Plan of the Talk

1. Motivation
2. Arithmetics of gauge group topology in F-theory compactifications
3. Constraining 8d gauge group topology with 1-form symmetry
4. Predicting (?) arithmetics of elliptic CY3
Quantum gravity: landscape vs swampland

• Framework of QFT most powerful in effective field theory (EFT) approach. RG-flow: details of UV-theory irrelevant in IR (usually).

• Different with gravity: UV-IR-mixing possible! What are imprints on EFT(s) of quantum gravity?

• Black hole arguments \(\rightarrow\) non-trivial consistency conditions on EFTs: No global symmetries, Completeness Hypothesis, Weak Gravity Conjecture, … culminates in Swampland program [Vafa ’05, Ooguri/Vafa ’06]:

Distinguishing UV-completable EFTs (landscape) from consistent EFTs not coming from UV-complete gravity (swampland)
Swampland principles from String Geometry

• String theory is (conjecturally) a consistent quantum gravity theory, should give consistent EFTs via *compactifications*:

  String theory on $\mathbb{R}^{1,D-1} \times X_d$: EFT in $\mathbb{R}^{1,D-1}$ determined by geometry of $X_d$.

• Geometric constraints not expected from perspective of EFT on $\mathbb{R}^{1,D-1}$. So perhaps compactifications of string theory not omnipotent?

• Counter-proposal: “String Universality”, or “String Lamppost Principle”: *String theory realizes all consistent quantum gravity models.*

• Leitmotif to Swampland program: find physical, *string-independent* conditions reflecting (geometric) constraints in string constructions.
Lessons from geometry

• String Universality for allowed gauge algebras in supergravity: “proof” in 10d [Adams/DeWolfe/Taylor ’10, Kim/Tarazi/Vafa ’19], strong evidence in 9d/8d/7d/6d [Kumar/Taylor ’09, García-Etxebarria/Hayashi/Ohmori/Tachikawa/Yonekura ’17, Montero/Vafa ’20, Cvetič/LL/Turner ’21].

• Recurring theme in EFT interpretation: symmetries and their quantum anomalies.

• “Arithmetics” of $X_d$ oftentimes give “discrete” constraints.

• To better understand these: need *generalized* notion of symmetries.
Generalized global symmetries

Point-like particles charged under ordinary (0-form) symmetries, $S \supset \int \gamma A^{(1)}$; extended objects are charged under higher-form symmetries, $S \supset \int \Sigma A^{(p+1)}$.

[Gaiotto/Kapustin/Seiberg/Willett '14]
Generalized global symmetries

• Pure gauge theories with gauge group $G$ have $Z(G)$ 1-form global symmetry.

• 1-form symmetries can describe (de-)confinement, useful to constrain phase structure at finite temperature, …

• Mixing of higher-form: higher group, non-invertible, categorical symmetries,…

• Also: $G = \tilde{G}/Z$ is equivalent to $\tilde{G}$ with *gaugged* $Z \subset Z(\tilde{G})$ 1-form symmetry.

→ Obstructions to gauging 1-form symmetry $\Leftrightarrow$ global form (topology) of $G$

$\Leftrightarrow$ arithmetics of elliptic Calabi—Yau manifolds.
F-theory in a nutshell

- F-theory: \textit{(compact)} elliptically fibered Calabi—Yau $X_d$\n\hspace{1cm} \iff \text{supergravity in } D = (12 - 2d) \text{ dimensions.}

- Local data (i.e., singular fibers) characterize non-Abelian gauge \textit{algebra} $\mathfrak{g}$.

- Global data (Mordell—Weil group) determines gauge \textit{group} $G = \tilde{G}/\mathbb{Z}$ \cite{Aspinwall/Morrison '98, Mayrhofer/Morrison/Till/Weigand '14, Cvetič/LL '17}.

- In the following, focus on torsional part of MW-group.
**Gauge group topology in F-theory**

(for simplicity, let $X$ have only $I_{n_i}$ fibers in codim 1, then $\mathfrak{g} = \bigoplus_i \mathfrak{su}(n_i)$)

- MW-group law $\rightarrow$ addition in $\prod_i \mathbb{Z}_{n_i} = Z(\prod_i SU(n_i)) = Z(\tilde{G})$

- In F-theory: MW-torsion $Z \Rightarrow$ non-Abelian gauge group is $\tilde{G}/Z$. 
Arithmetics of gauge group topologies in 8d F-theory

- In 8d $\mathcal{N}=1$ gauge theory, $\tilde{G}/\mathbb{Z}$ for any $\mathbb{Z}\subset\mathbb{Z}(\tilde{G})$ possible.

- In contrast: $\mathbb{Z}$ limited in F-theory on (compact) elliptic K3 $X$, due to [Miranda/Persson '89]:

Let $R \subset H_2(X, \mathbb{Z})$ be spanned by non-affine nodes of $I_{n_i}$ fibers. Then, $\mathbb{Z} \cong \text{Tor}(\text{MW}) \hookrightarrow (R^*)^*/R$ is isotropic w.r.t. $q(x) = \frac{1}{2}x \cdot x \mod \mathbb{Z}$.

- Consequence: $s \simeq (k_1, k_2, \ldots) \in \mathbb{Z}$ satisfy $\sum_i k_i^2 \frac{n_i - 1}{2n_i} \in \mathbb{Z}$.

- Note: $\mathbb{Z}(\tilde{G}) \cong (R^*)^*/R \cong \mathbb{Z}_{n_1} \times \mathbb{Z}_{n_2} \times \ldots$
\[ \sum_i k_i^2 \frac{n_i - 1}{2n_i} \in \mathbb{Z} \text{ as an anomaly condition…} \]

- \( \tilde{G}/\mathbb{Z} \) has \textit{fractional} instantons ['t Hooft '81]:'
  \[ \int_{\Sigma^4} \frac{1}{4} \text{Tr}(F^2) \equiv \int_{\Sigma^4} c_2(F) \in \frac{1}{2n} \mathbb{Z} \text{ for SU}(n)/\mathbb{Z}_n\text{-bundle with curvature } F. \]

- Equivalently: \( c_2(F) \equiv \alpha_{\tilde{G}} A^{(2)} \cup A^{(2)} \equiv \alpha_{\tilde{G}} (A^{(2)})^2 \mod \mathbb{Z} \), where \( A^{(2)} \) gauge field for \( Z; (A^{(2)})^2 \) integer, but \( \alpha_{\tilde{G}} \) fractional; e.g.; \( \alpha_{SU(n)} = \frac{n-1}{2n} \) [Kapustin/Seiberg '13]

- Gauging \( \mathbb{Z} \) 1-form: sum over \( \tilde{G}/\mathbb{Z} \) bundles in path integral for partition function.
  \( \Leftrightarrow \) sum over all configurations of \( A^{(2)} \).

- Only possible if partition function remains invariant under all (other) gauge symmetries!
\[ \sum_i k_i \frac{2n_i - 1}{2n_i} \in \mathbb{Z} \] as an anomaly condition in 8d

- In 8d $\mathcal{N} = 1$ SYM (only vector multiplet): no restrictions on $Z$ 1-form.
- But with gravity multiplet: $S \supset \int \sum_i c_2(F_i) \wedge B_4$ [Awada/Townsend ’85];
  with gauge field for $Z$ 1-form: $\Delta S \equiv \int \sum_i \alpha_{\tilde{G}_i}(A_i^{(2)})^2 \wedge B_4 \mod \mathbb{Z}$.
- For SUSY: $B_4$ enjoys $U(1)$ gauge symmetry w/ large gauge transformations $B_4 \rightarrow B_4 + b_4$.
- In $A_i^{(2)}$ background, partition function $\int \mathcal{D}[B_4, A^{(2)}, \ldots] \exp(2\pi i S[B_4, A^{(2)}, \ldots])$
  acquires phase $\exp(2\pi i \int \sum_i \alpha_{\tilde{G}_i}(A_i^{(2)})^2 \wedge b_4) \Rightarrow$ anomaly!
\[ \sum_i k_i^2 \frac{n_i - 1}{2n_i} \in \mathbb{Z} \text{ as an anomaly condition in } 8d \]

- Phase \( \exp(2\pi i \int \sum_i \alpha_{\tilde{G}_i}(A_i^{(2)})^2 \wedge b_4) \) in general non-trivial, so gauge group \( [\prod_i \mathbb{Z}(\tilde{G}_i)]/\mathbb{Z} \) inconsistent for generic \( Z \subset \prod_i \mathbb{Z}(\tilde{G}_i) \).

- Necessary consistency condition: no anomaly generated by \( (k_1, k_2, \ldots) \in \mathbb{Z} \).

- Take \( \tilde{G}_i = SU(n_i) \), with \( Z(\tilde{G}_i) = \mathbb{Z}_{n_i} \) and \( \alpha_i = (n_i - 1)/2n_i \). Then, for \( \mathbb{Z} \ell \subset \prod_i \mathbb{Z}_{n_i} \) with generator \( (k_1, k_2, \ldots) \), background field sets \( A_i^{(2)} = k_i A_i^{(2)} \) [Cordova/Freed/Lam/Seiberg ’19].

- Phase becomes:

\[
2\pi \int \sum_i \alpha_{\tilde{G}_i}(A_i^{(2)})^2 \wedge b_4 = 2\pi \left( \sum_i k_i^2 \frac{n_i - 1}{2n_i} \right) \times \int (A^{(2)})^2 \wedge b_4
\]
Allowed gauge groups in 8d $\mathcal{N} = 1$ supergravity

- “Anomalies” of non-$SU$ groups is integer sum of $SU$ groups \cite{Cordova/Freed/Lam/Seiberg ’19}.
- Solutions to $\sum_i k_i^2 n_i - \frac{1}{2n_i} \in \mathbb{Z}$, subject to $\sum_i n_i - 1 = 18$ \cite{Montero/Vafa ’20}, limited.
- E.g.: no $\tilde{G}/\mathbb{Z}_\ell$ with $\ell > 8$ anomaly-free; unique solutions for $\ell = 7,8$:
  \[ SU(7)^3/\mathbb{Z}_7 \text{ and } [SU(8)^2 \times SU(4) \times SU(2)]/\mathbb{Z}_8. \]
- With slight modifications: also makes predictions for rank 10 and 2 theories. Confirmed in string compactifications \cite{Cvetič/Dierigl/LL/Zhang ’21, ’22}.  

\[ \sum n_i - 1 = 18 \]
G/Z realizable in consistent 8d (s)QFT

G/Z consistent with gauged 1-form symmetry + gravity

G/Z realizable in string compactifications to 8d

Anomaly constraint for gauging 1-form symmetry (due to gravity!)

⇒ Geometric constraints in compactification
Compactifications to 6d / arithmetics of elliptic CY3

• In 6d: $c_2(F_i)$ couples to $B_2 \Leftrightarrow$ BPS strings, mechanism also present in non-gravitational theories / SCFTs [Apruzzi/Dierigl/LL ’20] ⇒ constrains global symmetry group [Heckman/Lawrie/LL/Zhang/Zoccarato ’22].

• In compact F-theory models / 6d $\mathcal{N} = (1,0)$ supergravity, there is a similar arithmetic constraint for $\text{Tor}(\text{MW})$:

Let $\pi : X \to B$ be smooth, flat elliptically fibered CY3, with $I_{n_i}$ fibers over $W_i \in H_2(B, \mathbb{Z})$. Let $s \in \text{Tor}(\text{MW})$ meet the $k_i$-th exceptional component of $I_{n_i}$.

Then, for any $D \in H_2(B, \mathbb{Z})$, one must have

$$\sum_i \frac{n_i - 1}{2n_i} k_i^2 \times (W_i \cdot B_2 D) \in \mathbb{Z}.$$
Conclusion

• Geometric constraints in string compactifications are features, not bugs: reflect UV-constraints on EFT description (“Swampland”).

• Can be quantified and sharpened — independent of string theory — using generalized symmetries.
Outlook

• Can generalized symmetries inspire new geometric insights?

• Interplay between geometry and further generalizations of symmetries (higher-groups, non-invertible, …) in quantum gravity or non-perturbative QFT?

Thank you!