Simons collaboration meeting, Oxford, January 9-13, 2023.
Algebraic Geometry, Gauge Theory and the Swampland Conjectures

Organizers: Dominic Joyce, Jason Lotay, Sakura Schafer-Nameki

Introduction: The meeting wil cover several topics: some differential geometry and gauge theory, some material on enumerative invariants including Virasoro relations and invariants counting coherent sheaves on Calabi-Yau 4-folds, and a strand on the Swampland Conjectures from String Theory. The conference will happen simultaneously, in the next door lecture theatre, to the ADK60 conference on 'Quivers, Clusters, Moduli and Stability' https://www.cantab.net/users/t.logvinenko/2023-qcms/index.html, in honour of the 60th birthday of Alastair King. Attendees at either conference are welcome to go to the talks at the other (but not to steal each others' biscuits). Six talks will be joint between both conferences. As a result, this Simons meeting will have more of an emphasis on Algebraic Geometry than usual.

Speakers:

- Luis Alvarez Consul (Madrid), joint with ADK60
- Gavin Ball (Wisconsin)
- Arkadij Bojko (ETH Zurich)
- Tom Bridgeland (Sheffield), joint with ADK60
- Oscar Garcia-Prada (Madrid), joint with ADK60
- Thomas Grimm (Utrecht)
- Victoria Hoskins (Radboud)
- Dominic Joyce (Oxford), joint with ADK60
- Thibault Langlais (Oxford)
- Woonam Lim (ETH Zurich)
- Miguel Moreira (ETH Zurich)
- Johannes Nordstrom (Bath)
- Boris Pioline (Paris)
- Jeff Streets (UC Irvine)
- Richard Thomas (Imperial), joint with ADK60
- Matt Turner (Bath)
- Timo Weigand (Hamburg)
- Katrin Wendland (Trinity College Dublin)

Schedule. Lecture Theatre 3 (or Lecture Theatre 2), Mathematical Institute, Oxford.

Time	Monday 9 January	Tuesday 10 January	Wednesday 11 January	Thursday 12 January	Friday 13 January
9.30-11.0	9.30-11.0 room L3 Johannes Nordstrom and Matt Turner	10.0-11.0 room L2 Dominic Joyce	10.0-11.0 room L2 Katrin Wendland	9.30-11.0 room L3 Thibault Langlais	9.30-11.0 room L3 Jeff Streets
11.0-11.30	11.0-11.30 tea break	11.0-11.30 tea break	11.0-11.30 tea break	11.0-11.30 tea break	11.0-11.30 tea break
11.30-12.30	11.30-12.30 room L3 Gavin Ball	11.30-12.30 room L2 Richard Thomas	11.30-12.30 room L3 Arkadij Bojko	11.30-12.30 room L3 Thomas Grimm	11.30-12.30 room L3 Geometric flows discussion (Jason Lotay to lead)
12.30-2.0	12.30-2.0 lunch	12.30-2.15 lunch	12.30-2.0 lunch	12.30-2.0 lunch	12.30- lunch, further discussions, departure.
2.0-3.0	2.0-3.0 room L3 Victoria Hoskins	2.15-3.15 room L2 Luis Alvarez-Consul	2.0-3.30 room L3 Woonam Lim and Miguel Moreira	2.0-3.0 room L3 Timo Weigand	
3.0-4.0	3.0-4.0 room L3 TBA	3.15-3.45 tea break		3.0-3.30 Tea break	
4.0-4.30	4.0-4.30 Tea break	3.45-4.45 room L2 Oscar Garcia-Prada	3.30-4.0 Tea break	3.30-4.30 room L3 Swampland discussion (Sakura Schafer-Nameki to lead)	
4.30-5.30	4.30-5.30 room L3 Differential Geometry and gauge theory discussion (Thomas Walpuski to lead)	4.45-5.0 break	4.0-5.30 room L3 Boris Pioline		
		5.0-6.0 room L2 Tom Bridgeland			
Evening			Conference dinner, Balliol College (by invitation only) 7.15 Drinks reception 8.0 dinner		

Practical arrangements.

- The conference will take place in Lecture Theatre 3 of the basement of the Mathematical Institute, Andrew Wiles Building, Radcliffe Observatory Quarter, Oxford, OX2 6GG, except for the lectures on Tuesday and the first lecture on Wednesday, which will happen in Lecture Theatre 2. See https://www.maths.ox.ac.uk/about-us/travel-maps for how to get here.
- Invited participants will be housed in the Linton Lodge Hotel, see https://www.bestwestern.co.uk/hotels/linton-lodge-hotel-bw-signature-collection-83647.
- Invited participants should arrange their own travel to Oxford, and claim (reasonable) travel costs back from the conference afterwards. Please keep all receipts if you intend to make a claim.
- The most convenient airport for Oxford is London Heathrow. A good way to get to and from the airport is to use a bus service called the Oxford Airline, see https://www.theairlineoxford.co.uk/. You can buy tickets on the bus – no need to do this in advance – and it is much cheaper to buy a 'period return' ticket rather than two single tickets. It is also possible to do the journey by train (go via the Elizabeth line), provided the trains are not on strike and have not melted down (e.g. wrong kind of rain, leaf on line).
- It is an easy walk to the Maths Institute from all of Linton Lodge, the bus station, and the train station.
- There is a cafe in the basement of the Mathematical Institute where you can buy your own lunch. There are also plenty of local shops and restaurants.
- Academics and PhD students at the Oxford Mathematical Institute or Physics department are welcome to attend the conference talks, no need to ask the organizers. Attendees at the ADK60 conference are also welcome to attend the conference talks.
- The conference is primarily an activity of the Simons Collaboration on Special Holonomy in Geometry, Analysis and Physics, https://sites.duke.edu/scshgap/, put on for the benefit of its members, and is not an open invitation conference. We do not have funding for inviting non-collaboration members, except invited speakers.
- Non-Oxford people who have not been invited to the conference, but would like to attend, should contact the organizers. We are not offering funding for this, nor to arrange travel or accommodation, this would be your problem. One limiting factor is the capacity of the lecture theatre.

Titles and abstracts.

Luis Alvarez-Consul

Title: Embedding superconformal vertex algebras from Killing spinors and (0,2) mirror symmetry

Abstract: I will construct embeddings of the N=2 superconformal vertex algebra under appropriate conditions inspired by the Killing spinor equations in supergravity. Firstly, when these equations are formulated on a quadratic Lie algebra, they become purely algebraic conditions that induce an embedding in the corresponding superaffine vertex algebra. Secondly, when they are formulated on a suitable class of Courant algebroids, they induce an embedding in the vertex algebra of global sections of the corresponding chiral de Rham complex. As an application, I will present an example of (0,2) mirror symmetry given by pairs of homogeneous Hopf surfaces equipped with a Bismut-flat pluriclosed metric. Joint work with Andoni De Arriba De La Hera and Mario Garcia-Fernandez (arxiv:2012.01851, to appear in IMRN, and further work in progress).

Gavin Ball

Title: Associative submanifolds of squashed 3-Sasakian manifolds

Abstract: I will describe work (joint with Jesse Madnick) on the study of associative submanifolds of 3-Sasakian 7-manifolds endowed with their canonical 1-parameter family of coclosed G_2-structures, with particular focus on the nearly parallel case. Associative submanifolds in this setting have first-order invariants and these invariants may be used to define certain subclasses of associatives. One of these subclasses turns out to consist of ruled associative submanifolds and these will be shown to be in correspondence with pseudo-holomorphic curves in an auxilliary 8-manifold. This general result will be applied to the cases of the squashed 7-sphere and the exceptional Aloff-Wallach space to prove the existence of infinitely many non-trivial associative submanifolds in these spaces.

Arkadij Bojko

Title: Wall-crossing for Calabi-Yau fourfolds and applications

Abstract: To count SU(4)-instantons on Calabi-Yau fourfolds, Borisov-Joyce and later on Oh-Thomas formulated a theory which counts Gieseker stable sheaves. An interesting question is what happens when one changes the stability condition and crosses walls where sheaves get destabilized. Relying on the ideas introduced by D. Joyce, I will explain a formulation of wall-crossing that is particularly useful in solving existing conjectures by discussing some of its applications.

Tom Bridgeland

Title: Clusters and twistors

Abstract: Given an ADE quiver Q I will explain how to construct a complex manifold Z with a map to $\mathbf{P}^1$ whose fibre over 0 is the stability space of the CY3 Ginzburg algebra of Q, quotiented by spherical twists, and whose general fibre is an etale cover of the cluster Poisson variety of Q. This is joint work with Helge Ruddat.

Oscar Garcia-Prada

Title: Vinberg pairs and Higgs bundles

Abstract: A finite order automorphism of a complex semisimple Lie group determines a cyclic grading of its Lie algebra. Vinberg's theory is concerned with the geometric invariant theory associated to this grading. Important examples include the case of involutions and representations of cyclic quivers. After reviewing some basic facts about Vinberg's theory, in this talk I will discuss about its relation to the geometry of moduli spaces of Higgs bundles over a compact Riemann surface.

Thomas Grimm

Title: Quantum gravity conjectures and asymptotic Hodge theory

Abstract: In this talk I will explain how asymptotic Hodge theory can be used to provide general evidence for some of the quantum gravity conjectures. In particular, I will describe how the orbit theorems of Hodge theory can be used in addressing the so-called Distance Conjecture. For Calabi-Yau manifolds, I will sketch the implied classification of asymptotic regions of the moduli space and comment on the special properties of infinite distance boundaries. I will highlight that the quantum gravity conjectures can actually lead to new mathematical theorems by presenting a finiteness theorem generalizing a famous result of Cattani, Deligne, and Kaplan on Hodge classes. This new result is based on work with B. Bakker, C. Schnell, and J. Tsimerman.

Vicky Hoskins

Title: Motivic mirror symmetry for Higgs bundles

Abstract: Moduli spaces of Higgs bundles for Langlands dual groups are conjecturally related by a form of mirror symmetry. For SL_n and PGL_n, Hausel and Thaddeus conjectured a topological mirror symmetry given by an equality of (twisted orbifold) Hodge numbers, which was proven by Groechenig-Wyss-Ziegler and also Maulik-Shen. We lift this to an isomorphism of Voevodsky motives, and thus in particular an equality of (twisted orbifold) rational Chow groups. Our method is based on Maulik and Shen's approach to the Hausel-Thaddeus conjecture, as well as showing certain motives are abelian, in order to use conservativity of the Betti realisation on abelian motives. This is joint work with Simon Pepin Lehalleur.

Dominic Joyce

Title: The structure of invariants counting coherent sheaves on complex surfaces.

Abstract: Let X be a complex projective surface with geometric genus $p_g > 0$. We can form moduli spaces $M_{(r,a,k)}^{\rm st} \subset M_{(r,a,k)}^{\rm ss}$ of Gieseker (semi)stable coherent sheaves on X with Chern character (r,a,k), where we take the rank r to be positive. In the case in which stable = semistable, there is a (reduced) perfect obstruction theory on $M_{(r,a,k)}^{\rm ss}$, giving a virtual class $[M_{(r,a,k)}^{\rm ss}]_{\rm virt}$ in homology.

By integrating universal cohomology classes over this virtual class, one can define enumerative invariants counting semistable coherent sheaves on X. These have been studied by many authors, and include Donaldson invariants, K-theoretic Donaldson invariants, Segre and Verlinde invariants, part of Vafa-Witten invariants, and so on.

In my paper https://arxiv.org/abs/2111.04694, in a more general context, I extended the definition of the virtual class $[M_{(r,a,k)}^{\rm ss}]_{\rm virt}$ to allow strictly semistables, proved wall-crossing formulae for these classes and associated "pair invariants", and gave an algorithm to compute the invariants $[M_{(r,a,k)}^{\rm ss}]_{\rm virt}$ by induction on the rank r, starting from data in rank 1, which is the Seiberg-Witten invariants of X and fundamental classes of Hilbert schemes of points on X. This is an algebro-geometric version of the construction of Donaldson invariants from Seiberg-Witten invariants; it builds on work of Mochizuki 2008.

This talk will report on a project to implement this algorithm, and actually compute the invariants $[M_{(r,a,k)}^{\rm ss}]_{\rm virt}$ for all ranks $r > 0$. I prove that the $[M_{(r,a,k)}^{\rm ss}]_{\rm virt}$ for fixed r and all a,k with a fixed mod r can be encoded in a generating function involving the Seiberg-Witten invariants and universal functions in infinitely many variables. I will spend most of the talk explaining the structure of this generating function, and what we can say about the universal functions, the Galois theory and algebraic numbers involved, and so on. This proves several conjectures in the literature by Lothar Gottsche, Martijn Kool, and others, and tells us, for example, the structure of U(r) and SU(r) Donaldson invariants of surfaces with $b^2_+ > 1$ for any rank $r > 1$.

Thibault Langlais

Title: An introduction to some aspects of the swampland distance conjectures

Abstract: The swampland program aims at distinguishing the quantum field theories which can be consistently coupled to quantum gravity at high energies from those which cannot. It leads to the formulation of many interesting problems at the intersection of geometry and physics. The first part of this talk will be an introduction to the distance conjectures, concerning the moduli spaces of vacua of the theories which admit a consistent quantum gravity completion. In the second part I will present ongoing work on twisted connected sum G_2-manifolds related to the distance conjectures.

Woonam Lim

Title: Virasoro constraints; history and moduli of sheaves

Abstract: Virasoro constraints were first conjectured for the moduli of stable curves (the Witten conjecture) and stable maps. These conjectures provide a set of universal relations among descendent invariants described by a representation of half of the Virasoro algebra. Recently, the analogous constraints were conjectured in several sheaf theoretic contexts. In joint work with A. Bojko and M. Moreira, we provide a unifying viewpoint to Virasoro constraints for general moduli of sheaves and prove the conjecture for torsion-free sheaves on curves and surfaces.

Miguel Moreira

Title: Virasoro constraints: vertex algebras and wall-crossing

Abstract: This talk will be the second one concerning the Virasoro constraints in moduli spaces of sheaves (see Woonam's abstract), based on joint work with A. Bojko and W. Lim. In this talk, I will focus on the connection between Virasoro constraints and the vertex algebra that D. Joyce recently introduced to study wall-crossing. It turns out that this vertex algebra can be endowed with a conformal element that induces the Virasoro operators that had appeared previously in the literature. In this language, our conjectures/results say that moduli of sheaves define physical/primary states in this vertex operator algebra. From this point of view and Joyce's theory, we can prove that the Virasoro constraints are compatible with wall-crossing. This is the main new technical tool that allows to prove the constraints for torsion-free sheaves on curves and surfaces by reducing everything to rank 1.

Johannes Nordstrom and Matt Turner

Title: Examples of asymptotically conical G_2-instanstons

Abstract: We present examples of G_2-instantons with dilation-invariant asymptotics on the "C_7" asymptotically conical G_2-metric on the anticanonical bundle of $\mathbf{CP}^1 \times \mathbf{CP}^1$. The examples have cohomogeneity one which reduces the problem to solving an ordinary differential equation. We find solutions to these equations using a dynamical systems approach. This is joint work with Karsten Matthies.

Boris Pioline

Title: Modularity of BPS indices on Calabi-Yau threefolds

Abstract: Unlike in cases with maximal or half-maximal supersymmetry, the spectrum of BPS states in type II string theory compactified on a Calabi-Yau threefold with generic SU(3) holonomy remains partially understood. Mathematically, the BPS indices coincide with the generalized Donaldson-Thomas invariants associated to the derived category of coherent sheaves, but they are rarely known explicitly. String dualities indicate that suitable generating series of rank 0 Donaldson-Thomas invariants counting D4-D2-D0 bound states should transform as vector-valued mock modular forms, in a precise sense. I will spell out and test these predictions in the case of one-modulus compact Calabi-Yau threefolds such as the quintic hypersurface in P^4, where rank 0 DT invariants can (at least in principle) be computed from Gopakumar-Vafa

invariants, using recent mathematical results by S. Feyzbakhsh and R. Thomas. Work in progress with S. Alexandrov, S. Feyzbakhsh, A. Klemm and T. Schimannek.

Jeff Streets

Title: The generalized Kahler Calabi-Yau problem

Abstract: In recent years generalized Kahler geometry has emerged as a natural extension of Kahler geometry with applications to complex, Poisson, and symplectic geometry, as well as mathematical physics. In this talk I will describe an extension of the Calabi-Yau problem to this setting. I will give a nearly complete picture of the existence and uniqueness of the relevant Calabi-Yau geometries using the generalized Kahler-Ricci flow, and explain a consequence for symplectomorphism groups on hyperKahler manifolds. This is joint work with V. Apostolov, X. Fu, and Y. Ustinovskiy.

Richard Thomas

Title: Counting sheaves and representations of CY4 quivers

Abstract: Borisov-Joyce found a way to define a count of sheaves on Calabi-Yau 4-folds, using real derived differential geometry. I will talk about joint work with Jeongseok Oh which gives a definition within algebraic geometry. To make things more interesting for Alastair I'll say something about the quivery version.

Timo Weigand

Title: Tower Counting for the Weak Gravity Conjecture

Abstract: This talk presents recent advances in our understanding of the Tower Weak Gravity Conjecture (WGC) in string compactifications with minimal supersymmetry. The underlying mathematics involves aspects of the Kahler and enumerative geometry of Calabi-Yau manifolds, in particular modular properties of partition functions of certain D4-D2-D0 bound states. The Tower Weak Gravity Conjecture predicts that any consistent gauge theory coupled to quantum gravity should exhibit an infinite tower of so-called super-extremal particles, i.e. of states whose charge-to-mass ratio exceeds that of an extremal black hole. While BPS states are automatically super-extremal, the Tower WGC is less obvious in those directions in the charge lattice that do not support towers of BPS states.
For time constraints we focus in this talk on M-theory compactifications on Calabi-Yau threefolds, but similar results hold for F-theory compactifications on Calabi-Yau three- or fourfolds. To deduce the presence of super-extremal towers, we first classify all weak coupling limits in M-theory compactifications on Calabi-Yau threefolds, extending an earlier classification of the possible infinite distance limits in the classical Kahler moduli space. We then show that every direction in the charge lattice dual to a gauge group with a weak coupling limit admits a tower of BPS or of superextremal non-BPS states at least asymptotically. To this end we translate the problem into a counting problem for certain D4-D2-

D0 bound states and make use of the modular properties of their partition function and results from Noether-Lefschetz theory. From a physics perspective, the asymptotic Tower WGC can be viewed as a consequence of the Emergent String Conjecture.

Katrin Wendland

Title: An application of folding ADE to BCFG

Abstract: We consider families of Calabi-Yau threefolds which are obtained from the deformation spaces of ADE type surface singularities. For these non-compact Calabi-Yau threefolds, Diaconescu, Donagi and Pantev discovered in 2007 that the associated Calabi-Yau integrable systems agree with the ADE type Hitchin integrable systems. In joint work with Beck and Donagi we show that these integrable systems allow `folding´ by automorphisms of the underlying ADE root systems, and we investigate the corresponding orbifoldings of Calabi-Yau threefolds.