lambda}{2m}, \end{cases}$$

where we also used $p_0 = 1$. Now we differentiate the above system with respect to α to get

$$\begin{cases} \frac{\partial \pi_E}{\partial \alpha} = \left(\frac{1-\rho}{2e}\right)\left[(2m\pi_M - \alpha\lambda)\frac{\partial \pi_M}{\partial \alpha} - \lambda\pi_M\right] \\ \frac{\partial \pi_M}{\partial \alpha} = \frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \alpha}\frac{\alpha\lambda}{2m} + \psi\frac{\lambda}{2m}. \end{cases}$$

We insert $\frac{\partial \pi_M}{\partial \alpha}$ from the second equation into the first one to get

$$\begin{aligned} \frac{\partial \pi_E}{\partial \alpha} &= \left(\frac{1-\rho}{2e}\right)\left[(2m\pi_M - \alpha\lambda)\left(\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \alpha}\frac{\alpha\lambda}{2m} + \psi\frac{\lambda}{2m}\right) - \lambda\pi_M\right] \\ &= \left(\frac{1-\rho}{2e}\right)\left[\left(\frac{2m}{\alpha\lambda}\pi_M - 1\right)\alpha\lambda\left(\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \alpha}\frac{\alpha\lambda}{2m} + \psi\frac{\lambda}{2m}\right) - \lambda\pi_M\right] \\ &= \left(\frac{1-\rho}{2e}\right)\left[(\psi - 1)\alpha\lambda\left(\frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \alpha}\frac{\alpha\lambda}{2m} + \psi\frac{\lambda}{2m}\right) - \lambda\pi_M\right]. \end{aligned}$$

Solving for $\frac{\partial \pi_E}{\partial \alpha}$ gives us

$$\begin{aligned} \frac{\partial \pi_E}{\partial \alpha} &= \frac{\left(\frac{1-\rho}{2e}\right)\left[-(1-\psi)\alpha\lambda\psi\frac{\lambda}{2m} - \lambda\pi_M\right]}{1 + \left(\frac{1-\rho}{2e}\right)(1-\psi)\frac{(\alpha\lambda)^2}{2m}\frac{\partial \psi}{\partial \pi_E}} \\ &= \frac{-\left(\frac{1-\rho}{2e}\right)\left[(2-\psi)\lambda\pi_M\right]}{1 + \left(\frac{1-\rho}{2e}\right)(1-\psi)\frac{(\alpha\lambda)^2}{2m}\frac{\partial \psi}{\partial \pi_E}}, \end{aligned}$$

where in the second equation we used $\psi\frac{\alpha\lambda}{2m} = \pi_M$. The denominator is positive by the same reasoning that we used in (37), and the numerator is obviously negative since $\psi < 1$, which concludes this part.

(ii) 3. We have $\frac{\partial \pi_M}{\partial \alpha} = \frac{\partial \psi}{\partial \pi_E}\frac{\partial \pi_E}{\partial \alpha}\frac{\alpha\lambda}{2m} + \psi\frac{\lambda}{2m}$, and from the previous part $\frac{\partial \pi_E}{\partial \alpha}$ is negative. Hence, the fact that $\frac{\partial \psi}{\partial \pi_E}$ is also negative gives us the result.

(ii) 4. It simply results from the fact that $P_F = \frac{\rho X}{\rho + (1-\rho)(1-\pi_E)}$ is increasing in evaluation, and the evaluation is decreasing in liquidity α. ■

Proof of the corollary 4: The fact that nothing changes if the financier liquidates is immediate because the two systems are the same when the financier liquidates.

(i) If the financier sells, by proposition 5, $p_0 = 1$ and her FOCs are the same as the fictitious social planner in the text (21), rewritten here again with $\psi = \frac{(1-\rho)(1-\pi_E)}{\rho + (1-\rho)(1-\pi_E)}$.

$$\begin{cases} \pi_E = \left(\frac{1-\rho}{2e}\right)(1 + m\pi_M^2 - \pi_M\alpha\lambda) \\ \pi_M = \psi\frac{\alpha\lambda}{2m}. \end{cases}$$

But in the first equation,

$$m\pi_M^2 - \pi_M\alpha\lambda = -\left(1 - \frac{\psi}{2}\right)\pi_M\alpha\lambda < 0,$$

which proves $\pi_E^{nc} < \frac{1-\rho}{2e} = \pi_E^* < \pi_E^c$. Recall that $\pi_E^* = \frac{1-\rho}{2e}$ since when the financier sells, the socially optimal level of monitoring is no monitoring at all, *i.e.* $\pi_M^* = 0$.

(ii) The second equation easily follows from part (i), because $\psi = \psi(\pi_E)$ is decreasing in π_E, and hence

$$\pi_M^* = 0 < \pi_M^c = \psi(\pi_E^c)\frac{\alpha\lambda}{2m} < \psi(\pi_E^{nc})\frac{\alpha\lambda}{2m} = \pi_M^{nc},$$

which concludes the result. ■