Money Management and Real Investment*

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Abstract

We propose and analyze an equilibrium model of money management in which the asset allocation decisions of money managers affect the production decisions of firms. The model produces two main results. First, comparing the performance of money managers to that of the overall market portfolio becomes less appropriate as investors (endogenously) choose to delegate more of their money to them. Indeed, as money managers control more money, their holdings get closer to the market portfolio, making it less likely that they outperform it. Second, although money managers may be outperformed by the market portfolio after their fees are taken into account, it is optimal for investors to hire their services. This is because money managers prompt a more efficient allocation of capital, making the economy more productive and firms more valuable in the process. In fact, as we show, the presence of money managers can improve the welfare of all investors, whether or not these investors choose to delegate their investment decisions to money managers.

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1. Introduction

Over the years, a great deal of attention has been devoted to the study of the performance of active money management. Much of this abundant literature, including the work of Jensen (1968), Malkiel (1995), Gruber (1996), Carhart (1997), Fama and French (2010) to name a few, concludes that actively managed mutual funds do not generate excess returns after accounting for fees and expenses. Wermers (2000), in particular, documents that mutual funds outperform the market before fees are taken into account, but that their net-of-fees returns underperform. French (2008) concludes that the annual return performance of the average investor in actively managed mutual funds would have improved by 67 basis points over 1980-2006 if they had instead opted for a passive market portfolio. At the same time, studies by Grinblatt, Titman, and Wermers (1995), Daniel et al. (1997), Ibbotson, Chen, and Zhu (2011), Petajisto (2013) all find some evidence that active money management yields positive alphas to investors. More than that, Berk and van Binsbergen (2015, 2016), arguing that the correct measure of fund manager skill is not alpha but value added, show that active money management creates value for investors despite average alphas being near zero.

A different strand of the literature is concerned with the informational and allocative role of financial market prices. Starting with the seminal work of Hayek (1945), this literature argues that the combined information of market participants is aggregated and reflected by asset prices, thereby allowing a more efficient allocation of resources and better real investment decisions. This role of financial markets is in fact nicely summarized by Fama and Miller (1972, p.335): "at any point in time, market prices of securities provide accurate signals for resource allocation; that is, firms can make production-investment decisions..." Since then, models of financial markets informing the real investment decisions of firms (through a "feedback effect") have been proposed by Dow and Gorton (1997), Dow and Rahi (2003), Goldstein and Guembel (2008), Goldstein, Ozdenoren, and Yuan (2013), and Sockin and Xiong (2015), and their validity has been confirmed by a number of empirical studies, including those of Chen, Goldstein, and Jiang (2007) and Bakke and Whited (2010) for example.[1] More broadly, Levine and Zervos (1998), Levine, Loayza, and Beck (2000), and Wurgler (2000) document that countries with better-functioning financial markets and financial intermediaries experience stronger long-term growth and better allocate their capital across firms.

The main objective of this paper is to combine these two literatures and show, within a rational expectations equilibrium model that includes money management and firm pro-

[1] Bond, Edmans, and Goldstein (2012) provide an extensive survey of this literature.

duction, how the feedback effect of financial markets onto the real investments of firms is affected by the presence of active money managers. Specifically, we propose and analyze a model in the same spirit as those of Grossman and Stiglitz (1980) and Hellwig (1980), in which prices partially reveal the aggregate information that some market participants possess.[2] We add two key components to the model. First, instead of assuming an exogenously specified payoff for each firm's securities, we assume that the stock of each firm is a claim on their production process, and that firms can affect this production by endogenously choosing their investment. Importantly, as in the aforementioned literature on the feedback effects of prices, we let firms learn from prices in financial markets as they make such decisions. Second, we assume that money managers can (imperfectly) learn about the production process of firms, and that the information that gets aggregated in financial markets originates from these money managers selling their services to otherwise uninformed investors.

The model goes as follows. First, a monopolistic fund manager who is imperfectly informed about the productivity of all N firms in the economy chooses the fee that he charges risk-averse investors for helping them with their portfolio of financial investments and who thus benefit from the fund manager's information. Investors choose whether or not to pay this fee knowing that equilibrium prices will partially reflect the fund manager's information, making it less valuable as more investors choose to do so. Next, before investors take positions in the financial market, a fraction of them (called hedgers) experience a nontradable shock that is correlated with the production process of firms. Ultimately, fund investors, hedgers, and uninformed investors trade in the financial market through a risk-neutral market maker and, subsequently, firms make their real investment decisions. Both investor portfolios and firms' real investment decisions are informed by equilibrium prices which partially reveal the fund's information.

The model produces three main results. One, the market portfolio and the performance of the (actively managed) fund relative to it both depend on the fraction of investors who choose to hire the fund manager. That is, as more investors delegate their financial investment to the fund, the market portfolio more closely resembles the fund's portfolio. As such, beating this market portfolio becomes harder for the fund. Two, in equilibrium, while the gross-of-fees alpha of the fund is positive, its net-of-fees alpha can be negative. This happens when the hedging needs of investors are larger, as the trades from such needs help the fund camouflage its own trading, making its information more valuable and expensive and thereby reducing its net-of-fees performance. This also hap-

[2] And these papers have their own antecedents in the work of Kihlstrom and Mirman (1975) and Grossman (1976), who show that prices can fully aggregate and reveal the private information of investors in rational expectations models.

pens when the productivity of firms is noisy and difficult to learn about, as investors then more greatly value the risk reduction provided by the fund.

Three, despite the fund's negative alpha, it is possible for all investors to be better off in an economy with money management than one without money management. This is because the fund's information about each firm's productivity percolates through financial market prices and eventually imperfectly informs their real investment decisions. This adjustment to their investment makes every firm more valuable and, while the fund manager captures some of the economic surplus that his information creates through the fees that he charges investors, he must leave some surplus on the table in order to compensate fund investors for the increased risk that the fund's trading strategy imposes on them. Thus, as in the work of Berk and van Binsbergen (2015, 2016), the proper measure of the fund's performance is not its alpha; in contrast to their work, however, the performance of active money management is best measured by the increase in welfare that its presence prompts, not just the value added. In other words, the fund manager's presence has an impact on productivity, value, and risk, all of which affect the investor's expected utility.

As mentioned above, our paper links the money management literature with the feedback effect literature to argue that the role of actively managed funds is to provide the economy with a better allocation of resources. The idea that active money management makes the financial market more efficient by impounding more information into equilibrium prices is also found in recent work by Gârleanu and Pedersen (2018, 2019), Bond and García (2020), Buss and Sundaresan (2020), and Lee (2020). As in our model, these papers endogenize the size of the active money management industry as well as the fees that it charges investors. Our model differs from theirs not only because it jointly endogenizes the production decisions of firms, but also because it provides a novel way to measure the value of active money management in a rational expectations equilibrium. Likewise, the idea that financial markets inform the production decisions of firms is a key aspect of the aforementioned work by Dow and Gorton (1997), Dow and Rahi (2003), Goldstein and Guembel (2008), Goldstein, Ozdenoren, and Yuan (2013), and Sockin and Xiong (2015), but none of these papers consider the endogenous role and impact of active money management on this feedback effect.[3]

Closer and complementary to our paper is the work by Baker, Chapman, and Gallmeyer (2021), who incorporate mutual fund intermediation into a dynamic model of a production economy. As in our work, their model shows that active money manage-

[3] It is worth noting that David, Hopenhayn, and Venkateswaran (2016), Sockin (2017), Benhabib, Liu, and Wang (2019), and Neuhann and Sockin (2020) also propose and analyze models of financial markets that affect the real economy in various manners.

ment can improve productive efficiency and that this can benefit even those investors who do not invest in actively managed funds. The channel is different, however, as the better allocation of capital is not done through the feedback effect of stock prices in financial markets but through the direct intervention of activist funds. Likewise, the emphasis of their paper is different from ours: while we focus on the performance and role of active money management, they concentrate on the interactions and externalities between active and passive money management.

In terms of modeling techniques, our paper is probably closest to that of Dow and Rahi (2003) who add hedgers, risk-neutral market making, and firms' real investment decisions to the classic rational expectations models of Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981). As in their model, the replacement of supply noise by hedgers enables a formal welfare analysis, while the risk-neutral market maker is introduced in such a way that it not only makes the treatment of endogenous firm production decisions tractable, but it also allows investors to submit (limit) orders that condition on prices. In our case, we follow Gârleanu and Pedersen (2018, 2019) and also add the possibility of an informed market participant (i.e., an active fund manager) to charge uninformed participants an endogenously chosen fee for his services.

The rest of the paper is organized as follows. We set up the model in section 2. Then, in section 3, we solve for the equilibrium in financial markets, the equilibrium fraction of investors who choose to invest in the fund, and the fund manager's equilibrium choice of fees. Finally, section 4 contains the derivation and discussion of our results concerning the fund's performance relative to the market portfolio, as well as the welfare and performance of investors. This section also contrasts an economy with money management to one without money management. The proofs to all the results are contained in Appendix A.

2. Model Setup

The economy has three dates. At date 0, investors who are endowed with the market portfolio decide whether or not to delegate their subsequent portfolio choice to an active money manager. At date 1, the firms' shares are traded in a competitive market-making system. Based on the observed stock prices, firms simultaneously make a real investment decision. At date 2, payoffs are realized and consumption takes place. Throughout, the risk-free rate is normalized to zero.

There are N firms, indexed by $n \in \{1, \ldots, N\}$. The date-2 payoff of firm n is given by

$$v_n = k_n(\theta_n + \varepsilon_n) - \tfrac{c}{2}k_n^2, \qquad [1]$$

where k_n represents the firm's real investment choice, $\theta_n + \varepsilon_n$ denotes its profitability per unit of investment, and $c > 0$ is an investment cost parameter. The manager of firm n chooses the investment level k_n to maximize the firm's expected payoff after observing the firm's date-1 stock price p_n and total trading activity x_n, both of which will be endogenized later. The manager cannot directly observe θ_n or ε_n, which are known to be independently normally distributed, with each other and across all N firms:

$$\theta_n \sim \mathcal{N}\left(\tfrac{\mu}{\sqrt{N}}, \tfrac{\sigma_\theta^2}{N}\right) \quad \text{and} \quad \varepsilon_n \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right).$$

We assume that $\mathbb{E}(\theta_n + \varepsilon_n) = \mathbb{E}(\theta_n) = \frac{\mu}{\sqrt{N}}$ goes to zero at the same rate as $\frac{1}{\sqrt{N}}$ so that the expected value of the total economy remains constant as the number N of firms increases. This happens because the equilibrium investment choice k_n made by each firm will itself be proportional to $\frac{1}{\sqrt{N}}$ making the product of productivity and investment proportional to $\frac{1}{N}$. Also, as we will see later, the ratio $\frac{\mathrm{Var}(\theta_n)}{\mathrm{Var}(\varepsilon_n)} = \frac{1}{N}\frac{\sigma_\theta^2}{\sigma_\varepsilon^2}$ effectively measures the informational advantage of the money manager as it relates to each stock. With these distributions, therefore, the informational advantage of the money manager about any one stock goes to zero as N grows large but, since he is then informed about a larger number of stocks, his overall informational advantage stays constant. As we show later, this will also ensure that the fees charged by the money manager in equilibrium remain finite.

The fact that θ_n, ε_n, and $\theta_n + \varepsilon_n$ can take on negative values is immaterial for the model. In particular, the production function in [1] can be thought of as an adjustment that each firm n can make in its investment policy to become more productive and more valuable. That is, $\theta_n + \varepsilon_n$ essentially represents the changes in firm n's profitability and, if this quantity is expected to be negative, the firm can increase its value by divesting (i.e., choosing $k_n < 0$) at a cost of $\frac{c}{2}k_n^2$. In this light, in fact, a choice of $k_n = 0$ simply represents the status quo in which the firm does not see any reason to change its existing investment policy.[4]

There is a continuum of risk-averse investors, indexed by $i \in [0, 1]$. Investors have CARA preferences with a common risk aversion coefficient of $\gamma > 0$; that is, investor i's utility over final wealth W_i is

$$u(W_i) = -e^{-\gamma W_i}.$$

[4]In the same spirit, although we write the paper with $\mu > 0$ in mind, the analysis applies and our results obtain even if $\mu < 0$.

Each investor is endowed with $z > 0$ shares of each firm. Effectively, since all the firms are identical ex ante, each investor is endowed with the market portfolio, which is equally-weighted across all N firms. At date 0, investors decide whether or not to delegate their portfolio decision to a money manager who has some private information about the profitability of all firms in the economy. In particular, we assume that the money manager observes θ_n for all $n = 1, \ldots, N$ before the financial market opens at date 1 and that he shares this information with investors for a fee $f \geqslant 0$, to be endogenized later.

Before the financial market opens at date 1, each investor potentially receives a portfolio of nontradable assets that are correlated with the N firms' production processes (those who do are referred to as *hedgers*). Specifically, with probability $\omega \in (0, 1)$, each investor receives a stochastic portfolio $\{\eta_1, \ldots, \eta_N\}$ in which η_n represents the number of units of an asset that generates a stochastic payoff of ξ_n at date 2. We assume that each ξ_n is normally distributed with mean zero and variance σ_ξ^2, and that the correlation of ξ_n with ε_n is $\rho \in [0, 1]$; that is, ρ measures the extent to which the nontradable endowment is correlated with the economy's real assets.[5] Finally, we assume that the η_n's are iid and normally distributed with mean zero and variance $\frac{\sigma_\eta^2}{N}$, and that they are independent of all θ_n's and ε_n's. The η_n's are not observable by any market participants other than the hedgers. The fact that the variance of η_n is normalized by N simply implies that the variance of the portfolio shock that hedgers experience does not grow with N.

As we will demonstrate below, investors who experience an endowment shock do not benefit from learning the profitability θ_n of every firm. We therefore assume that investors who hire a fund manager only pay the fee f if they do not experience an endowment shock and hence can in fact benefit from the fund manager's services.[6] The money manager sets his fee f to maximize his expected profit. We let λ denote the fraction of investors who hire the money manager. At date 1, each firm's shares are traded in a competitive market-making system similar to that of Kyle (1985). For each of the N firms, every investor i submits a demand schedule $x_{in}(p_n)$ to a risk-neutral, competitive market maker. For every stock $n \in \{1, \ldots, N\}$, the market maker observes the aggregate demand schedule $x_n(p_n) = \int_0^1 x_{in}(p_n)\, di$ and sets its price to $p_n = \mathbb{E}(v_n \mid x_1, \ldots, x_N) = \mathbb{E}(v_n \mid x_n)$.

To summarize, the model's time line is as follows. First, the money manager posts

[5] We could also allow for some correlation between ξ_n and θ_n, $n \in \{1, \ldots, N\}$, but, as will become clear below, any correlation between ξ_n and θ_n is irrelevant because investors with hedging needs can infer θ_n from the equilibrium price and trading activity.

[6] This assumption is made purely for expositional and tractability reasons. It is also without loss of generality: if investors knew that the fee they incur to hire the money manager would be wasted a fraction ω of the time, this would simply lower the fee that the money manager could charge for his services. Alternatively, we could also assume that the fees charged by the money manager could be refunded should an investor wish to pull out of the arrangement before financial markets open.

the fee f that investors must pay in order to use his investment services (and information), and each investor decides whether or not to pay this fee. Second, the money manager observes a vector of signals $\{\theta_n\}_{n=1}^{N}$ about the productivity of each firm. At the same time, a fraction ω of investors experience a nontradable endowment shock $\{\eta_n\}_{n=1}^{N}$ that is correlated with $\{\varepsilon_n\}_{n=1}^{N}$, the unobservable component of each firm's productivity. Third, all traders submit their demand schedule for each of the N stocks to a risk-neutral, competitive market maker who clears the market. Observing this trading activity, each firm n simultaneously chooses its investment policy k_n. Finally, all payoffs are realized and all investors consume their final wealth.

3. Equilibrium

In this section, we solve for the equilibrium of the model from section 2 in three steps. First, we assume that the fraction λ of investors who choose to hire the money manager is exogenously given, and solve for the equilibrium that prevails in the financial market. Second, we fix the fee f charged by the fund manager and solve for the endogenous fraction of investors who choose to pay this fee and hire him. Finally, because the trading equilibrium can be anticipated by the fund manager for any f that he chooses to charge, we solve for the fee that maximizes his expected profits.[7] In the process, we also characterize the limit economy as the number N of firms grows infinitely large.

3.1. Financial Market Equilibrium

We first solve for a rational expectations equilibrium for any given $\lambda \in (0, 1]$. We conjecture that, for each stock $n \in \{1, \ldots, N\}$, observing the equilibrium price p_n and trading activity x_n is informationally equivalent to observing the signal

$$\tau_n = \theta_n - \frac{\mu}{\sqrt{N}} - \phi\, \eta_n, \qquad [2]$$

where $\phi > 0$ is a constant that reflects the informativeness of p_n and x_n. In the ensuing analysis, we derive an equilibrium in which this conjecture is confirmed to be correct.

[7] As mentioned before, our main analysis assumes monopolistic pricing by the money management industry. Given that this allows this industry to capture the most economic surplus, our subsequent results about the investment performance and welfare of investors will effectively represent lower bounds on these quantities.

From the projection theorem,[8] we have

$$\mathbb{E}(\theta_n \mid p_n, x_n) = \mathbb{E}(\theta_n \mid \tau_n) = \tfrac{\mu}{\sqrt{N}} + \delta\tau_n, \quad \text{and} \tag{3}$$

$$\mathrm{Var}(\theta_n \mid p_n, x_n) = \mathrm{Var}(\theta_n \mid \tau_n) = (1-\delta)\,\tfrac{\sigma_\theta^2}{N}, \tag{4}$$

where

$$\delta = \frac{\frac{\sigma_\theta^2}{N}}{\frac{\sigma_\theta^2}{N} + \phi^2\frac{\sigma_\eta^2}{N}} = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \phi^2\sigma_\eta^2}. \tag{5}$$

Firm n chooses its investment level k_n to maximize its value, which is the expected value of its final payoff. Importantly, the information that the financial market simultaneously reveals about its productivity, is taken into account in this decision; that is, each firm n chooses k_n conditional on the information revealed by p_n and x_n:

$$\max_{k_n}\ \mathbb{E}(v_n \mid p_n, x_n) = k_n\,\mathbb{E}(\theta_n \mid \tau_n) - \tfrac{c}{2}k_n^2.$$

Thus, firm n chooses

$$k_n = \tfrac{1}{c}\,\mathbb{E}(\theta_n \mid \tau_n) \overset{[3]}{=} \tfrac{1}{c}\left(\tfrac{\mu}{\sqrt{N}} + \delta\tau_n\right). \tag{6}$$

Based on the above conjecture, the market maker learns the signal τ_n from the aggregate demand schedule of investors. The market maker therefore sets the date-1 price of stock n equal to

$$p_n = \mathbb{E}(v_n \mid \tau_n) = k_n\,\mathbb{E}(\theta_n \mid \tau_n) - \tfrac{c}{2}k_n^2 \overset{[6]}{=} \tfrac{1}{2c}\big[\mathbb{E}(\theta_n \mid \tau_n)\big]^2 \overset{[3]}{=} \tfrac{1}{2c}\left(\tfrac{\mu}{\sqrt{N}} + \delta\tau_n\right)^2. \tag{7}$$

Let x_{in} denote the number of shares of stock n that investor i buys in the financial market equilibrium ($x_{in} < 0$ denotes a sale). The (gross-of-fees) terminal wealth of investor $i \in [0,1]$ is given by

$$W_i = \sum_{n=1}^{N}\Big[(z + x_{in})\,v_n - x_{in}\,p_n + \tilde{\eta}_{in}\,\xi_n\Big], \tag{8}$$

where $\tilde{\eta}_{in} \in \{\eta_n, 0\}$, depending on whether or not this investor received an endowment shock. Since the investment level k_n is (p_n, x_n)-measurable, W_i is normally distributed conditional on the investor's information set $\mathcal{F}_i$ (which includes at least p_n and x_n for every $n \in \{1, \ldots, N\}$ for all investors). Thus, given the investor's CARA preferences, his optimal demand schedule solves

$$\max_{x_{i1},\ldots,x_{iN}}\ \mathbb{E}(W_i \mid \mathcal{F}_i) - \tfrac{\gamma}{2}\,\mathrm{Var}(W_i \mid \mathcal{F}_i). \tag{9}$$

[8] See Result 1 from Appendix B.

The result of this maximization problem is the subject of the following lemma. Specifically, the lemma derives the demand schedule for three types of investors: informed fund investors who have hired a money manager (F), hedgers who have experienced an endowment shock (H), and uninformed investors without a money manager or shock (U).

Lemma 1. *The demand schedules for fund investors, hedgers, and uninformed investors are given by*

$$x_n^{\mathrm{F}} = \frac{k_n\theta_n - \frac{c}{2}k_n^2 - p_n}{\gamma\, k_n^2\, \sigma_\varepsilon^2} - z, \tag{10}$$

$$x_n^{\mathrm{H}} = \frac{k_n\theta_n - \frac{c}{2}k_n^2 - p_n - \gamma\, k_n\, \rho\, \sigma_\varepsilon\, \sigma_\xi\, \eta_n}{\gamma\, k_n^2\, \sigma_\varepsilon^2} - z, \quad \textit{and} \tag{11}$$

$$x_n^{\mathrm{U}} = -z, \tag{12}$$

respectively.

Because investors are risk-averse and the market maker is risk-neutral, we can see from the last term in each of [10], [11], and [12] that it is optimal for investors to use the financial market to hedge their initial position z in each stock. Fund investors also use the financial market to take advantage of the information θ_n that they receive from the money manager by increasing (decreasing) their position in stock n when the expected value of its output, $k_n\theta_n - \frac{c}{2}k_n^2$, is greater (smaller) than the equilibrium price. Such positions are larger when their risk aversion (γ) is small and when the conditional variance of the firm's output, $k_n^2\sigma_\varepsilon^2$, is small. Hedgers effectively learn θ_n for each stock n through equilibrium prices and trading activity. Indeed, under our conjecture that the information jointly contained in p_n and x_n is equivalent to that in τ_n as defined in [2], hedgers can use the fact that they know the common shock η_n to infer θ_n from τ_n. As such, they too can use this information to speculate about each firm. In addition, hedgers use the stock of firm n to hedge their endowment shock η_n (which is correlated with ε_n).

The investors' aggregate demand schedule for stock n, x_n, is obtained by integrating over all investors: $x_n \equiv \int_0^1 x_{in} di$. Since a fraction ω of them are hedgers, $(1-\omega)\lambda$ are fund investors, and $(1-\omega)(1-\lambda)$ are uninformed, we can use [10], [11], and [12] to write this aggregate demand as

$$\begin{aligned} x_n &= \omega x_n^{\mathrm{H}} + (1-\omega)\lambda x_n^{\mathrm{F}} + (1-\omega)(1-\lambda)x_n^{\mathrm{U}} \\ &= \frac{\left[\omega + (1-\omega)\lambda\right]\left(k_n\theta_n - \frac{c}{2}k_n^2 - p_n\right) - \omega\,\gamma\, k_n\, \rho\, \sigma_\varepsilon\, \sigma_\xi\, \eta_n}{\gamma\, k_n^2\, \sigma_\varepsilon^2} - z. \end{aligned} \tag{13}$$

This last expression confirms our conjecture that the aggregate order flow for each stock n

reveals a linear combination of θ_n and η_n to market participants (i.e., to investors, the market marker, and firms), as specified in [2], where

$$\phi = \frac{\omega\,\gamma\,\rho\,\sigma_\varepsilon\,\sigma_\xi}{\omega + (1-\omega)\lambda}. \tag{14}$$

We can now solve for the model's financial market equilibrium.

Proposition 1 (Financial Market Equilibrium)**.** *There exists an equilibrium in which the price of stock n is given by* [7] *and the equilibrium aggregate order flow is given by*

$$x_n = \frac{\left[\omega + (1-\omega)\lambda\right] c\,(1-\delta)\,\tau_n}{\gamma\left(\frac{\mu}{\sqrt{N}} + \delta\tau_n\right)\sigma_\varepsilon^2} - z, \tag{15}$$

where δ and ϕ are as in [5] *and* [14]*, respectively.*

Notice from [7] and [15] that the equilibrium price p_n and order flow x_n for each stock n jointly convey τ_n.[9] That is, as initially conjectured, equilibrium prices and trading activity reveal a linear combination of θ_n and η_n to all market participants. Because fund investors learn θ_n and because hedgers learn η_n, they are able to disentangle θ_n and η_n from τ_n, but the market maker, uninformed traders, and firms can only condition on τ_n when they make (pricing or investment) decisions.

In contrast to most rational expectations models of financial markets, our model shows how active money management comes to affect the real investment decisions of firms through its impact on financial markets. Specifically, the information θ_n that the money manager uncovers about the productivity of each firm gets imperfectly reflected in asset prices and (at least partially) used by firms when they choose k_n. In fact, we can see from [6] that, in equilibrium, the investment decision of firm n is effectively linearly related to θ_n by a factor of $\frac{\delta}{c}$. It is obvious from [5] that δ is decreasing in ϕ and from [14] that ϕ is decreasing in λ and increasing in ω, ρ, and γ. This implies that firms' investment decisions will better incorporate the information that is available about their future productivity when λ is large, and when ω, ρ, and γ are small. That is, firms make more accurate investment decisions when more investors hire active money managers (λ large), when the hedging needs of investors are small (ω and ρ small), and when investors are less risk-averse (γ small). In turn, this makes firms more valuable.

To quantify the effect that money management has on the value of firms, we calculate the ex ante value of each firm, $\mathbb{E}(v_n)$. This is the value that anticipates the real decisions of firms as well as the information about θ_n that will percolate into such deci-

[9] The proof of Proposition 1 formally establishes this result.

sions.

Proposition 2. *The ex ante value of each firm $n \in \{1, \dots, N\}$ is given by*

$$\mathbb{E}(v_n) = \frac{1}{2cN}\left(\mu^2 + \frac{\sigma_\theta^4}{\sigma_\theta^2 + \phi^2\sigma_\eta^2}\right). \quad [16]$$

It is clear from [16] that the ex ante value of each firm is decreasing in ϕ which, given [14], implies that it is increasing in λ. Since each firm is ex ante identical and since investors are endowed with an equal-weighted portfolio of all N firms, the combined value of all N firms effectively represents the ex ante value of the market portfolio:

$$V \equiv \sum_{n=1}^{N} \mathbb{E}(v_n) = \frac{1}{2c}\left(\mu^2 + \frac{\sigma_\theta^4}{\sigma_\theta^2 + \phi^2\sigma_\eta^2}\right). \quad [17]$$

Both [16] and [17] highlight the fact that the value of firms comes from two sources in this economy. In both expressions, the first term (μ^2) captures the no-information value of firms: without knowing anything about θ_n, we can see from setting $\tau_n = 0$ in [6] and [7] that each firm n chooses $k_n = \frac{\mu}{c\sqrt{N}}$ and that this leads to a value of $\frac{\mu^2}{2cN}$. The extra value in the second term comes from the fact that each firm is able to learn more about its productivity through financial markets. The informativeness of this information is decreasing in ϕ and so increasing in λ, the extent to which investors delegate their investment decisions to active money management.

Finally, using [4], we can see that the variance of θ_n conditional on the information that emanates from the trading process is decreasing in δ, and so decreasing in λ. When more investors hire money managers, the aggregate position taken by active funds more strongly reflect the information they have about firms. This allows even uninformed market participants to benefit from this information: it lowers the uncertainty that market makers face regarding each firm's productivity and, hence, reduces their informational disadvantage vis-à-vis fund investors. As we show in the next section, this in turn affects the willingness of fund investors to pay for this information in the first place, in the same way that it did in the work of Grossman and Stiglitz (1980). In other words, the services of money managers are less valuable to traders when many other investors already use them.

3.2. Money Management Equilibrium

We now turn to finding the equilibrium value of λ. This value will be affected by the fee f charged by the money manager for his services. Indeed, when choosing whether or not to delegate their financial investment decisions, investors will compare their expected (utility) payoffs from doing or not doing so. Also, as pointed out above, because the financial market reflects more of the money managers' information when more investors hire them, this tradeoff is itself affected by λ.

For now, let us take both the fraction λ of investors who hire the fund manager and the fee f charged by the fund manager as given. Let us denote the (gross-of-fees) final wealth of fund investors, hedgers, and uninformed investors by W^{F}, W^{H}, and W^{U}, respectively. These quantities are obtained by replacing x_{in} in [8] by the demand schedules derived in Lemma 1 for each type of investor (and by setting $\tilde{\eta}_{in}$ equal to η_n if investor i is a hedger and to zero otherwise). The expected utility of an investor who chooses not to hire an active fund manager at the outset is given by

$$U^{\mathrm{NM}} = \omega\, \mathbb{E}\Big[u(W^{\mathrm{H}})\Big] + (1-\omega)\, \mathbb{E}\Big[u(W^{\mathrm{U}})\Big]. \quad [18]$$

Similarly, the expected utility of an investor who does hire a fund manager is given by

$$U^{\mathrm{FM}} = \omega\, \mathbb{E}\Big[u(W^{\mathrm{H}})\Big] + (1-\omega)\, \mathbb{E}\Big[u(W^{\mathrm{F}} - f)\Big]. \quad [19]$$

The difference between [19] and [18],

$$U^{\mathrm{FM}} - U^{\mathrm{NM}} = (1-\omega)\Big(\mathbb{E}\big[u(W^{\mathrm{F}} - f)\big] - \mathbb{E}\big[u(W^{\mathrm{U}})\big]\Big), \quad [20]$$

then measures the expected utility gain from hiring a fund manager. As the following lemma shows, this difference is decreasing in λ: the gain from active money management is smaller when many other investors also hire a fund manager. This in turn leads to an equilibrium fraction λ of investors who choose to hire the fund manager.

Lemma 2. *Suppose that the money manager charges a fee $f > 0$ for his services. Then the equilibrium fraction λ of investors who hire him is as follows:*

(i) If $f \leqslant \underline{f}$ (where $\underline{f}$ is defined in the proof), then $\lambda = 1$.

(ii) If $f \geqslant \bar{f}$ (where $\bar{f}$ is defined in the proof), then $\lambda = 0$.

(iii) If $f \in (\underline{f}, \bar{f})$, then $\lambda \in (0,1)$ and it satisfies

$$e^{\gamma f} = \left(1 + \frac{\phi^2 \sigma_\eta^2 \sigma_\theta^2}{N\sigma_\varepsilon^2(\sigma_\theta^2 + \phi^2\sigma_\eta^2)}\right)^{N/2}, \quad [21]$$

where ϕ is given by [14].

When the money manager sets a sufficiently low fee ($f \leqslant \underline{f}$), then all investors improve their welfare by paying it and by using his information in their investment decisions. Likewise, when the money manager sets a prohibitively high fee ($f \geqslant \bar{f}$), then no investor finds it beneficial to rely on his services. Otherwise, some investors hire the money manager and some do not. Since the right-hand side of [21] is increasing in ϕ which is itself decreasing in λ, larger money management fees f lead to a lower fraction λ of investors who hire him. It is also easy to verify that λ, as implicitly defined in [21], is decreasing in σ_ε and increasing in σ_θ, σ_η, σ_ξ, ω, and ρ: investors are more willing to pay the fund manager's fees when his information advantage is greater (i.e., when σ_θ is large), when the risk associated with exploiting the fund manager's information is smaller (i.e., when σ_ε is small), and when the hedging motives of other traders are stronger (i.e., when σ_η, σ_ξ, ω, and/or ρ are large). In all three cases, this is because the fund manager's information is effectively more valuable. While this is clear for the first result, the reason for the second result is that a reduction in the residual risk σ_ε enables investors to trade more aggressively on the fund manager's information. The last result comes from the fact that hedgers trade more aggressively when they are more motivated, which improves the camouflage that such trading provides to informed traders.

Of course, as we discuss below, it is never optimal for the fund manager to set f strictly smaller than $\underline{f}$ as he can collect more fees when $f = \underline{f}$ and still attract all of the investors. Also, the fund manager never sets $f \geqslant \bar{f}$, as he does not collect any fee at all if $\lambda = 0$. Thus, it will always be the case that [21] holds in equilibrium.

3.3. The Limit Economy

Before we solve for the fund manager's optimal choice of f, we consider the limit economy as the number N of firms grows to infinity. As we will see, this will render the expression for λ as a function of f that is implicitly defined in [21] more tractable, and allow a more insightful characterization of equilibrium fees.

Recall from section 2 that, as $N \to \infty$, $\mathrm{Var}(\theta_n)$ and $\mathrm{Var}(\eta_n)$ both go to zero at the same rate as $\frac{1}{N}$, and that $\mathbb{E}(\theta_n)$ goes to zero at the same rate as $\frac{1}{\sqrt{N}}$. Notice also from [14] that the informativeness of stock prices, ϕ, is unaffected by this changes in N. Likewise, we can see from [5] that δ is unaffected by this normalization. Indeed, because the noise in the firm's productivity stays proportional to the hedgers' endowment shock, the informativeness of a firm's stock price (or, equivalently, of the signal τ_n as defined in [2]) is unaffected.

As discussed at the end of section 3.2, it must be the case that [21] will hold in equilibrium. It is easy to verify that, in the limit economy as $N \to \infty$, [21] converges to

$$e^{\gamma f} = \exp\left\{ \frac{\phi^2 \sigma_\eta^2 \sigma_\theta^2}{2\left(\sigma_\theta^2 + \phi^2 \sigma_\eta^2\right)\sigma_\varepsilon^2} \right\}$$

or, equivalently,

$$\gamma f = \frac{\phi^2 \sigma_\eta^2 \sigma_\theta^2}{2\left(\sigma_\theta^2 + \phi^2 \sigma_\eta^2\right)\sigma_\varepsilon^2}. \qquad [22]$$

We can now insert [14] for ϕ in this last expression and solve for the fraction of investors who choose to hire the money manager:

$$\lambda = \frac{\omega}{1-\omega}\left(\gamma\, \rho\, \sigma_\xi\, \sigma_\eta \sqrt{\frac{1}{2\gamma f} - \frac{\sigma_\varepsilon^2}{\sigma_\theta^2}} - 1 \right). \qquad [23]$$

Of course, this expression is only applicable in case of an interior solution. If the fee f is sufficiently large (resp. small) so that the right-hand side of [23] becomes negative (resp. exceeds one), the fraction of investors who hire the fund manager equals zero (resp. one).

The expression in [23] shows that more investors hire the money manager when the probability ω that they experience a portfolio shock is large, when the shocks they experience are larger (large σ_η) and they have a larger impact on their wealth (large σ_ξ), when the money manager learns relatively more about the firms' productivity (large $\frac{\sigma_\theta}{\sigma_\varepsilon}$), when the shocks are highly correlated with the firms' payoffs (large ρ), and when the fees f charged by the money manager are small. This closed-form expression also allows us to tractably characterize the fee f that the fund manager will choose to charge investors in equilibrium; this is the object of the next section.

3.4. The Fund Manager's Choice of Fees

The fund manager sets the fee f to maximize his expected profits, which are given by

$$\pi(f) = f(1-\omega)\lambda \overset{[23]}{=} f\omega\left(\gamma\rho\sigma_\xi\sigma_\eta \sqrt{\frac{1}{2\gamma f} - \frac{\sigma_\varepsilon^2}{\sigma_\theta^2}} - 1 \right) = \omega\left(\gamma\rho\sigma_\xi\sigma_\eta \sqrt{\frac{f}{2\gamma} - \frac{f^2\sigma_\varepsilon^2}{\sigma_\theta^2}} - f \right). \qquad [24]$$

As in Lemma 2, we let $\underline{f}$ (resp. $\bar{f}$) denote the fee that makes the right-hand side of [23] equal to one (resp. zero).[10] Clearly, it cannot be optimal for the fund manager to choose

[10]In fact, these quantities are the limits of the expressions defined in [A10] and [A11] of the proof of Lemma 2 as $N \to \infty$, respectively.

a fee $f \geqslant \bar{f}$, as such a fee would not attract any investors to the fund and hence would generate no profits for the manager. Similarly, it cannot be optimal to set a fee $f < \underline{f}$ because the fund manager could then increase his profits by raising his fee to $\underline{f}$ without losing any clients. The optimal fee therefore maximizes the manager's objective function in [24] over the interval $[\underline{f}, \bar{f})$. This is the object of the following proposition.

Proposition 3. *In equilibrium, the monopolistic fund manager sets the fee for his services to*

$$f = \max\left\{ \frac{\sigma_\theta^2}{4\gamma\sigma_\varepsilon^2}\left(1 - \sqrt{\frac{1}{1+\chi}}\right), \underline{f} \right\}, \qquad [25]$$

where

$$\underline{f} = \frac{\sigma_\theta^2}{2\gamma\sigma_\varepsilon^2\left(1 + \frac{1}{\omega^2\chi}\right)} \qquad \text{and} \qquad \chi = \frac{\gamma^2\rho^2\sigma_\varepsilon^2\sigma_\xi^2\sigma_\eta^2}{\sigma_\theta^2}. \qquad [26]$$

It is straightforward to verify that [25] is increasing in σ_θ: a greater informational advantage allows the fund manager to more aggressively capitalize on it. It is also the case that [25] is increasing in σ_ξ, σ_η, and ρ: when investors are particularly worried about their hedging needs, the information that the fund manager shares with his clients is more easily camouflaged by the more aggressive trading that comes from hedging.

The last step is to solve for the equilibrium fraction λ of traders who will hire the fund manager with this optimal fee. This is done by inserting [25] into [23], which yields the following proposition.

Proposition 4. *In equilibrium, the fraction of investors who hire the fund manager is*

$$\lambda = \min\left\{ \frac{\omega}{1-\omega}\sqrt{1+\chi}, 1 \right\}, \qquad [27]$$

where χ is defined in [26].

The above argument that $f \in [\underline{f}, \bar{f})$ implies that the equilibrium fraction of investors who hire the fund manager is strictly positive. Furthermore, it is straightforward to verify that, when f is strictly greater than $\underline{f}$, λ is increasing in ω, γ, ρ, σ_ξ, σ_η, and σ_ε, and it is decreasing in σ_θ. Thus, while the fund manager charges more when $\frac{\sigma_\theta}{\sigma_\varepsilon}$ is large, fewer investors actually pay for his services. That is, in equilibrium, it is more advantageous for the fund manager to collect higher fees from fewer investors when his information advantage is greater. This is akin to the result in Admati and Pfleiderer (1986) that a monopolistic information seller may optimally restrict the number of buyers when his information advantage is large relative to the amount of noise trading in markets.

4. Results

In this section, we analyze and characterize various properties of the equilibrium of section 3. First, in section 4.1, we analyze the return performance of money management relative to that of the overall market. Obviously, the fund manager's information allows him to perform better than the rest of the market. Less obvious is whether investors who hire the money manager fare better than the market, given that their performance is negatively affected by the fees they pay. Indeed, we show that while the fund's gross alpha is always positive, it can be the case that its net alpha is negative.

The rest of the section is dedicated to assessing the role and value of the money management industry. We start this analysis in section 4.2 by first deriving the model's equilibrium under the alternative assumption that there is no active money manager in the economy. We then use this equilibrium to contrast the return performance (in section 4.4) and welfare (in section 4.3) of individual investors with and without the active money management industry. We finish in section 4.5 with results about the performance and welfare of passive index investors who, as we show, could easily be added to the model without any effect on the model's equilibrium.

4.1. Fund Performance

In this section, we analyze the performance of active money management using the model of section 2 and the equilibrium of section 3.

For each investor type $t \in \{\mathrm{F}, \mathrm{H}, \mathrm{U}\}$, the expected gross-of-fees (dollar) return from the economy's N stocks is given by[11]

$$R^t = \sum_{n=1}^{N} \left[(z + x_n^t)(v_n - p_n) + z p_n \right],$$

where x_n^{F}, x_n^{H}, and x_n^{U} are as derived in Lemma 1. Of those investors who hire the fund manager, a fraction ω experience a nontradable endowment shock and a fraction $1-\omega$ do not. Thus, the gross return of investors who hire the fund manager is given by

$$R^{\mathrm{FM}} = \omega R^{\mathrm{H}} + (1-\omega) R^{\mathrm{F}} = \sum_{n=1}^{N} \left(\left[\omega (z + x_n^{\mathrm{H}}) + (1-\omega)(z + x_n^{\mathrm{F}}) \right] (v_n - p_n) + z p_n \right), \quad [28]$$

[11]We exclude the nontradable endowment shocks from this definition as, technically, the end-of-period wealth resulting from these is not from the N stocks. Regardless, their inclusion would not affect our result as their ex ante expected value is zero anyway.

while that of investors who do not is

$$R^{\mathrm{NM}} = \omega R^{\mathrm{H}} + (1-\omega)R^{\mathrm{U}} = \sum_{n=1}^{N}\left(\left[\omega(z+x_n^{\mathrm{H}}) + (1-\omega)(z+x_n^{\mathrm{U}})\right](v_n - p_n) + zp_n\right)$$

$$\overset{[12]}{=} \sum_{n=1}^{N}\left[\omega(z+x_n^{\mathrm{H}})(v_n-p_n) + zp_n\right]. \qquad [29]$$

Because a fraction λ of investors hire the fund manager, the return of the market as a whole is the weighted average of [28] and [29]:[12]

$$R^{\mathrm{Mkt}} = \lambda R^{\mathrm{FM}} + (1-\lambda)R^{\mathrm{NM}} = \sum_{n=1}^{N}\left(\left[\omega(z+x_n^{\mathrm{H}}) + \lambda(1-\omega)(z+x_n^{\mathrm{F}})\right](v_n - p_n) + zp_n\right).$$

The gross-of-fees alpha of the fund is therefore given by

$$\alpha_{\mathrm{Gross}}^{\mathrm{FM}} \equiv \mathbb{E}(R^{\mathrm{FM}} - R^{\mathrm{Mkt}}) = \mathbb{E}\left[\sum_{n=1}^{N}\Big((1-\lambda)(1-\omega)(z+x_n^{\mathrm{F}})(v_n-p_n)\Big)\right]. \qquad [30]$$

Since a fraction $(1-\omega)$ of investors who commit to the fund end up paying the fee f charged by the fund manager for his services, the net-of-fees alpha of the fund is given by

$$\alpha_{\mathrm{Net}}^{\mathrm{FM}} \equiv \alpha_{\mathrm{Gross}}^{\mathrm{FM}} - (1-\omega)f. \qquad [31]$$

The following lemma derives the fund's gross- and net-of-fees alphas with exogenous values for f and λ. We insert the equilibrium expressions derived in sections 3.2-3.4 for these variables later.

Lemma 3. *In equilibrium, the fund's gross-of-fees alpha is given by*

$$\alpha_{\mathrm{Gross}}^{\mathrm{FM}} = \frac{(1-\lambda)(1-\omega)(1-\delta)\sigma_\theta^2}{\gamma\sigma_\varepsilon^2}, \qquad [32]$$

and its net-of-fees alpha is given by

$$\alpha_{\mathrm{Net}}^{\mathrm{FM}} = (1-\omega)\left(\frac{(1-\lambda)(1-\delta)\sigma_\theta^2}{\gamma\sigma_\varepsilon^2} - f\right). \qquad [33]$$

It is clear from both [32] and [33] that the performance of money management is decreasing in λ: as the fund's services get employed by a larger fraction of investors, the

[12] Note that we do not include the return of the market maker in this definition. This is immaterial as the fact that they are risk neutral and competitive implies that their ex ante expected payoff is zero and so does not affect the expected performance of the market as a whole.

fund's portfolio becomes closer to the market, and thus it becomes more more difficult for the fund manager to outperform the market. In fact, in the limit when $\lambda = 1$, the fund effectively becomes the market and so cannot outperform it: the gross alpha is zero and the net alpha is negative. The question is whether and under what conditions this happens with the equilibrium values for f and λ. This is the object of the following proposition.

Proposition 5. *In the limit economy with f and λ given by* [25] *and* [27]*, respectively, the fund's gross-of-fees alpha is always nonnegative, while its net-of-fees alpha is negative if and only if*

$$\frac{\omega}{1-\omega}\sqrt{1+\chi} > \frac{1}{2}, \qquad [34]$$

where χ is defined in [26]*.*

Recall that χ is positive. Thus, a sufficient condition for [34] to hold is that $\omega > \frac{1}{3}$. Also, since χ is increasing in γ, ρ, σ_η, σ_ξ, and σ_ε, and decreasing in σ_θ, the fund's net-of-fees alpha tends to be negative when the hedging needs of investors are large (ω, γ, ρ, σ_η, and σ_ξ are large), when the noise in the firm's productivity is large (σ_ε is large), and when the fund manager's information is more limited (σ_θ is small).

Why might investors hire a fund manager despite its net-of-fees alpha being negative? The answer lies in the economic surplus that the fund manager creates and in the economic surplus that he appropriates for himself. On the positive side, because the fund manager's information (imperfectly) informs the real investment decisions of firms, his presence creates additional value.[13] Of course, the manager captures a fraction of this bigger pie through the fees that he charges investors. When the investors' hedging motives are large, their trading is more intense and this creates the noise required to make informed trading particularly valuable. This is when the fund manager charges more than what he creates for his information.

4.2. Equilibrium Without Money Management

Next, we are interested in measuring the impact of active money management on investors' welfare and performance. Specifically, we use the equilibrium from section 3 and contrast it with that of an economy that is without money managers but otherwise identical. In order to make this comparison, this section solves for this economy without money managers. In terms of notation, we use hats on every equilibrium variable to denote the fact that they are derived in this alternative scenario.

[13] In fact, as we will see in section 4.2, the total value created by the manager is equal to $\frac{\sigma_\theta^4}{2c(\sigma_\theta^2+\phi^2\sigma_\eta^2)}$.

The first thing to note in solving for the equilibrium is that, because no market participant is informed about θ_n, the trading process cannot reveal anything about it either. That is, each firm n then makes its investment decision according to its priors about all the model's random variables:

$$\max_{\hat{k}_n} \mathbb{E}(v_n) = \hat{k}_n \mathbb{E}(\theta_n + \varepsilon_n) - \tfrac{c}{2}\hat{k}_n^2 = \hat{k}_n \tfrac{\mu}{\sqrt{N}} - \tfrac{c}{2}\hat{k}_n^2.$$

This leads to

$$\hat{k}_n = \frac{\mu}{c\sqrt{N}}. \qquad [35]$$

Likewise, the market maker cannot learn anything about θ_n from the aggregate demand schedule of investors. As such, he sets the price of stock n equal to

$$\hat{p}_n = \mathbb{E}(v_n) = \hat{k}_n \mathbb{E}(\theta_n + \varepsilon_n) - \tfrac{c}{2}\hat{k}_n^2 = \hat{k}_n \tfrac{\mu}{\sqrt{N}} - \tfrac{c}{2}\hat{k}_n^2 \overset{[35]}{=} \tfrac{\mu^2}{2cN}. \qquad [36]$$

Since no investor can hire the services of a money manager to get informed about θ_n, there are now only two types on investors: hedgers (fraction ω) and uninformed investors (fraction $1-\omega$). Their maximization problem is as in [9] and, since prices do not reveal any information about θ_n, we have

$$\mathbb{E}(W_i \mid \mathcal{F}_i) = \sum_{n=1}^{N} \Big[(z + \hat{x}_{in})\mathbb{E}(v_n) - \hat{x}_{in}\hat{p}_n \Big] \overset{[36]}{=} \sum_{n=1}^{N} z\mathbb{E}(v_n) \overset{[36]}{=} \frac{z\mu^2}{2c}, \qquad [37]$$

and

$$\begin{aligned}
\mathrm{Var}(W_i \mid \mathcal{F}_i) &= \sum_{n=1}^{N} \Big[(z + \hat{x}_{in})^2 \mathrm{Var}(v_n) + 2(z + \hat{x}_{in})\tilde{\eta}_{in}\mathrm{Cov}(v_n, \xi_n) + \tilde{\eta}_{in}^2 \mathrm{Var}(\xi_n) \Big] \\
&= \sum_{n=1}^{N} \left[(z + \hat{x}_{in})^2 \, \hat{k}_n^2 \left(\tfrac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2 \right) + 2(z + \hat{x}_{in})\tilde{\eta}_{in}\hat{k}_n \rho \sigma_\varepsilon \sigma_\xi + \tilde{\eta}_{in}^2 \sigma_\xi^2 \right]
\end{aligned} \qquad [38]$$

where, as before, $\tilde{\eta}_{in} \in \{\eta_n, 0\}$, depending on whether or not investor i experiences a nontradable endowment shock. The solution to this maximization problem, along with the rest of the equilibrium, is the subject of the following proposition.

Proposition 6. *Without a money manager, the demand schedules for hedgers and uninformed investors are given by*

$$\hat{x}_n^{\mathrm{H}} = -\frac{\rho\,\sigma_\varepsilon\,\sigma_\xi\,\eta_n}{\hat{k}_n \left(\frac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2 \right)} - z, \quad \textit{and} \qquad [39]$$

$$\hat{x}_n^{\mathrm{U}} = -z, \qquad [40]$$

respectively. In equilibrium, the optimal real investment decision of all firms is $\hat{k}_n = \frac{\mu}{c\sqrt{N}} \equiv \hat{k}$, *and the price and ex ante value of all firms is* $\hat{p}_n = \frac{\mu^2}{2cN} \equiv \hat{p}$.

Before we make use of this equilibrium to assess the impact of money management on the economy, it is worth noting that the ex ante value of firms is lower in this economy without money management. The combined ex ante value of all N firms in the absence of active fund management is

$$\hat{V} \equiv N\hat{p} = \frac{\mu^2}{2c}. \quad [41]$$

Comparing this value to V in [17], one can see that the presence of the fund manager increases the ex ante value of firms by $\frac{\sigma_\theta^4}{2c(\sigma_\theta^2+\phi^2\sigma_\eta^2)}$, which is increasing in the fraction λ of investors who use this service. That is, by impounding his information into prices and in turn into the real investment decisions of firms, the money manager makes the economy more productive and more valuable. And while the monopolistic fund manager undoubtedly captures some of the economic surplus that he creates, the question is whether some of this surplus ends up in the hands of investors. This is the question we turn to next.

4.3. Welfare Analysis

In this section, we seek to analyze the effects that money management has on welfare. First, note that the expected utility of the risk-neutral market maker is unaffected by the presence of a fund manager. The market maker breaks even in expectation, whether or not there are informed investors present in the market. As such, we can exclude him from the welfare analysis. We also exclude the fund manager from our welfare calculations. We do this for two reasons. First, we are primarily interested in knowing if investors benefit from active money management, and so it would be inappropriate to include the surplus that the fund manager captures in this measure. In fact, as we will show, the fund manager's presence can actually make all investors better off despite the fact that he collects fees from them. Second, given that the surplus captured by the money management industry will never exceed that of the monopolistic fund manager, our measure of welfare effectively represents a lower bound on how well off investors are when their money can be actively managed.

Thus, to assess the impact of money management on welfare, we must calculate the ex ante expected utility of investors in an economy with and without money management. Specifically, for each of the three investor types $t \in \{\mathrm{F}, \mathrm{H}, \mathrm{U}\}$, we calculate

$$U_m^t \equiv \mathbb{E}\big[u(W^t - f\mathbb{1}_{\{t=F\}}\big],$$

where W^{F}, W^{H}, and W^{U} are as defined in section 3.2 and $\mathbb{1}_{\{\}}$ is an indicator function equal to one if the condition in brackets is satisfied and zero otherwise. We calculate the expected utility of investors with and without active money management (m = MM or m = WO).[14] In all cases, we perform this calculation for the limit economy as $N \to \infty$. The following lemma shows the product of these calculations.

Lemma 4. *In the limit-economy equilibrium with active money management (from section 3), the ex ante expected utility of fund investors, uninformed investors, and hedgers is given by*

$$U^{\mathrm{F}}_{\mathrm{MM}} = U^{\mathrm{U}}_{\mathrm{MM}} = -\exp\left\{-\gamma z V\right\}, \quad \textit{and} \tag{42}$$

$$U^{\mathrm{H}}_{\mathrm{MM}} = -\exp\left\{-\gamma\left(zV - \frac{\gamma}{2}(1-\rho^2)\sigma_\eta^2\sigma_\xi^2 - \left(1 + 2\lambda\,\frac{1-\omega}{\omega}\right)f\right)\right\}, \tag{43}$$

respectively, where V is given by [17], f by [25], and λ by [27]. In the limit-economy equilibrium without money management (from section 4.2), the ex ante expected utility of uninformed investors and hedgers is given by

$$U^{\mathrm{U}}_{\mathrm{WO}} = -\exp\left\{-\gamma z\hat{V}\right\}, \quad \textit{and} \tag{44}$$

$$U^{\mathrm{H}}_{\mathrm{WO}} = -\exp\left\{-\gamma\left(z\hat{V} - \frac{\gamma}{2}(1-\rho^2)\sigma_\eta^2\sigma_\xi^2\right)\right\}, \tag{45}$$

respectively, where $\hat{V}$ is given by [41].

Before we proceed with the welfare comparisons with and without money management, it is worth making a few observations about Lemma 4. First, we note from [42] that the welfare of fund investors and uninformed investors is identical. This is not surprising as the equilibrium λ is derived precisely to make these two quantities equal. Second, the hedgers' welfare is smaller than that of other investors in both cases. This is also not surprising as these traders use the financial market not only to hedge their initial endowment of the stock but also their nontradable endowment shock. Because ε_n and ξ_n are imperfectly correlated, the trading of stock n to hedge this risk does not allow hedgers to fully eliminate it. The risk that remains lowers the hedgers' welfare. In fact, we can see this effect clearly from [45]: when $\rho = 1$, perfect hedging is possible, and the second term in that equation goes to zero; that is, the hedgers' ex ante expected utility is then indeed the same as that of other traders.

A simple comparison of [42] and [44] immediately yields the result that fund investors and uninformed investors are always better off with an active money management

[14]Of course, there are no fund investors in the economy without a fund manager.

industry than without one. The reason is intuitive. Through financial market prices, the active money management industry imparts some of its information about firm productivity to the firms themselves. In turn, these firms make better real investment decisions, and this creates value. In fact, using [17] and the equilibrium from section 4.2, it is easy to verify that the total value that this industry creates is $\frac{\sigma_\theta^4}{2c(\sigma_\theta^2+\phi^2\sigma_\eta^2)}$. This is precisely the extra term that shows up in the expression for the fund investors' and uninformed investors' welfare in [42].

The impact of money management on the welfare of hedgers is not as clear cut. In addition to benefitting from the extra value that this industry creates, hedgers also face the familiar Hirshleifer (1971) effect when they use the financial market to hedge their nontradable shocks. Specifically, the fact that they face informed prices for their hedging needs creates additional risk, which lowers their ex ante welfare. This effect is made even worse by the fact that the impact that hedgers have on equilibrium prices goes precisely in the wrong direction. Indeed, to hedge a negative endowment shock $\eta_n < 0$, hedgers must purchase stock n, but we can see from [2] and [3] that τ_n and $\mathbb{E}[\theta_n \mid \tau_n]$ are then both larger. The resulting welfare loss is captured by the last term in [43]. Ultimately, the net impact of the money management industry on the hedgers' welfare involves a comparison of these two effects. This is done in the following proposition.

Proposition 7. *In equilibrium, the presence of active money management always improves the welfare of fund investors and uninformed investors, and it improves the welfare of hedgers if and only if*

$$\frac{z}{c} > \frac{2}{\sigma_\theta^2}\left(1 + 2\lambda\,\frac{1-\omega}{\omega}\right)\left[1 + \left(\frac{\omega}{\omega + (1-\omega)\lambda}\right)^2 \chi\right] f, \qquad [46]$$

where f is given by [25], χ by [26], and λ by [27].

Focusing first on the left-hand side of [46], we can see that the money management industry makes hedgers better off when z is large and when c is small. This is because this industry has a large impact on firms' real investment decision when the input costs required for this production are low (c is small) and when investors initially own a large proportion of this production (z is large). That is, the information about θ_n that financial markets reveal leads to large gains in value, which directly feed into the portfolio with which investors are endowed.

Turning to the right-hand-side of [46], we see that stronger hedging motives (large γ, ρ, σ_η, σ_ξ, and ω) lead hedgers to favor an economy without money managers as this frees them from the aforementioned Hirshleifer effect.[15] Finally, like a reduction in c,

[15]Of course, all of these variables appear in the equilibrium expression for λ in [27] but, because the term

an increase in σ_θ, and thus an increase in the value of the fund manager's information about θ_n, contributes to firms being more productive, and this also benefits hedgers when money management is available.

4.4. Investor Performance

As discussed in section 4.3, the net effect that money management has on the welfare of traders comes from its impact on both the value and risk of their portfolios. In this section, we abstract from the impact on risk and restrict our attention to the return performance of investors. Specifically, for $t \in \{F, H, U\}$, we are interested in comparing

$$r_m^t \equiv \mathbb{E}\big[W^t - f\mathbb{1}_{\{t=F\}}\big]$$

with and without active money management ($m = \text{MM}$ or $m = \text{WO}$).[16] These quantities are calculated in the following lemma.

Lemma 5. *In the limit-economy equilibrium with active money management (from section 3), the ex ante expected performance of fund investors, hedgers, and uninformed investors is given by*

$$r_{\text{MM}}^{\text{F}} = zV + f, \tag{47}$$

$$r_{\text{MM}}^{\text{H}} = zV - 2\lambda\left(\frac{1-\omega}{\omega}\right)f, \quad \textit{and} \tag{48}$$

$$r_{\text{MM}}^{\text{U}} = zV, \tag{49}$$

respectively, where V is given by [17], f by [25], and λ by [27]. In the limit-economy equilibrium without money management (from section 4.2), the ex ante expected performance of hedgers and uninformed investors is given by

$$r_{\text{WO}}^{\text{H}} = r_{\text{WO}}^{\text{U}} = z\hat{V}, \tag{50}$$

where $\hat{V}$ is given by [41].

A comparison of [49] with [50] immediately reveals that uninformed traders perform better in an economy with active money management, even though they do not hire the fund manager themselves. The reason is as before: they benefit from some information

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in parentheses in [46] is bounded below by zero and bounded above by one, the right-hand side of [46] can be made arbitrarily large or small using the variables outside of the parentheses.

[16]As discussed in section 4.1, the performance of hedgers could exclude the expected payoff from their nontradable endowment shock but, since the expected payoff of this shock is zero, it does not affect expected performance.

about θ_n's being used by firms in their real investment decisions. We also see from comparing [47] and [49] that the services of the fund manager improves the performance of fund investors. Although this result is intuitive, it is not obvious, as the fund manager could in theory appropriate the full value that his information creates. The reason he cannot is that, in equilibrium, risk-averse investors must earn extra returns for the risk that their more aggressive portfolios create. Finally, the presence of money management comes with a performance tradeoff for hedgers, just like it did for welfare in section 4.3.

Proposition 8. *In equilibrium, the presence of active money management always improves the expected performance of fund investors and uninformed investors, and it improves the expected performance of hedgers if and only if*

$$\frac{z}{c} > \frac{4}{\sigma_\theta^2}\left(\frac{1-\omega}{\omega}\right)\left[1+\left(\frac{\omega}{\omega+(1-\omega)\lambda}\right)^2\chi\right]\lambda f, \tag{51}$$

where f is given by [25], χ by [26], and λ by [27].

The interpretation of [51] as the various parameters change is similar to that for [46] in section 4.3. One difference is the fact that ω sufficiently close to zero and ω sufficiently close to one both result in hedgers performing better with a money manager. In the former case, this is because a small ω makes ϕ small, and so the adverse impact that hedgers' trades have on prices is small. In the latter case, this is because a large ω makes it appealing for investors to hire the money manager (as seen by the fact that λ is increasing in ω in [27]) and the more informative prices that this leads to allows firms to make better real investment decisions.

4.5. Index Investors

As shown in Lemma 1, all three types of investors seek to hedge their initial position z in each stock. That is, all investors actively optimize their positions in the stock market based on their endowment, risk aversion, and information. In this section, we consider a fourth type of investor who is also endowed with an equally-weighted portfolio of all stocks at the outset, but chooses to remain passive; we refer to these investors as *indexers*. Such investors could easily be introduced into the model of section 2 by assuming that investors must pay some (potentially heterogenous) utility cost for paying attention and actively optimizing their portfolio in the financial market equilibrium.

Because such investors do not make any changes to their initial position, their end-of-period wealth is simply

$$W^{\mathrm{I}} = z \sum_{n=1}^{N} v_n.$$

That is, these investors are endowed with the market portfolio at the outset (since every firm is identical ex ante) and stick to it thereafter. In the next proposition, we are interested in the return performance and welfare of these investors and how they are affected by the presence of active money management, even if they never use it.

Proposition 9. *The expected performance of indexers is always greater in the presence of active money management. The welfare of indexers is greater in the presence of active money management if and only if*

$$\gamma \sigma_\varepsilon^2 < \frac{c}{z}. \tag{52}$$

As the proof of Proposition 9 shows, the expected performance of indexers is identical to that of uninformed investors, with or without money management. The reason is simple. All these investors are endowed with the same market portfolio. While uninformed investors use the financial market to liquidate this position and indexers stick to it, the fact that the market maker is risk-neutral and so on average charges the initial value of the investors' portfolio when they hedge it ensures that the expected final payoffs of all these investors is the same.

The welfare result is more subtle. For one thing, the welfare of indexers is not as great as that of uninformed investors, as the risk that they retain negatively affects their expected utility. In fact, condition [52] says that this risk will be sufficiently small when the endowed market portfolio is dispersed (z is small), when firms find it costly to adjust their investment decisions (c is large), and when the investment decisions of firms does not create too much additional risk through ε_n (σ_ε is small).

5. Conclusion

In this paper, we develop an equilibrium model of money management in which the production decisions of firms are affected by outcomes in financial markets. In essence, the model incorporates a money management component to a rational expectations model of the financial market with real feedback effects. Specifically, a money manager is endowed with the ability to learn about the production process of firms, and sets his fees for managing the money of investors, and for thereby sharing his information with them. Investors, who may or may not have hedging needs, can choose to invest on their own or delegate their investment decisions to the money manager. Indeed, in equilibrium, they are indifferent between doing so and not doing so.

In this context, the model shows two main results. First, the performance of the money manager relative to the market portfolio depends on the extent to which investors delegate their money to him. In particular, because the money manager's portfolio more closely resembles the market portfolio when he acts on behalf of a large fraction of investors, his performance as measured by the excess returns that he generates is decreasing in the number of investors that his fund attracts (and this quantity is itself larger when the fund's fees are small).

Second, while this can lead the net-of-fee performance of the fund to be lower than that of the market portfolio, this does not always equate to a decrease in welfare; in fact, in some cases, all investors are made better off by the presence of money management. This is due to the fact that the manager's information affects the production decision of firms. That is, even though the money manager collects fees, the better production decisions that firms make by gleaning the information that transpires in financial markets from the impact of his trades allows for a more productive and less risky economy. In sum, investors are ultimately getting a smaller share of a bigger pie, and this can work in their favor.

Appendices

A. Proofs

This appendix contains the proofs to all of the paper's results.

Proof of Lemma 1. All three types of investors use their information set $\mathcal{F}_i$, which includes equilibrium prices p_n and nontradable endowments η_n (if trader i has experienced an endowment shock), to calculate

$$\mathbb{E}(W_i \mid \mathcal{F}_i) = \sum_{n=1}^{N} \mathbb{E}\big[(z + x_{in})v_n - x_{in}\, p_n + \tilde{\eta}_{in}\xi_n \,\big|\, \mathcal{F}_i\big] = \sum_{n=1}^{N} \big[(z + x_{in})\, \mathbb{E}(v_n \mid \mathcal{F}_i) - x_{in}\, p_n\big],$$

and

$$\begin{aligned}\mathrm{Var}(W_i \mid \mathcal{F}_i) &= \sum_{n=1}^{N} \mathrm{Var}\big[(z + x_{in})v_n - x_{in}\, p_n + \tilde{\eta}_{in}\xi_n \,\big|\, \mathcal{F}_i\big] \\ &= \sum_{n=1}^{N} \Big[(z + x_{in})^2\, \mathrm{Var}(v_n \mid \mathcal{F}_i) + 2(z + x_{in})\tilde{\eta}_{in}\mathrm{Cov}(v_n, \xi_n \mid \mathcal{F}_i) + \tilde{\eta}_{in}^2 \mathrm{Var}(\xi_n \mid \mathcal{F}_i)\Big].\end{aligned}$$

We can now use these expressions in the maximization problem [9]. The first-order condition yields

$$x_{in} = \frac{\mathbb{E}(v_n \mid \mathcal{F}_i) - p_n - \gamma \tilde{\eta}_{in} \mathrm{Cov}(v_n, \xi_n \mid \mathcal{F}_i)}{\gamma \mathrm{Var}(v_n \mid \mathcal{F}_i)} - z. \quad \text{[A1]}$$

Let us first consider the fund investors and hedgers. Fund investors know the profitability parameter θ_n of each firm $n = 1, \ldots, N$, as this is the service they purchase from the money manager. Through equilibrium prices and trading activity, this also allows them to infer the demand shocks η_n, $n = 1, \ldots, N$, experienced by the hedgers. Although hedgers do not observe θ_n per se, the fact that they learn $\{\eta_n\}_{n=1}^N$, which is perfectly correlated across hedgers, allows them to infer θ_n from equilibrium prices and trading activity. This implies that these two types of investors effectively have the same information set, $\mathcal{F}_i = \{p_n, x_n, \theta_n, \eta_n\}_{n=1}^N$, which includes the information τ_n learned by each firm n through the trading process and equilibrium prices, and therefore their investment decision k_n. Thus, for these investors, we have

$$\mathbb{E}(v_n \mid \mathcal{F}_i) = k_n \mathbb{E}(\theta_n + \varepsilon_n \mid \mathcal{F}_i) - \tfrac{1}{2} k_n^2 = k_n \theta_n - \tfrac{1}{2} k_n^2,$$

$$\mathrm{Cov}(v_n, \xi_n \mid \mathcal{F}_i) = k_n \mathrm{Cov}(\varepsilon_n, \xi_n \mid \mathcal{F}_i) = k_n \rho \sigma_\varepsilon \sigma_\xi,$$

and

$$\mathrm{Var}(v_n \mid \mathcal{F}_i) = k_n^2 \mathrm{Var}(\theta_n + \varepsilon_n \mid \mathcal{F}_i) = k_n^2 \sigma_\varepsilon^2.$$

Using these expressions (along with $\tilde{\eta}_{in} = 0$ for fund investors and $\tilde{\eta}_{in} = \eta_n$ for hedgers) in [A1] yields [10] and [11]. Finally, since the uninformed investors' information set is the same as that of the market maker and all firms, i.e., $\mathcal{F}_i = \{p_n, x_n\}$, they use $\mathbb{E}(v_n \mid \mathcal{F}_i) = p_n$ (along with $\tilde{\eta}_{in} = 0$) in [A1], which yields [12]. □

Proof of Proposition 1. To show this result, we insert the equilibrium price p_n from [7] into [13], and simplify using [2], [3], [6], and [14]:

$$\begin{aligned}
x_n &= \frac{\left[\omega + (1-\omega)\lambda\right] k_n \left[\theta_n - \mathbb{E}(\theta_n \mid \tau_n)\right] - \omega\, \gamma\, k_n\, \rho\, \sigma_\varepsilon\, \sigma_\xi\, \eta_n}{\gamma\, k_n^2\, \sigma_\varepsilon^2} - z \\
&\overset{[3]}{=} \frac{\left[\omega + (1-\omega)\lambda\right] \left(\theta_n - \frac{\mu}{\sqrt{N}} - \delta \tau_n\right) - \omega\, \gamma\, \rho\, \sigma_\varepsilon\, \sigma_\xi\, \eta_n}{\gamma\, k_n\, \sigma_\varepsilon^2} - z \\
&\overset{[14]}{=} \frac{\left[\omega + (1-\omega)\lambda\right] \left(\theta_n - \frac{\mu}{\sqrt{N}} - \delta \tau_n - \phi \eta_n\right)}{\gamma\, k_n\, \sigma_\varepsilon^2} - z \\
&\overset{[6]}{=} \frac{\left[\omega + (1-\omega)\lambda\right] \left(\theta_n - \frac{\mu}{\sqrt{N}} - \delta \tau_n - \phi \eta_n\right)}{\gamma\, \frac{1}{c} \left(\frac{\mu}{\sqrt{N}} + \delta\, \tau_n\right) \sigma_\varepsilon^2} - z
\end{aligned}$$

$$\overset{[2]}{=} \frac{\left[\omega + (1-\omega)\lambda\right] c\,(1-\delta)\,\tau_n}{\gamma\left(\frac{\mu}{\sqrt{N}} + \delta\,\tau_n\right)\sigma_\varepsilon^2} - z.$$

We can see from this last expression that, as a function of τ_n, $x_n(\tau_n)$ has a discontinuity at $\tau_n = -\frac{\mu}{\delta\sqrt{N}}$. Nevertheless, $x_n(\tau_n)$ is a bijective function since (i) $x_n(\tau_1) > x_n(\tau_2)$ for all $\tau_1 \in \left(-\infty, -\frac{\mu}{\delta\sqrt{N}}\right)$ and $\tau_2 \in \left(-\frac{\mu}{\delta\sqrt{N}}, \infty\right)$, and (ii) $x_n(\tau_n)$ is strictly increasing over the intervals $\left(-\infty, -\frac{\mu}{\delta\sqrt{N}}\right)$ and $\left(-\frac{\mu}{\delta\sqrt{N}}, \infty\right)$:

$$\frac{dx_n}{d\tau_n} = \frac{\left[\omega + (1-\omega)\lambda\right] c\,(1-\delta)\,\frac{\mu}{\sqrt{N}}}{\gamma\left(\frac{\mu}{\sqrt{N}} + \delta\,\tau_n\right)^2 \sigma_\varepsilon^2} > 0.$$

This implies that, for each stock $n \in \{1, \ldots, N\}$, the equilibrium trading activity x_n reveals the signal τ_n to market participants, as initially conjectured. □

***Proof of Proposition* 2.** To calculate the date-0 expected value of a share of stock n, we can take the ex ante expected value of [7]:

$$\begin{aligned}
\mathbb{E}(v_n) = \mathbb{E}(p_n) &= \tfrac{1}{2c}\,\mathbb{E}\left[\left(\tfrac{\mu}{\sqrt{N}} + \delta\tau_n\right)^2\right] = \tfrac{1}{2c}\left[\tfrac{\mu^2}{N} + \tfrac{2\mu\delta}{\sqrt{N}}\,\mathbb{E}(\tau_n) + \delta^2\,\mathbb{E}\left(\tau_n^2\right)\right] \\
&= \tfrac{1}{2c}\left[\tfrac{\mu^2}{N} + \delta^2\,\mathrm{Var}(\tau_n)\right] \overset{[2]}{=} \tfrac{1}{2c}\left[\tfrac{\mu^2}{N} + \tfrac{\delta^2}{N}\left(\sigma_\theta^2 + \phi^2\sigma_\eta^2\right)\right] \overset{[5]}{=} \tfrac{1}{2c}\left(\tfrac{\mu^2}{N} + \tfrac{\delta\sigma_\theta^2}{N}\right) \\
&\overset{[5]}{=} \frac{1}{2cN}\left(\mu^2 + \frac{\sigma_\theta^4}{\sigma_\theta^2 + \phi^2\sigma_\eta^2}\right).
\end{aligned} \qquad \textbf{[A7]}$$

This completes the proof. □

***Proof of Lemma* 2.** Using [8], [10], and [12] in [20], we can write

$$\begin{aligned}
\frac{U^{\mathrm{FM}} - U^{\mathrm{NM}}}{1-\omega} &= \mathbb{E}\left[u\left(\sum_{n=1}^{N}\left[(z + x_n^{\mathrm{F}})(v_n - p_n) + zp_n\right] - f\right)\right] - \mathbb{E}\left[u\left(\sum_{n=1}^{N} zp_n\right)\right] \\
&= -\mathbb{E}\left[\left(e^{\gamma f}\exp\left\{-\gamma\sum_{n=1}^{N}(z + x_n^{\mathrm{F}})(v_n - p_n)\right\} - 1\right)\exp\left\{-\gamma z\sum_{n=1}^{N} p_n\right\}\right] \\
&= -\mathbb{E}\left[\mathbb{E}\left[\left(e^{\gamma f}\exp\left\{-\gamma\sum_{n=1}^{N}(z + x_n^{\mathrm{F}})(v_n - p_n)\right\} - 1\right)\exp\left\{-\gamma z\sum_{n=1}^{N} p_n\right\} \,\Big|\, \{\tau_n\}_{n=1}^{N}\right]\right]
\end{aligned}$$

$$= -\mathbb{E}\left[\left(e^{\gamma f}\,\mathbb{E}\left[\exp\left\{-\gamma\sum_{n=1}^{N}(z+x_n^{\mathrm{F}})(v_n-p_n)\right\}\,\middle|\,\{\tau_n\}_{n=1}^{N}\right]-1\right)\exp\left\{-\gamma z\sum_{n=1}^{N}p_n\right\}\right], \tag{A8}$$

where the last equality comes from the fact that p_n is known conditional on knowing τ_n, as can be seen from [7]. Focusing now on the inside conditional expectation, we have

$$\begin{aligned}
&\mathbb{E}\left[\exp\left\{-\gamma\sum_{n=1}^{N}(z+x_n^{\mathrm{F}})(v_n-p_n)\right\}\,\middle|\,\{\tau_n\}_{n=1}^{N}\right]\\
&\quad= \mathbb{E}\left[\mathbb{E}\left[\exp\left\{-\gamma\sum_{n=1}^{N}(z+x_n^{\mathrm{F}})(v_n-p_n)\right\}\,\middle|\,\{\tau_n,\theta_n\}_{n=1}^{N}\right]\,\middle|\,\{\tau_n\}_{n=1}^{N}\right]\\
&\quad= \mathbb{E}\left[\exp\left\{-\gamma\sum_{n=1}^{N}(z+x_n^{\mathrm{F}})\left[\mathbb{E}(v_n\mid\theta_n)-p_n\right]+\frac{\gamma^2}{2}\sum_{n=1}^{N}(z+x_n^{\mathrm{F}})^2\,k_n^2\,\sigma_\varepsilon^2\right\}\,\middle|\,\{\tau_n\}_{n=1}^{N}\right]\\
&\quad\overset{[1]}{=} \mathbb{E}\left[\exp\left\{-\gamma\sum_{n=1}^{N}(z+x_n^{\mathrm{F}})\left(k_n\theta_n-\tfrac{c}{2}k_n^2-p_n\right)+\frac{\gamma^2}{2}\sum_{n=1}^{N}(z+x_n^{\mathrm{F}})^2\,k_n^2\,\sigma_\varepsilon^2\right\}\,\middle|\,\{\tau_n\}_{n=1}^{N}\right]\\
&\quad\overset{[10]}{=} \mathbb{E}\left[\exp\left\{-\sum_{n=1}^{N}\frac{k_n\theta_n-\frac{c}{2}k_n^2-p_n}{k_n^2\,\sigma_\varepsilon^2}\left(k_n\theta_n-\tfrac{c}{2}k_n^2-p_n\right)\right.\right.\\
&\qquad\qquad\left.\left.+\frac{1}{2}\sum_{n=1}^{N}\frac{\left(k_n\theta_n-\frac{c}{2}k_n^2-p_n\right)^2}{k_n^2\,\sigma_\varepsilon^2}\right\}\,\middle|\,\{\tau_n\}_{n=1}^{N}\right]\\
&\quad= \mathbb{E}\left[\exp\left\{-\frac{1}{2}\sum_{n=1}^{N}\frac{\left(k_n\theta_n-\frac{c}{2}k_n^2-p_n\right)^2}{k_n^2\,\sigma_\varepsilon^2}\right\}\,\middle|\,\{\tau_n\}_{n=1}^{N}\right]\\
&\quad\overset{[7]}{=} \mathbb{E}\left[\exp\left\{-\frac{1}{2}\sum_{n=1}^{N}\frac{\left[k_n\theta_n-k_n\mathbb{E}(\theta_n\mid\tau_n)\right]^2}{k_n^2\,\sigma_\varepsilon^2}\right\}\,\middle|\,\{\tau_n\}_{n=1}^{N}\right]\\
&\quad= \mathbb{E}\left[\exp\left\{-\frac{1}{2}\sum_{n=1}^{N}\frac{\left[\theta_n-\mathbb{E}(\theta_n\mid\tau_n)\right]^2}{\sigma_\varepsilon^2}\right\}\,\middle|\,\{\tau_n\}_{n=1}^{N}\right]
\end{aligned}$$

$$= \prod_{n=1}^{N} \mathbb{E}\left[\exp\left\{ -\frac{\left[\theta_n - \mathbb{E}(\theta_n \mid \tau_n)\right]^2}{2\sigma_\varepsilon^2} \right\} \,\middle|\, \tau_n \right] \overset{[B14]}{=} \prod_{n=1}^{N} \sqrt{\frac{\sigma_\varepsilon^2}{\mathrm{Var}(\theta_n \mid \tau_n) + \sigma_\varepsilon^2}}$$

$$\overset{[4]}{=} \prod_{n=1}^{N} \sqrt{\frac{\sigma_\varepsilon^2}{(1-\delta)\frac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2}} \overset{[5]}{=} \prod_{n=1}^{N} \sqrt{\frac{\sigma_\varepsilon^2}{\frac{\phi^2\sigma_\eta^2}{\sigma_\theta^2+\phi^2\sigma_\eta^2}\frac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2}} = \prod_{n=1}^{N} \sqrt{\frac{(\sigma_\theta^2+\phi^2\sigma_\eta^2)\sigma_\varepsilon^2}{(\sigma_\theta^2+\phi^2\sigma_\eta^2)\sigma_\varepsilon^2 + \frac{1}{N}\phi^2\sigma_\eta^2\sigma_\theta^2}},$$

where we have used Result 2 from Appendix B to calculate the expectation of the exponential of the quadratic term of θ_n. We can now reinsert this last expression into [A8] to get

$$\frac{U^{\mathrm{FM}} - U^{\mathrm{NM}}}{1-\omega} = -\mathbb{E}\left[\left(e^{\gamma f} \prod_{n=1}^{N} \sqrt{\frac{(\sigma_\theta^2+\phi^2\sigma_\eta^2)\sigma_\varepsilon^2}{(\sigma_\theta^2+\phi^2\sigma_\eta^2)\sigma_\varepsilon^2 + \frac{1}{N}\phi^2\sigma_\eta^2\sigma_\theta^2}} - 1 \right) e^{-\gamma e \sum_{n=1}^{N} p_n} \right]$$

$$= \left[1 - e^{\gamma f} \left(\frac{(\sigma_\theta^2+\phi^2\sigma_\eta^2)\sigma_\varepsilon^2}{(\sigma_\theta^2+\phi^2\sigma_\eta^2)\sigma_\varepsilon^2 + \frac{1}{N}\phi^2\sigma_\eta^2\sigma_\theta^2} \right)^{N/2} \right] \mathbb{E}\left[e^{-\gamma e \sum_{n=1}^{N} p_n} \right]. \qquad \textbf{[A9]}$$

It is easy to verify that this last expression is strictly decreasing in λ, as it is strictly increasing in ϕ which, as seen in [14], is itself strictly decreasing in λ. Thus, for a given fee $f > 0$, the equilibrium fraction λ of investors who hire the fund manager is uniquely determined as follows. If the expression in [A9] is positive for $\lambda = 1$, all investors hire the fund manager (i.e., $\lambda = 1$). It is straightforward to verify that this happens if $f \leqslant \underline{f}$, where

$$\underline{f} \equiv \frac{N}{2\gamma} \log\left(1 + \frac{1}{N\sigma_\varepsilon^2 \left(\frac{1}{\underline{\phi}^2\sigma_\eta^2} + \frac{1}{\sigma_\theta^2} \right)} \right) \qquad \textbf{[A10]}$$

and $\underline{\phi} \equiv \omega\gamma\rho\sigma_\varepsilon\sigma_\xi$. If the expression is negative for $\lambda = 0$, no investor hires the fund manger (i.e., $\lambda = 0$). This happens if $f \geqslant \bar{f}$, where

$$\bar{f} \equiv \frac{N}{2\gamma} \log\left(1 + \frac{1}{N\sigma_\varepsilon^2 \left(\frac{1}{\bar{\phi}^2\sigma_\eta^2} + \frac{1}{\sigma_\theta^2} \right)} \right) \qquad \textbf{[A11]}$$

and $\bar{\phi} \equiv \gamma\rho\sigma_\varepsilon\sigma_\xi$. Otherwise, there is an interior solution $\lambda \in (0, 1)$ that makes investors indifferent between hiring and not hiring the fund manager. In this case, λ must make the expression in [A9] equal to zero or, equivalently, it must satisfy [21]. □

***Proof of Proposition* 3.** Note that $\pi(f)$ is strictly concave over $[\underline{f}, \bar{f})$ and that $\pi'(\bar{f}) < 0$.

Thus, if $\pi'(\underline{f}) \geqslant 0$, the optimal fee follows immediately from the first-order condition,

$$0 = \pi'(f) = \omega \left(\frac{\gamma\rho\sigma_\xi\sigma_\eta \left(\frac{1}{2\gamma} - \frac{2f\sigma_\varepsilon^2}{\sigma_\theta^2} \right)}{2\sqrt{\frac{f}{2\gamma} - \frac{f^2\sigma_\varepsilon^2}{\sigma_\theta^2}}} - 1 \right), \quad \text{[A12]}$$

which reduces to the first term from the max function in [25]. If $\pi'(\underline{f}) < 0$, then the f resulting from solving [A12] is smaller than $\underline{f}$. In this case, it is optimal for the manager to set $f = \underline{f}$, as discussed in the paragraph preceding the proposition. Finally, $\underline{f}$ is the f that makes [23] equal to one; it is easy to show that this is equivalent to [26]. □

Proof of Proposition 4. This result follows immediately from substituting [25] into [23]:

$$\begin{aligned}
\lambda &= \min\left\{ \frac{\omega}{1-\omega} \left(\gamma\rho\sigma_\xi\sigma_\eta \sqrt{\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}\left(\frac{2}{1-\sqrt{\frac{1}{1+\chi}}} - 1 \right)} - 1 \right), 1 \right\} \\
&= \min\left\{ \frac{\omega}{1-\omega} \left(\frac{\gamma\rho\sigma_\xi\sigma_\eta\sigma_\varepsilon}{\sigma_\theta} \sqrt{\frac{\sqrt{1+\chi}+1}{\sqrt{1+\chi}-1}} - 1 \right), 1 \right\} \\
&= \min\left\{ \frac{\omega}{1-\omega} \left(\sqrt{\chi\left(\frac{\sqrt{1+\chi}+1}{\sqrt{1+\chi}-1} \right)} - 1 \right), 1 \right\} \\
&= \min\left\{ \frac{\omega}{1-\omega} \sqrt{1+\chi}, 1 \right\}.
\end{aligned}$$

□

Proof of Lemma 3. From the definition of the fund's gross-of-fees alpha in [30], we have

$$\begin{aligned}
\alpha_{\text{Gross}}^{\text{FM}} &= \mathbb{E}\left[\sum_{n=1}^{N} (1-\lambda)(1-\omega)(z + x_n^{\text{F}})(v_n - p_n) \right] \\
&= (1-\lambda)(1-\omega)\,\mathbb{E}\left[\sum_{n=1}^{N} \mathbb{E}\left[\mathbb{E}\left((z + x_n^{\text{F}})(v_n - p_n) \,\middle|\, \theta_n, \tau_n \right) \,\middle|\, \tau_n \right] \right] \\
&= (1-\lambda)(1-\omega)\,\mathbb{E}\left[\sum_{n=1}^{N} \mathbb{E}\left[(z + x_n^{\text{F}}) \left[\mathbb{E}(v_n \mid \theta_n) - p_n \right] \,\middle|\, \tau_n \right] \right]
\end{aligned}$$

$$\stackrel{[1]}{=} (1-\lambda)(1-\omega)\,\mathbb{E}\left[\sum_{n=1}^{N}\mathbb{E}\left[(z+x_n^{\mathrm{F}})\left(k_n\theta_n-\tfrac{c}{2}k_n^2-p_n\right)\,\middle|\,\tau_n\right]\right]$$

$$\stackrel{[10]}{=} (1-\lambda)(1-\omega)\,\mathbb{E}\left[\sum_{n=1}^{N}\mathbb{E}\left[\frac{k_n\theta_n-\frac{c}{2}k_n^2-p_n}{\gamma k_n^2\sigma_\varepsilon^2}\left(k_n\theta_n-\tfrac{c}{2}k_n^2-p_n\right)\,\middle|\,\tau_n\right]\right]$$

$$\stackrel{[7]}{=} (1-\lambda)(1-\omega)\,\mathbb{E}\left[\sum_{n=1}^{N}\mathbb{E}\left[\frac{\left[k_n\theta_n-k_n\mathbb{E}(\theta_n\mid\tau_n)\right]^2}{\gamma k_n^2\sigma_\varepsilon^2}\,\middle|\,\tau_n\right]\right]$$

$$= \frac{(1-\lambda)(1-\omega)}{\gamma\sigma_\varepsilon^2}\,\mathbb{E}\left[\sum_{n=1}^{N}\mathbb{E}\left[\left(\theta_n-\mathbb{E}(\theta_n\mid\tau_n)\right)^2\,\middle|\,\tau_n\right]\right]$$

$$= \frac{(1-\lambda)(1-\omega)}{\gamma\sigma_\varepsilon^2}\,\mathbb{E}\left[\sum_{n=1}^{N}\mathrm{Var}(\theta_n\mid\tau_n)\right]$$

$$\stackrel{[4]}{=} \frac{(1-\lambda)(1-\omega)(1-\delta)\sigma_\theta^2}{\gamma\sigma_\varepsilon^2}.$$

The net-of-fees alpha in [31] is then given by

$$\alpha_{\mathrm{Net}}^{\mathrm{FM}} = \alpha_{\mathrm{Gross}}^{\mathrm{FM}} - (1-\omega)f = (1-\omega)\left(\frac{(1-\lambda)(1-\delta)\sigma_\theta^2}{\gamma\sigma_\varepsilon^2} - f\right).$$

□

Proof of Proposition 5. Since $\lambda \in (0,1]$, it immediately follows from [32] that the fund's gross-of-fees alpha is nonnegative. Its net-of-fees alpha is clearly negative if, in equilibrium, $\lambda = 1$ and hence $f = \underline{f} > 0$. Thus, to derive a (necessary and sufficient) condition for $\alpha_{\mathrm{Net}}^{\mathrm{FM}} < 0$, we can restrict our attention to the case where $\lambda < 1$ and $f > \underline{f}$. In this case, we can use [22] to express the equilibrium fee f as a function of λ:

$$f = \frac{\phi^2\sigma_\eta^2\sigma_\theta^2}{2\gamma\left(\sigma_\theta^2+\phi^2\sigma_\eta^2\right)\sigma_\varepsilon^2} = \frac{(1-\delta)\sigma_\theta^2}{2\gamma\sigma_\varepsilon^2}.$$

Substituting this expression into [33], we can write the condition $\alpha_{\mathrm{Net}}^{\mathrm{FM}} < 0$ as

$$\frac{(1-\lambda)(1-\delta)\sigma_\theta^2}{\gamma\sigma_\varepsilon^2} < \frac{(1-\delta)\sigma_\theta^2}{2\gamma\sigma_\varepsilon^2},$$

or, equivalently, as $\lambda > \frac{1}{2}$. Since $\lambda = \min\left\{\frac{\omega}{1-\omega}\sqrt{1+\chi}, 1\right\}$ in equilibrium (Proposition 4), a necessary and sufficient condition for the fund's net-of-fees alpha to be negative is thus that $\frac{\omega}{1-\omega}\sqrt{1+\chi} > \frac{1}{2}$. □

Proof of Proposition 6. Substituting [37] and [38] into the investor's optimization problem [9], we derive the first-order condition for a maximum as

$$\hat{x}_{in} = -\frac{\rho\,\sigma_\varepsilon\,\sigma_\xi\,\tilde{\eta}_{in}}{\hat{k}_n\left(\frac{\sigma_\theta^2}{N}+\sigma_\varepsilon^2\right)} - z.$$

Setting $\tilde{\eta}_{in} = \eta_n$ for hedgers and $\tilde{\eta}_{in} = 0$ for uninformed investors yields [39] and [40]. The optimal investment level $\hat{k}_n$ and stock price $\hat{p}_n$ of each firm are derived in [35] and [36], respectively. □

Proof of Lemma 4. Let us first consider the expected utility of investors in the presence of a money manager. In equilibrium, the expected utility of fund investors has to be equal to the expected utility of uninformed investors.[17] That is, for equilibrium values of f and λ, the expected utility of these investor types in the limit economy is given by

$$\begin{aligned}
U^{\mathrm{F}}_{\mathrm{MM}} = U^{\mathrm{U}}_{\mathrm{MM}} &= \lim_{N\to\infty}\mathbb{E}\left[u\left(\sum_{n=1}^{N}\left[(z+x_n^{\mathrm{U}})(v_n-p_n)+zp_n\right]\right)\right] \overset{[12]}{=} \lim_{N\to\infty}\mathbb{E}\left[u\left(\sum_{n=1}^{N}zp_n\right)\right] \\
&\overset{[7]}{=} \lim_{N\to\infty} -\mathbb{E}\left[\exp\left\{-\frac{\gamma z}{2c}\sum_{n=1}^{N}\left[\mathbb{E}(\theta_n\mid\tau_n)\right]^2\right\}\right] \\
&= \lim_{N\to\infty} -\prod_{n=1}^{N}\mathbb{E}\left[\exp\left\{-\frac{\gamma z}{2c}\left[\mathbb{E}(\theta_n\mid\tau_n)\right]^2\right\}\right].
\end{aligned}$$

The random variable $\mathbb{E}(\theta_n\mid\tau_n)$ in the above expression is normally distributed with mean $\frac{\mu}{\sqrt{N}}$ and variance $\frac{\delta\sigma_\theta^2}{N}$. Using Result 2 from Appendix B, we can thus write this expression as

$$\begin{aligned}
U^{\mathrm{F}}_{\mathrm{MM}} = U^{\mathrm{U}}_{\mathrm{MM}} &= \lim_{N\to\infty} -\prod_{n=1}^{N}\left(1+\frac{\gamma z\delta\sigma_\theta^2}{cN}\right)^{-\frac{1}{2}}\exp\left\{-\frac{\gamma z\mu^2}{2cN}+\frac{\frac{\gamma^2z^2\mu^2}{c^2N}}{2\left(\frac{N}{\delta\sigma_\theta^2}+\frac{\gamma z}{c}\right)}\right\} \\
&= \lim_{N\to\infty} -\left(1+\frac{\gamma z\delta\sigma_\theta^2}{cN}\right)^{-\frac{N}{2}}\exp\left\{-\frac{\gamma z\mu^2}{2c}+\frac{\gamma^2z^2\mu^2\delta\sigma_\theta^2}{2c(cN+\gamma z\delta\sigma_\theta^2)}\right\} \qquad \textbf{[A13]}
\end{aligned}$$

[17] Even if the fund manager decides to sell his services to all investors (i.e., if $\lambda = 1$ in equilibrium), the fund manager optimally charges a fee f such that investors are indifferent between hiring and not hiring him.

$$= -\exp\left\{-\frac{\gamma z}{2c}\left(\mu^2 + \delta\sigma_\theta^2\right)\right\} \overset{[17]}{=} -\exp\left\{-\gamma z V\right\}.$$

The above derivation shows that the second term in the exponential function in [A13] is irrelevant in the limit economy as $N \to \infty$: Since both the squared mean and the variance of $\mathbb{E}(\theta_n \mid \tau_n)$ are of order $1/N$, the term is also of order $1/N$ when the exponential function is raised to the power of N and hence converges to zero as $N \to \infty$.

The expected utility of hedgers in the presence of a money manager is given by

$$\begin{aligned}
U_{\mathrm{MM}}^{\mathrm{H}} &= \lim_{N\to\infty} \mathbb{E}\left[u\left(\sum_{n=1}^{N} \Big[(z + x_n^{\mathrm{H}})(v_n - p_n) + z p_n + \eta_n \xi_n \Big] \right) \right] \\
&= \lim_{N\to\infty} -\prod_{n=1}^{N} \mathbb{E}\left[\mathbb{E}\left(\exp\left\{ -\gamma \Big[(z + x_n^{\mathrm{H}})(v_n - p_n) + z p_n + \eta_n \xi_n \Big] \right\} \middle| \theta_n, \eta_n \right) \right] \\
&\overset{[7]}{=} \lim_{N\to\infty} -\prod_{n=1}^{N} \mathbb{E}\left[\exp\left\{ -\gamma \Big[(z + x_n^{\mathrm{H}}) k_n [\theta_n - \mathbb{E}(\theta_n \mid \tau_n)] + \frac{z}{2c} [\mathbb{E}(\theta_n \mid \tau_n)]^2 \Big] \right. \right. \\
&\qquad \left. \left. + \frac{\gamma^2}{2} \Big[(z + x_n^{\mathrm{H}})^2 k_n^2 \sigma_\varepsilon^2 + 2(z + x_n^{\mathrm{H}}) \eta_n k_n \rho \sigma_\varepsilon \sigma_\xi + \eta_n^2 \sigma_\xi^2 \Big] \right\} \right] \\
&\overset{[10]}{=} \lim_{N\to\infty} -\prod_{n=1}^{N} \mathbb{E}\left[\exp\left\{ -\frac{[\theta_n - \mathbb{E}(\theta_n \mid \tau_n)]^2}{2\sigma_\varepsilon^2} - \frac{\gamma z}{2c} [\mathbb{E}(\theta_n \mid \tau_n)]^2 \right. \right. \\
&\qquad \left. \left. + \frac{\gamma \rho \sigma_\xi}{\sigma_\varepsilon} [\theta_n - \mathbb{E}(\theta_n \mid \tau_n)] \eta_n + \frac{\gamma^2}{2} (1 - \rho^2) \sigma_\xi^2 \eta_n^2 \right\} \right].
\end{aligned}$$

The random variables $\left(\mathbb{E}(\theta_n \mid \tau_n), \theta_n - \mathbb{E}(\theta_n \mid \tau_n), \eta_n\right)$ in the above expression are jointly normally distributed with mean vector $\left(\frac{\mu}{\sqrt{N}}, 0, 0\right)$ and covariance matrix

$$\Sigma = \frac{1}{N} \begin{pmatrix} \delta\sigma_\theta^2 & \delta(1-\delta)\sigma_\theta^2 - \delta^2\phi^2\sigma_\eta^2 & -\delta\phi\sigma_\eta^2 \\ \delta(1-\delta)\sigma_\theta^2 - \delta^2\phi^2\sigma_\eta^2 & (1-\delta)^2\sigma_\theta^2 + \delta^2\phi^2\sigma_\eta^2 & \delta\phi\sigma_\eta^2 \\ -\delta\phi\sigma_\eta^2 & \delta\phi\sigma_\eta^2 & \sigma_\eta^2 \end{pmatrix}.$$

Using Result 2 from Appendix B, we have

$$U_{\mathrm{MM}}^{\mathrm{H}} = \lim_{N\to\infty} \left| I + \Sigma \begin{pmatrix} \frac{\gamma z}{c} & 0 & -\frac{\gamma\rho\sigma_\xi}{\sigma_\varepsilon} \\ 0 & \frac{1}{\sigma_\varepsilon^2} & 0 \\ -\frac{\gamma\rho\sigma_\xi}{\sigma_\varepsilon} & 0 & -\gamma^2(1-\rho^2)\sigma_\xi^2 \end{pmatrix} \right|^{-\frac{N}{2}} \exp\left\{ -\frac{\gamma z \mu^2}{2c} + \mathcal{O}(1/N) \right\},$$

where I is the 3×3 identity matrix and $\mathcal{O}(1/N)$ is a term of order $1/N$ (because the covariance matrix Σ is of order $1/N$). Taking the limit as $N \to \infty$, we can express the expected utility of hedgers in the limit economy as

$$\begin{aligned}
U_{\text{MM}}^{\text{H}} &= -\exp\left\{-\frac{\gamma z}{2c}\left(\mu^2 + \delta\sigma_\theta^2\right) + \frac{\gamma^2}{2}(1-\rho^2)\sigma_\eta^2\sigma_\xi^2 - \frac{\delta\phi\sigma_\eta^2}{2\sigma_\varepsilon^2}\left(\phi - 2\gamma\rho\sigma_\varepsilon\sigma_\xi\right)\right\} \\
&\overset{[17]}{=} -\exp\left\{-\gamma\left(zV - \frac{\gamma}{2}(1-\rho^2)\sigma_\eta^2\sigma_\xi^2 + \frac{\delta\phi\sigma_\eta^2}{2\gamma\sigma_\varepsilon^2}\left(\phi - 2\gamma\rho\sigma_\varepsilon\sigma_\xi\right)\right)\right\} \\
&\overset{[14]}{=} -\exp\left\{-\gamma\left(zV - \frac{\gamma}{2}(1-\rho^2)\sigma_\eta^2\sigma_\xi^2 + \frac{\delta\phi^2\sigma_\eta^2}{2\gamma\sigma_\varepsilon^2}\left(1 - 2\,\frac{\omega + (1-\omega)\lambda}{\omega}\right)\right)\right\} \\
&\overset{[22]}{=} -\exp\left\{-\gamma\left(zV - \frac{\gamma}{2}(1-\rho^2)\sigma_\eta^2\sigma_\xi^2 - \left(1 + 2\lambda\,\frac{1-\omega}{\omega}\right)f\right)\right\}.
\end{aligned}$$

We next consider the investors' expected utility in the absence of active money management. In the limit economy, the expected utility of uninformed investors is equal to

$$\begin{aligned}
U_{\text{WO}}^{\text{U}} &= \lim_{N\to\infty} \mathbb{E}\left[u\left(\sum_{n=1}^{N}\left[(z + \hat{x}_n^{\text{U}})(v_n - \hat{p}_n) + z\hat{p}_n\right]\right)\right] \overset{[40]}{=} \lim_{N\to\infty} \mathbb{E}\left[u\left(\sum_{n=1}^{N} z\hat{p}_n\right)\right] \\
&\overset{[36]}{=} \lim_{N\to\infty} -\mathbb{E}\left[\exp\left\{-\frac{\gamma z}{2c}\sum_{n=1}^{N}\frac{\mu^2}{N}\right\}\right] = -\exp\left\{-\frac{\gamma z\mu^2}{2c}\right\} \overset{[41]}{=} -\exp\left\{-\gamma z\hat{V}\right\}.
\end{aligned}$$

The expected utility of hedgers in the absence of money management is given by

$$\begin{aligned}
U_{\text{WO}}^{\text{H}} &= \lim_{N\to\infty} \mathbb{E}\left[u\left(\sum_{n=1}^{N}\left[(z + \hat{x}_n^{\text{H}})(v_n - \hat{p}_n) + z\hat{p}_n + \eta_n\xi_n\right]\right)\right] \\
&= \lim_{N\to\infty} -\prod_{n=1}^{N}\mathbb{E}\left[\mathbb{E}\left(\exp\left\{-\gamma\left[(z + \hat{x}_n^{\text{H}})(v_n - \hat{p}_n) + z\hat{p}_n + \eta_n\xi_n\right]\right\}\,\middle|\,\eta_n\right)\right] \\
&\overset{[36]}{=} \lim_{N\to\infty} -\prod_{n=1}^{N}\mathbb{E}\left[\exp\left\{-\frac{\gamma z\mu^2}{2cN} + \frac{\gamma^2}{2}\left[(z + \hat{x}_n^{\text{H}})^2\hat{k}_n^2\left(\frac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2\right)\right.\right.\right.
\end{aligned}$$

$$+ 2(z + \hat{x}_n^{\mathrm{H}})\eta_n \hat{k}_n \rho\sigma_\varepsilon\sigma_\xi + \eta_n^2\sigma_\xi^2 \Big] \Big\} \Big]$$

$$\overset{[39]}{=} \lim_{N\to\infty} - \prod_{n=1}^{N} \mathbb{E}\left[\exp\left\{-\frac{\gamma z\mu^2}{2cN} + \frac{\gamma^2\sigma_\xi^2}{2}\left(1 - \frac{\rho^2\sigma_\varepsilon^2}{\frac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2}\right)\eta_n^2\right\}\right].$$

Using Result 2 from Appendix B, we have

$$U_{\mathrm{WO}}^{\mathrm{H}} = \lim_{N\to\infty} - \prod_{n=1}^{N}\left[1 - \gamma^2\sigma_\xi^2\left(1 - \frac{\rho^2\sigma_\varepsilon^2}{\frac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2}\right)\frac{\sigma_\eta^2}{N}\right]^{-\frac{1}{2}} \exp\left\{-\frac{\gamma z\mu^2}{2cN}\right\}$$

$$= \lim_{N\to\infty} - \left[1 - \frac{\gamma^2\left[\sigma_\theta^2 + (1-\rho^2)N\sigma_\varepsilon^2\right]\sigma_\eta^2\sigma_\xi^2}{N\left(\sigma_\theta^2 + N\sigma_\varepsilon^2\right)}\right]^{-\frac{N}{2}} \exp\left\{-\frac{\gamma z\mu^2}{2c}\right\}$$

$$= -\exp\left\{-\frac{\gamma z\mu^2}{2c} + \frac{\gamma^2}{2}(1-\rho^2)\sigma_\eta^2\sigma_\xi^2\right\} \overset{[41]}{=} -\exp\left\{-\gamma\left(z\hat{V} - \frac{\gamma}{2}(1-\rho^2)\sigma_\eta^2\sigma_\xi^2\right)\right\}.$$

This completes the proof. □

Proof of Proposition 7. Comparing [42] with [44] immediately reveals that $U_{\mathrm{MM}}^{\mathrm{F}} = U_{\mathrm{MM}}^{\mathrm{U}} > U_{\mathrm{WO}}^{\mathrm{U}}$. Thus, the presence of active money management improves the welfare of fund investors and uninformed investors.

Comparing [43] with [45], we find that the presence of active money management makes hedgers better off if and only if

$$U_{\mathrm{MM}}^{\mathrm{H}} > U_{\mathrm{WO}}^{\mathrm{H}} \iff zV - \left(1 + 2\lambda\frac{1-\omega}{\omega}\right)f > z\hat{V}$$

$$\overset{[17],[41]}{\iff} \frac{z}{2c}\left(\frac{\sigma_\theta^4}{\sigma_\theta^2 + \phi^2\sigma_\eta^2}\right) > \left(1 + 2\lambda\frac{1-\omega}{\omega}\right)f$$

Using the equilibrium value of ϕ in [14], we can express this condition as

$$\frac{z}{c} > \frac{2}{\sigma_\theta^2}\left(1 + 2\lambda\frac{1-\omega}{\omega}\right)\left[1 + \left(\frac{\omega}{\omega + (1-\omega)\lambda}\right)^2\chi\right]f,$$

where f is given by [25], χ by [26], and λ by [27]. □

Proof of Lemma 5. We first consider the performance of investors in the presence of a money manager. In the limit economy, the expected return of fund investors is given by

$$
\begin{aligned}
r^{\mathrm{F}}_{\mathrm{MM}} &= \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}(z + x^{\mathrm{F}}_n)(v_n - p_n) + z p_n\right] - f \\
&\overset{[A7]}{=} \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}(z + x^{\mathrm{F}}_n)(v_n - p_n)\right] + \frac{z}{2c}\left(\mu^2 + \frac{\sigma_\theta^4}{\sigma_\theta^2 + \phi^2\sigma_\eta^2}\right) - f \\
&\overset{[17]}{=} \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}(z + x^{\mathrm{F}}_n)(v_n - p_n)\right] + zV - f \\
&\overset{[7]}{=} \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}(z + x^{\mathrm{F}}_n)k_n[\theta_n - \mathbb{E}(\theta_n \mid \tau_n)]\right] + zV - f \\
&\overset{[10]}{=} \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}\frac{[\theta_n - \mathbb{E}(\theta_n \mid \tau_n)]^2}{\gamma\sigma_\varepsilon^2}\right] + zV - f \\
&= \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}\frac{\mathbb{E}\big[[\theta_n - \mathbb{E}(\theta_n \mid \tau_n)]^2 \mid \tau_n\big]}{\gamma\sigma_\varepsilon^2}\right] + zV - f \\
&= \lim_{N\to\infty} \sum_{n=1}^{N}\frac{\mathrm{Var}(\theta_n \mid \tau_n)}{\gamma\sigma_\varepsilon^2} + zV - f \\
&\overset{[4]}{=} \frac{\phi^2\sigma_\eta^2\sigma_\theta^2}{\gamma\sigma_\varepsilon^2(\sigma_\theta^2 + \phi^2\sigma_\eta^2)} + zV - f \\
&\overset{[22]}{=} zV + f.
\end{aligned}
$$

The expected return of hedgers in the presence of active money management is given by

$$
\begin{aligned}
r^{\mathrm{H}}_{\mathrm{MM}} &= \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}(z + x^{\mathrm{H}}_n)(v_n - p_n) + z p_n + \eta_n\xi_n\right] \\
&= \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}(z + x^{\mathrm{H}}_n)(v_n - p_n) + z p_n\right] \\
&\overset{[A7]}{=} \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}(z + x^{\mathrm{H}}_n)(v_n - p_n)\right] + \frac{z}{2c}\left(\mu^2 + \frac{\sigma_\theta^4}{\sigma_\theta^2 + \phi^2\sigma_\eta^2}\right) \\
&\overset{[17]}{=} \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}(z + x^{\mathrm{H}}_n)(v_n - p_n)\right] + zV
\end{aligned}
$$

$$\overset{[7]}{=} \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}(z + x_n^{\mathrm{H}})k_n[\theta_n - \mathbb{E}(\theta_n \mid \tau_n)]\right] + zV$$

$$\overset{[11]}{=} \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N} \frac{[\theta_n - \mathbb{E}(\theta_n \mid \tau_n)]^2}{\gamma\sigma_\varepsilon^2} - \frac{\rho\sigma_\xi}{\sigma_\varepsilon}\,[\theta_n - \mathbb{E}(\theta_n \mid \tau_n)]\,\eta_n\right] + zV$$

$$\overset{[2],[3]}{=} \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N} \frac{\mathbb{E}\big[[\theta_n - \mathbb{E}(\theta_n \mid \tau_n)]^2 \mid \tau_n\big]}{\gamma\sigma_\varepsilon^2} - \frac{\rho\sigma_\xi}{\sigma_\varepsilon}\,\delta\phi\,\mathrm{Var}(\eta_n)\right] + zV$$

$$\overset{[5]}{=} \lim_{N\to\infty} \sum_{n=1}^{N}\left[\frac{\mathrm{Var}(\theta_n \mid \tau_n)}{\gamma\sigma_\varepsilon^2} - \frac{\rho\sigma_\xi\sigma_\theta^2\phi\sigma_\eta^2}{\sigma_\varepsilon(\sigma_\theta^2 + \phi^2\sigma_\eta^2)N}\right] + zV$$

$$\overset{[4]}{=} \frac{\phi^2\sigma_\eta^2\sigma_\theta^2}{\gamma\sigma_\varepsilon^2(\sigma_\theta^2 + \phi^2\sigma_\eta^2)} - \frac{\rho\sigma_\xi\sigma_\theta^2\phi\sigma_\eta^2}{\sigma_\varepsilon(\sigma_\theta^2 + \phi^2\sigma_\eta^2)} + zV$$

$$= \frac{\phi\sigma_\eta^2\sigma_\theta^2(\phi - \gamma\rho\sigma_\varepsilon\sigma_\xi)}{\gamma\sigma_\varepsilon^2(\sigma_\theta^2 + \phi^2\sigma_\eta^2)} + zV$$

$$\overset{[14]}{=} \frac{\phi^2\sigma_\eta^2\sigma_\theta^2}{\gamma\sigma_\varepsilon^2(\sigma_\theta^2 + \phi^2\sigma_\eta^2)}\left(1 - \frac{\omega + (1-\omega)\lambda}{\omega}\right) + zV$$

$$\overset{[22]}{=} -2\lambda\left(\frac{1-\omega}{\omega}\right) f + zV.$$

The expected return of uninformed investors in the presence of active money management is equal to

$$r_{\mathrm{MM}}^{\mathrm{U}} = \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}(z + x_n^{\mathrm{U}})(v_n - p_n) + zp_n\right] \overset{[12]}{=} \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N} zp_n\right]$$

$$\overset{[A7]}{=} \frac{z}{2c}\left(\mu^2 + \frac{\sigma_\theta^4}{\sigma_\theta^2 + \phi^2\sigma_\eta^2}\right) \overset{[17]}{=} zV.$$

Finally, let us consider the investors' expected return without money management. In this case, the expected return of hedgers is equal to that of uninformed investors because

$$r_{\mathrm{WO}}^{\mathrm{H}} = \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}(z + \hat{x}_n^{\mathrm{H}})(v_n - \hat{p}_n) + z\hat{p}_n + \eta_n\xi_n\right]$$

$$= \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N} \mathbb{E}[(z + \hat{x}_n^{\mathrm{H}})(v_n - \hat{p}_n) \mid \eta_n] + z\hat{p}_n\right]$$

$$\overset{[39]}{=} \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}(z+\hat{x}_n^{\mathrm{H}})\mathbb{E}[v_n - \hat{p}_n \mid \eta_n] + z\hat{p}_n\right] \overset{[36]}{=} \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N} z\hat{p}_n\right]$$

$$\overset{[40]}{=} \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N}(z+\hat{x}_n^{\mathrm{U}})(v_n - \hat{p}_n) + z\hat{p}_n\right] = r_{\mathrm{WO}}^{\mathrm{U}}$$

Furthermore, since $\hat{p}_n = \frac{\mu^2}{2cN}$ (Proposition 6), we have

$$r_{\mathrm{WO}}^{\mathrm{H}} = r_{\mathrm{WO}}^{\mathrm{U}} = \lim_{N\to\infty}\sum_{n=1}^{N} z\hat{p}_n = \frac{z\mu^2}{2c} \overset{[41]}{=} z\hat{V}.$$

□

***Proof of Proposition** 8.* Comparing [47] and [49] with [50] immediately reveals that $r_{\mathrm{MM}}^{\mathrm{F}} > r_{\mathrm{MM}}^{\mathrm{U}} > r_{\mathrm{WO}}^{\mathrm{U}}$. Thus, the presence of active money management increases the expected return of fund investors and uninformed investors.

Comparing [48] with [50], we find that the presence of active money management improves the expected performance of hedgers if and only if

$$r_{\mathrm{MM}}^{\mathrm{H}} > r_{\mathrm{WO}}^{\mathrm{H}} \qquad \Longleftrightarrow \qquad zV - 2\lambda\left(\frac{1-\omega}{\omega}\right) f > z\hat{V}$$

$$\overset{[17],[41]}{\Longleftrightarrow} \quad \frac{z}{2c}\left(\frac{\sigma_\theta^4}{\sigma_\theta^2 + \phi^2\sigma_\eta^2}\right) > 2\lambda\left(\frac{1-\omega}{\omega}\right) f.$$

Using the equilibrium value of ϕ in [14], we can express this condition as

$$\frac{z}{c} > \frac{4}{\sigma_\theta^2}\left(\frac{1-\omega}{\omega}\right)\left[1 + \left(\frac{\omega}{\omega + (1-\omega)\lambda}\right)^2 \chi\right]\lambda f,$$

where f is given by [25], χ by [26], and λ by [27]. □

***Proof of Proposition** 9.* The expected return of indexers in the presence of active money management is given by

$$r_{\mathrm{MM}}^{\mathrm{I}} = \lim_{N\to\infty}\mathbb{E}\left[\sum_{n=1}^{N} zv_n\right] = \lim_{N\to\infty}\mathbb{E}\left[\sum_{n=1}^{N} z\mathbb{E}(v_n \mid \tau_n)\right] \overset{[7]}{=} \lim_{N\to\infty}\mathbb{E}\left[\sum_{n=1}^{N} zp_n\right]$$

$$\overset{[A7]}{=} \frac{z}{2c}\left(\mu^2 + \frac{\sigma_\theta^4}{\sigma_\theta^2 + \phi^2\sigma_\eta^2}\right) \overset{[17]}{=} zV = r_{\mathrm{MM}}^{\mathrm{U}}.$$

In the absence of active money management, indexers earn an expected return of

$$r^{\text{I}}_{\text{WO}} = \lim_{N\to\infty} \mathbb{E}\left[\sum_{n=1}^{N} z v_n\right] \overset{[36]}{=} \lim_{N\to\infty} \sum_{n=1}^{N} z\hat{p}_n \overset{[36]}{=} \frac{z\mu^2}{2c} \overset{[41]}{=} z\hat{V} = r^{\text{U}}_{\text{WO}}.$$

Thus, the expected performance of indexers is identical to that of uninformed investors, with or without money management. This implies that indexers earn higher expected returns in an economy with active money management.

The expected utility of indexers in the presence of active money management is given by

$$\begin{aligned}
U^{\text{I}}_{\text{MM}} &= \lim_{N\to\infty} \mathbb{E}\left[u\left(\sum_{n=1}^{N} z v_n\right)\right] = \lim_{N\to\infty} -\prod_{n=1}^{N} \mathbb{E}\left[\exp\left\{-\gamma z v_n\right\}\right] \\
&\overset{[1]}{=} \lim_{N\to\infty} -\prod_{n=1}^{N} \mathbb{E}\left[\mathbb{E}\left(\exp\left\{-\gamma z\left(k_n(\theta_n+\varepsilon_n) - \tfrac{c}{2}k_n^2\right)\right\} \,\Big|\, \tau_n\right)\right] \\
&= \lim_{N\to\infty} -\prod_{n=1}^{N} \mathbb{E}\left[\exp\left\{-\gamma z\left(k_n\mathbb{E}(\theta_n \mid \tau_n) - \tfrac{c}{2}k_n^2\right) + \tfrac{1}{2}\gamma^2 z^2 k_n^2\left(\text{Var}(\theta_n \mid \tau_n) + \sigma_\varepsilon^2\right)\right\}\right] \\
&\overset{[4]}{=} \lim_{N\to\infty} -\prod_{n=1}^{N} \mathbb{E}\left[\exp\left\{-\gamma z\left(k_n\mathbb{E}(\theta_n \mid \tau_n) - \tfrac{c}{2}k_n^2\right) + \tfrac{1}{2}\gamma^2 z^2 k_n^2\left((1-\delta)\tfrac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2\right)\right\}\right] \\
&\overset{[6]}{=} \lim_{N\to\infty} -\prod_{n=1}^{N} \mathbb{E}\left[\exp\left\{\underbrace{-\frac{\gamma z}{2c}\left(1 - \frac{\gamma z}{c}\left((1-\delta)\tfrac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2\right)\right)}_{\equiv a^I}\left[\mathbb{E}(\theta_n \mid \tau_n)\right]^2\right\}\right]
\end{aligned}$$

The random variable $\mathbb{E}(\theta_n \mid \tau_n)$ in the above expression is normally distributed with mean $\frac{\mu}{\sqrt{N}}$ and variance $\frac{\delta\sigma_\theta^2}{N}$. Furthermore, the term $1 - 2a^I\delta\sigma_\theta^2/N$ is positive for N large enough. Thus, we can use Result 2 from Appendix B to write this expression as

$$\begin{aligned}
U^{\text{I}}_{\text{MM}} = \lim_{N\to\infty} -\prod_{n=1}^{N} &\left[1 + \frac{\gamma z}{c}\left(1 - \frac{\gamma z}{c}\left((1-\delta)\tfrac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2\right)\right)\frac{\delta\sigma_\theta^2}{N}\right]^{-\frac{1}{2}} \\
&\times \exp\left\{-\frac{\gamma z}{2c}\left(1 - \frac{\gamma z}{c}\left((1-\delta)\tfrac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2\right)\right)\frac{\mu^2}{N}\right\}
\end{aligned}$$

$$= \lim_{N\to\infty} - \left[1 + \frac{\gamma z}{c}\left(1 - \frac{\gamma z}{c}\left((1-\delta)\tfrac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2\right)\right)\frac{\delta\sigma_\theta^2}{N}\right]^{-\frac{N}{2}}$$

$$\times \exp\left\{-\frac{\gamma z}{2c}\left(1 - \frac{\gamma z}{c}\left((1-\delta)\tfrac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2\right)\right)\mu^2\right\}$$

$$= -\exp\left\{-\frac{\gamma z}{2c}\left(1 - \frac{\gamma z \sigma_\varepsilon^2}{c}\right)\left(\mu^2 + \delta\sigma_\theta^2\right)\right\}$$

$$\overset{[17]}{=} -\exp\left\{-\gamma z V\left(1 - \frac{\gamma z \sigma_\varepsilon^2}{c}\right)\right\}.$$

In the absence of active money management, the expected utility of indexers is given by

$$U_{\mathrm{WO}}^{\mathrm{I}} = \lim_{N\to\infty} \mathbb{E}\left[u\left(\sum_{n=1}^{N} z v_n\right)\right] = \lim_{N\to\infty} - \prod_{n=1}^{N} \mathbb{E}\left[\exp\left\{-\gamma z v_n\right\}\right]$$

$$\overset{[1]}{=} \lim_{N\to\infty} - \prod_{n=1}^{N} \mathbb{E}\left[\exp\left\{-\gamma z\left(\hat{k}_n(\theta_n + \varepsilon_n) - \tfrac{c}{2}\hat{k}_n^2\right)\right\}\right]$$

$$= \lim_{N\to\infty} - \prod_{n=1}^{N} \exp\left\{-\gamma z\left(\hat{k}_n \tfrac{\mu}{\sqrt{N}} - \tfrac{c}{2}\hat{k}_n^2\right) + \tfrac{1}{2}\gamma^2 z^2 k_n^2\left(\tfrac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2\right)\right\}$$

$$\overset{[35]}{=} \lim_{N\to\infty} - \prod_{n=1}^{N} \exp\left\{-\frac{\gamma z \mu^2}{2cN}\left(1 - \frac{\gamma z}{c}\left(\frac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2\right)\right)\right\}$$

$$= \lim_{N\to\infty} -\exp\left\{-\frac{\gamma z \mu^2}{2c}\left(1 - \frac{\gamma z}{c}\left(\frac{\sigma_\theta^2}{N} + \sigma_\varepsilon^2\right)\right)\right\}$$

$$= -\exp\left\{-\frac{\gamma z \mu^2}{2c}\left(1 - \frac{\gamma z \sigma_\varepsilon^2}{c}\right)\right\}$$

$$\overset{[41]}{=} -\exp\left\{-\gamma z \hat{V}\left(1 - \frac{\gamma z \sigma_\varepsilon^2}{c}\right)\right\}.$$

Comparing $U_{\mathrm{MM}}^{\mathrm{I}}$ and $U_{\mathrm{WO}}^{\mathrm{I}}$ (and recalling that $V > \hat{V}$), we find that indexers are better of in an economy with active money management if and only if $1 - \frac{\gamma z \sigma_\varepsilon^2}{c} > 0$ or, equivalently, if and only if $\gamma\sigma_\varepsilon^2 < c/z$. □

B. Multivariate Normal Distribution

In this appendix, we list a number of results about the multivariate normal distribution that are used throughout the paper.

Result 1 (Projection Theorem)**.** *Suppose that X is an $m \times 1$ vector, that Y is an $n \times 1$ random vector, and that they have the following multivariate distribution:*

$$\begin{bmatrix} X \\ Y \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix}, \begin{bmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{bmatrix}\right).$$

Then the distribution of X conditional on Y is

$$X \mid Y \sim \mathcal{N}\left(\mu_x + \Sigma_{xy}\Sigma_{yy}^{-1}(Y - \mu_y), \Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{xy}^{\top}\right).$$

Result 2. *Suppose that $X \in \mathbb{R}^n$ is a normally distributed random vector with mean (vector) μ and covariance matrix Σ. Then, as long as $I - 2\Sigma A$ is positive definite, we have*

$$\begin{aligned} &\mathbb{E}\left[e^{X^\top AX + b^\top X + c}\right] \\ &\quad = |I - 2\Sigma A|^{-\frac{1}{2}} \exp\left\{c + b^\top \mu + \mu^\top A\mu + \tfrac{1}{2}(b + 2A\mu)^\top(\Sigma^{-1} - 2A)^{-1}(b + 2A\mu)\right\}, \end{aligned} \quad \text{[B14]}$$

where A is a symmetric $n \times n$ matrix, b is an $n \times 1$ vector, c is a scalar, and I is the $n \times n$ identity matrix.

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