An introduction to some aspects of the swampland distance conjectures

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Introduction

(Compact) manifolds with special geometric structures play an important role in the construction of quantum gravity vacua.

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Given the above correspondence, they lead to the formulation of many interesting problems at the intersection of geometry and physics.
Plan

Motivation for the swampland program

Low-energy limits and moduli spaces

The distance conjectures

Relation to special holonomy
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This contrasts with the *huge number* of possible vacua, leading to an equally huge number of possible low-energy effective field theories.

**Question**

Is any self-consistent effective field theory the low-energy limit of a consistent quantum gravity theory, for some choice of vacuum?
Landscape vs swampland

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**Question**

How to distinguish the theories forming the landscape from those belonging to the swampland?
The swampland program

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- As swampland criteria are formulated in terms of the EFT itself, they yield *well-defined* mathematical problems.

Conversely, mathematical objects that were developed a priori without relation to physics were found to be powerful tools in the exploration of the swampland program.
Motivation for the swampland program

Low-energy limits and moduli spaces

The distance conjectures

Relation to special holonomy
Energy and length scales

The action of a string $X : (\Sigma^2, h) \to (M^D, g)$ moving in space-time is:

$$S[X] = -\frac{1}{4\pi\alpha'} \int h^{ab} g_{\mu\nu}(X) \frac{\partial X^\mu}{\partial \sigma^a} \frac{\partial X^\nu}{\partial \sigma^b} \sqrt{-h} d\sigma^2.$$
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The parameter $\alpha'$ has dimension $(\text{length})^2 = (\text{mass})^{-2}$. Define:
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Consequence

The massive states in the string spectrum have mass of order $m_s$. Thus at energies $E \ll m_s$ only the massless modes are relevant.
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- $\gamma_{\mu\nu}$ symmetric 2-tensor, $B_{\mu\nu}$ 2-form, $\Phi$ scalar
- $\text{graviton}$, $\text{Ramond-Kalb field}$, $\text{dilaton}$
Low-energy limits and moduli spaces

Low-energy limit

Effective field theory

Low-energy physics is given by a Lagrangian theory of fields which only describes the dynamics of the massless modes.
Low-energy limits and moduli spaces

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In the limit of low energies $E \ll m_s$ and small curvature backgrounds $l_s \ll L$ the effective action is:

$$S_{\text{eff}}[g, B, \Phi] = \frac{1}{2\kappa_0^2} \int_M e^{-2\Phi} \left( R_g - \frac{1}{12} |dB|^2 + 4|d\Phi|^2 \right) d\text{Vol}_g.$$
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Possible string vacua correspond to configurations $(\langle g \rangle, \langle B \rangle, \langle \Phi \rangle)$ that solve the corresponding Euler-Lagrange equations.
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$$g = \langle g \rangle + \delta g, \quad B = \langle B \rangle + \delta B, \quad \Phi = \langle \Phi \rangle + \delta \Phi$$
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\]

After redefinition of the space-time metric \( g \rightarrow \tilde{g} \) to absorb the factor \( e^{-2\Phi} \) the effective action can be put in the form:

\[
S_D = \frac{1}{2\kappa^2} \int_M \left( R_{\tilde{g}} - \frac{1}{12} |dB|^2 - C_D |d\Phi|^2 \right) d\text{Vol}_{\tilde{g}}
\]
Dimensional reduction

To obtain a field theory in 4 dimensions, take $M^D = \mathbb{R}^4 \times Y^{D-4}$ with:

$$\langle g \rangle = \eta^{(4)} \times g_Y, \quad \langle B \rangle \in \mathcal{H}^2(Y), \quad \langle \Phi \rangle \equiv \text{cst.}$$
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The effective action in $\mathbb{R}^4$ is obtained by integrating along $Y$. 

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**Example:** modes generated by $B_{\delta B}(x, y) = P \phi_n(x) \beta_n(y) + \cdots$ where $\beta_n$ is an $L^2$-basis of $\Omega^2(Y)$ with $\Delta_Y \beta_n = \lambda_n \beta_n$, $\|\beta_n\|_{L^2} = 1$ and $d^* \beta_n = 0$.

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**Remark 1:** Kaluza-Klein states. Expansions of $B$ and $\Phi$ yield towers of states with masses determined by the spectrum of $\Delta_Y$. Harmonic forms correspond to massless states.
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each mode $\phi_n$ corresponds to a scalar field of mass $m_n^2 = \lambda_n$.  

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**Remark 2: Kaluza-Klein reduction**

At low energies, the massive modes can be discarded. Only a finite number of massless fields in 4D remain.
Moduli spaces

The massless scalars generated by $B$ correspond to perturbations $\delta B = \sum \phi^k(x) \beta_k(y)$, with $\beta_1, \ldots, \beta_{b^2(Y)}$ harmonic 2-forms on $Y$. 

The moduli spaces are parametrised by the vacuum expectation value of scalar fields. The effective Lagrangian for such variations reads (up to constant):

$$L(\phi^k) = -g^{k \ell} \partial_\mu \phi^k(x) \partial^\mu \phi^\ell(x),$$

with $g^{k \ell} = \langle \beta_k, \beta_\ell \rangle$.

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The effective Lagrangian for such variations reads (up to constant):
\[ \mathcal{L}(\phi^k) = -g_{kl} \partial_\mu \phi^k(x) \partial^\mu \phi^l(x), \quad g_{kl} = \langle \beta_k, \beta_l \rangle_{L^2} \]
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Moduli spaces come naturally equipped with a metric, corresponding to the kinetic term in the effective Lagrangian in 4D.
Motivation for the swampland program

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The distance conjectures

Relation to special holonomy
Infinite distance limits

Conjecture 1

*Moduli spaces of field theories that admit a consistent completion into quantum gravity at high energies have infinite diameter.*
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An interesting problem is to test it for moduli spaces that have a geometric origin (complex structure moduli spaces on CY manifolds, moduli spaces of Kähler forms, moduli spaces of $G_2$-manifolds, etc.).
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▶ Is there always an infinite distance limit?
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- Can such limits be characterised?
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The rest of the distance conjectures are concerned with what happens near an infinite distance limit in such moduli space $M$. 
Volume and curvatures

Conjecture 2

Near the boundary at infinity in $\mathcal{M}$, the scalar curvature becomes non-positive (negative if the dimension is greater than 1).

Remark

There are moduli spaces for which the scalar curvature can take positive values. Thus this is really an asymptotic statement.

A related fact is that the relevant moduli spaces often have finite volume.

Moreover, there are other notions of curvature worth considering (Ricci curvature, sectional and holomorphic sectional curvatures).
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▶ Asymptotic geometry?
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▶ Asymptotic geometry?
▶ What can be said about the total volume, and about the positivity of various curvatures?
Towers of light states

Conjecture 3

As $d(P, P_0) \to \infty$ in $\mathcal{M}$, the EFT breaks down due to the appearance of an infinite tower of particles becoming light, with mass scale

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This means that the EFT cannot be trusted near a point at infinite distance, and corrections have to be taken into account in this limit.
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Understanding these towers of states involves many interesting problems:

- What is the moduli dependence of the eigenvalues of the Laplacian acting on differential forms?
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This means that the EFT cannot be trusted near a point at infinite distance, and corrections have to be taken into account in this limit.

Understanding these towers of states involves many interesting problems:

- What is the moduli dependence of the eigenvalues of the Laplacian acting on differential forms?
- Which homology classes are represented by calibrated submanifolds (e.g. $SLag$ in $CY3$, associatives or co-associatives in $G_2$)?
Motivation for the swampland program

Low-energy limits and moduli spaces

The distance conjectures

Relation to special holonomy
Calabi-Yau threefold compactifications

An important class of string backgrounds are of the form $\mathbb{R}^4 \times X^6$, where the internal manifold is a compact Calabi-Yau threefold

- $X$ complex threefold, Kähler, with trivial canonical bundle.
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According to mirror symmetry, there should be a form of duality between them, on different CY3 $X$ and $\hat{X}$. 
Moduli space of complex structures

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- The local universal family of deformations of complex structures on $X$ is smooth and has dimension $b^{2,1}(X)$. 

Much information on the deformations of $X$ is captured in the variations of the Hodge decomposition:

$$H^3(X; \mathbb{C}) = H^3,0(X) \oplus H^2,1(X) \oplus H^1,2(X) \oplus H^0,3(X).$$

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Support for the swampland conjectures

This period mapping and the general theory of variations of Hodge structures provides a lot of information on the geometry of the moduli space of complex structures on $X$, and many tools in the exploration of the swampland conjectures:

1. There are (partial) characterisations of infinite distance limits in terms of singularities of the period mapping.
2. The moduli spaces of polarized Calabi-Yau threefolds (or $n$-folds, $n \geq 4$) have finite Weil-Petersson volume.
3. The integral of the scalar curvature is also finite.
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The low-energy limit of M-theory is expected to be given by the action:

$$S[g, C] = \frac{1}{2} \int R_g \, d\text{Vol}_g - \frac{1}{4} \int dC \wedge \star dC - \frac{1}{12} \int C \wedge dC \wedge dC$$
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Two bosonic fields:

- $g$ is the space-time metric.
- $C$ is a 3-form. As the action is invariant under $C \to C + dB$ one may impose $d\ast C = 0$. Possible vacuum expectation values $(\langle g \rangle, \langle C \rangle)$ on $M_{11} = \mathbb{R}^4 \times Y^7$ are:
  - $\langle g \rangle = \eta(4) \times g_\phi$, with $\phi$ torsion-free $G_2$-structure on $Y$.
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Complexified moduli spaces of $G_2$-manifolds

The moduli space of torsion-free $G_2$-structures on $Y$ has dimension $b^3(Y)$, and is locally parametrised by $\varphi \mapsto [\varphi] \in H^3(Y)$. Thus the moduli space $\mathcal{M}$ of the vevs $(\langle \varphi \rangle, \langle C \rangle)$ is locally modelled on $H^3(Y) \oplus H^3(Y)$. 

This moduli space has a natural complex structure $J(\eta, \xi) = (-\xi, \eta)$, $\eta, \xi \in H^3(Y)$.

The relevant metric to consider is:

$$\| (\eta, \xi) \|^2 = \| \eta \|^2_{L^2} + \| \xi \|^2_{L^2} \operatorname{Vol}(\varphi), \quad \eta, \xi \in H^3(Y)$$

This metric is Kähler, and admits a global potential given (up to constant) by $\log \operatorname{Vol}(\varphi)$.

Question: Global / asymptotic geometry of $\mathcal{M}$?
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