An introduction to some aspects of the swampland distance conjectures

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January 12, 2023

Introduction

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- *e.g.* string theory on $\mathbb{R}^4 \times CY_3$, M-theory on $\mathbb{R}^4 \times G_2$, etc.

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Given the above correspondence, they lead to the formulation of many interesting problems at the intersection of geometry and physics.

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Question

Is any self-consistent effective field theory the low-energy limit of a consistent quantum gravity theory, for some choice of vacuum?

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Question

How to distinguish the theories forming the landscape from those belonging to the swampland?

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Conversely, mathematical objects that were developed a priori without relation to physics were found to be powerful tools in the exploration of the swampland program.

Motivation for the swampland program

Low-energy limits and moduli spaces

The distance conjectures

Relation to special holonomy

Energy and length scales

The action of a string $X : (\Sigma^2, h) \to (M^D, g)$ moving in space-time is:

$$S[X] = -\frac{1}{4\pi\alpha'} \int h^{ab} g_{\mu\nu}(X) \frac{\partial X^\mu}{\partial \sigma^a} \frac{\partial X^\nu}{\partial \sigma^b} \sqrt{-h} d\sigma^2.$$

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$\gamma_{\mu\nu}$ symmetric 2-tensor,	$B_{\mu\nu}$ 2-form,	Φ scalar
graviton	*Ramond-Kalb field*	*dilaton*

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$$S_{eff}[g, B, \Phi] = \frac{1}{2\kappa_0^2} \int_M e^{-2\Phi} \left(R_g - \frac{1}{12}|dB|^2 + 4|d\Phi|^2 \right) d\,\mathrm{Vol}_g \,.$$

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$$g = \langle g \rangle + \delta g, \quad B = \langle B \rangle + \delta B, \quad \Phi = \langle \Phi \rangle + \delta \Phi$$

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After redefinition of the space-time metric $g \to \widetilde{g}$ to absorb the factor $e^{-2\Phi}$ the effective action can be put in the form:

$$S_D = \frac{1}{2\kappa^2} \int_M \left(R_{\widetilde{g}} - \frac{1}{12}|dB|^2 - C_D|d\Phi|^2 \right) d\,\mathrm{Vol}_{\widetilde{g}}$$

Dimensional reduction

To obtain a field theory in 4 dimensions, take $M^D = \mathbb{R}^4 \times Y^{D-4}$ with:

$$\langle g \rangle = \eta^{(4)} \times g_Y, \quad \langle B \rangle \in \mathcal{H}^2(Y), \quad \langle \Phi \rangle \equiv cst.$$

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Example: modes generated by B

$\delta B(x,y) = \sum \phi_n(x)\beta_n(y) + \cdots$ where β_n is an L^2-basis of $\Omega^2(Y)$ with $\Delta_Y \beta_n = \lambda_n \beta_n$, $\|\beta_n\|_{L^2} = 1$ and $d^*\beta_n = 0$.

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Remark 1: Kaluza-Klein states

Expansions of B and Φ yield towers of states with masses determined by the spectrum of Δ_Y. Harmonic forms correspond to massless states.

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Remark 2: Kaluza-Klein reduction

At low energies, the massive modes can be discarded. Only a finite number of massless fields in 4D remain.

Moduli spaces

The massless scalars generated by B correspond to perturbations $\delta B = \sum \phi^k(x)\beta_k(y)$, with $\beta_1, \ldots, \beta_{b^2(Y)}$ harmonic 2-forms on Y.

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The effective Lagrangian for such variations reads (up to constant):

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Metric

Moduli spaces come naturally equipped with a metric, corresponding to the *kinetic term* in the effective Lagrangian in 4D.

Infinite distance limits

Conjecture 1

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The rest of the distance conjectures are concerned with what happens near an infinite distance limit in such moduli space $\mathcal{M}$.

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- Asymptotic geometry?
- What can be said about the total volume, and about the positivity of various curvatures?

Towers of light states

Conjecture 3

As $d(P, P_0) \to \infty$ in $\mathcal{M}$, the EFT breaks down due to the appearance of an infinite tower of particles becoming light, with mass scale

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- What is the moduli dependence of the eigenvalues of the Laplacian acting on differential forms?
- Which homology classes are represented by calibrated submanifolds (e.g. SLag in CY3, associatives or co-associatives in G_2)?

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Calabi-Yau threefold compactifications

An important class of string backgrounds are of the form $\mathbb{R}^4 \times X^6$, where the internal manifold is a compact Calabi-Yau threefold

- X complex threefold, Kähler, with trivial canonical bundle.

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- The moduli space of complex structures on X (relevant for type IIB).
- The moduli space of complexified Kähler forms (in type IIA).

According to mirror symmetry, there should be a form of duality between them, on different CY3 X and $\widehat{X}$.

Moduli space of complex structures

Let X be a compact CY3 with $\pi_1(X)$ finite, and fix a Kähler class.

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In fact g_{WP} can be expressed as the curvature of the first Hodge bundle.

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- The integral of the scalar curvature is also finite.
- There is also a lot of work using the machinery of asymptotic Hodge theory to study various aspects of the swampland program.

Compactifications on G_2-manifolds

The low-energy limit of M-theory is expected to be given by the action:

$$S[g,C] = \frac{1}{2}\int R_g \, d\,\mathrm{Vol}_g - \frac{1}{4}\int dC \wedge *dC - \frac{1}{12}\int C \wedge dC \wedge dC$$

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- $\langle C \rangle$ harmonic 3-form on Y.

Complexified moduli spaces of G_2-manifolds

The moduli space of torsion-free G_2-structures on Y has dimension $b^3(Y)$, and is locally parametrised by $\varphi \mapsto [\varphi] \in H^3(Y)$. Thus the moduli space $\mathcal{M}$ of the vevs $(\langle\varphi\rangle, \langle C\rangle)$ is locally modelled on $H^3(Y) \oplus H^3(Y)$.

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Question

Global / asymptotic geometry of $\mathcal{M}$?