Gravity, Geometry, Generalized symmetries

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Plan of the Talk

1. Motivation
2. Arithmetics of gauge group topology in F-theory compactifications
3. Constraining 8d gauge group topology with 1-form symmetry
4. Predicting (?) arithmetics of elliptic CY3

Quantum gravity: landscape vs swampland

- Framework of QFT most powerful in *effective field theory* (EFT) approach. RG-flow: details of UV-theory irrelevant in IR (usually).
- Different with gravity: UV-IR-mixing possible! What are imprints on EFT(s) of quantum gravity?
- Black hole arguments $\Longrightarrow$ non-trivial consistency conditions on EFTs: No global symmetries, Completeness Hypothesis, Weak Gravity Conjecture, … culminates in *Swampland program* [Vafa ’05, Ooguri/Vafa ’06]:

Distinguishing UV-completable EFTs (landscape) from consistent EFTs *not* coming from UV-complete gravity (swampland)

Swampland principles from String Geometry

- String theory is (conjecturally) a consistent quantum gravity theory, should give consistent EFTs via *compactifications*:

  String theory on $\mathbb{R}^{1,D-1} \times X_d$: EFT in $\mathbb{R}^{1,D-1}$ determined by geometry of X_d.

- Geometric constraints not expected from perspective of EFT on $\mathbb{R}^{1,D-1}$.
  So perhaps compactifications of string theory not omnipotent?

- Counter-proposal: “String Universality”, or “String Lamppost Principle”:
  String theory realizes all consistent quantum gravity models.

- Leitmotif to Swampland program: find physical, *string-independent* conditions reflecting (geometric) constraints in string constructions.

Lessons from geometry

- String Universality for allowed gauge algebras in supergravity:
  “proof” in 10d [Adams/DeWolfe/Taylor ’10, Kim/Tarazi/Vafa ’19],
  strong evidence in 9d/8d/7d/6d [Kumar/Taylor ’09, García-Etxebarria/Hayashi/Ohmori/Tachikawa/Yonekura ’17, Montero/Vafa ’20, Cvetič/LL/Turner ’21].

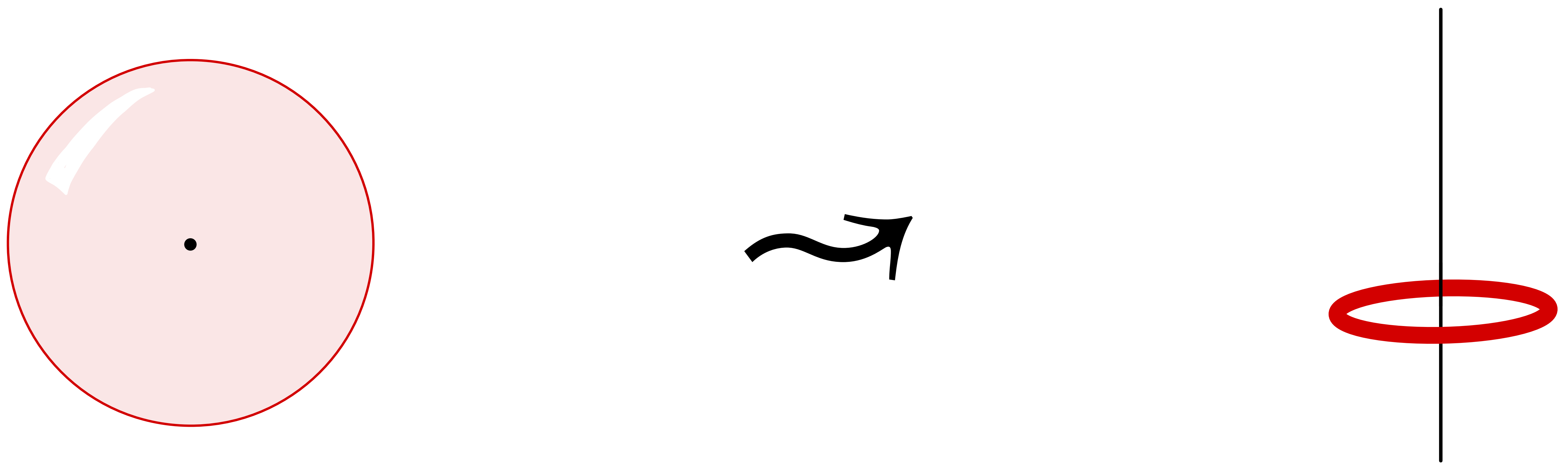
- Recurring theme in EFT interpretation: symmetries and their quantum anomalies.

- “Arithmetics” of X_d oftentimes give “discrete” constraints.

- To better understand these: need *generalized* notion of symmetries.

Generalized global symmetries

Point-like particles charged under ordinary (0-form) symmetries, $S \supset \int_\gamma A^{(1)}$;
extended objects are charged under *higher-form* symmetries, $S \supset \int_\Sigma A^{(p+1)}$.
[Gaiotto/Kapustin/Seiberg/Willett '14]



Generalized global symmetries

- Pure gauge theories with gauge group G have $Z(G)$ 1-form global symmetry.
- 1-form symmetries can describe (de-)confinement, useful to constrain phase structure at finite temperature, …
- Mixing of higher-form: higher group, non-invertible, categorical symmetries,…
- Also: $G = \tilde{G}/Z$ is equivalent to $\tilde{G}$ with *gauged* $Z \subset Z(\tilde{G})$ 1-form symmetry.

➡ Obstructions to gauging 1-form symmetry ⇔ global form (topology) of G

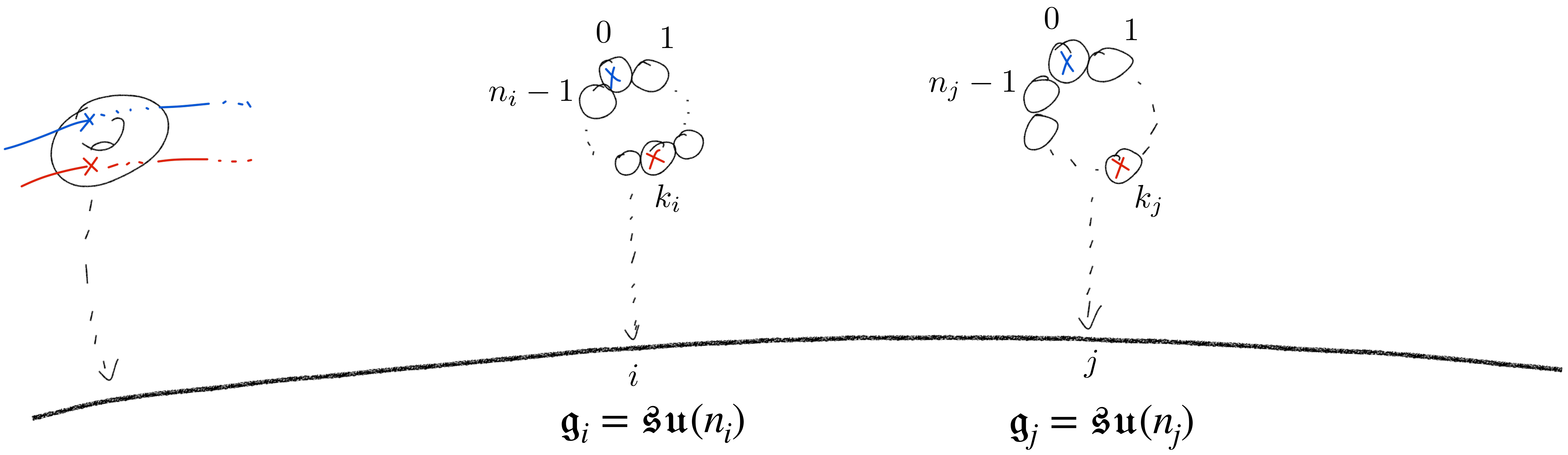
⇔ arithmetics of elliptic Calabi—Yau manifolds.

F-theory in a nutshell

- F-theory: (*compact*) elliptically fibered Calabi—Yau X_d
  $\Longleftrightarrow$ supergravity in $D = (12 - 2d)$ dimensions.
- Local data (i.e., singular fibers) characterize non-Abelian gauge *algebra* $\mathfrak{g}$.
- Global data (Mordell—Weil group) determines gauge *group* $G = \tilde{G}/Z$
  [Aspinwall/Morrison '98, Mayrhofer/Morrison/Till/Weigand '14, Cvetič/LL '17].
- In the following, focus on torsional part of MW-group.

Gauge group topology in F-theory

(for simplicity, let X have only I_{n_i} fibers in codim 1, then $\mathfrak{g} = \bigoplus_i \mathfrak{su}(n_i)$)



- MW-group law → addition in $\prod_i \mathbb{Z}_{n_i} = Z(\prod_i SU(n_i)) = Z(\tilde{G})$
- In F-theory: MW-torsion $Z \Rightarrow$ non-Abelian gauge group is $\tilde{G}/Z$.

Arithmetics of gauge group topologies in 8d F-theory

- In 8d $\mathcal{N} = 1$ gauge theory, $\tilde{G}/Z$ for any $Z \subset Z(\tilde{G})$ possible.

- In contrast: Z limited in F-theory on (compact) elliptic K3 X, due to [Miranda/Persson '89]:

  Let $R \subset H_2(X, \mathbb{Z})$ be spanned by non-affine nodes of I_{n_i} fibers. Then,
  $Z \cong \mathrm{Tor(MW)} \hookrightarrow (R^*)^*/R$ is isotropic w.r.t. $q(x) = \frac{1}{2}x \cdot x \mod \mathbb{Z}$.

- Consequence: $s \simeq (k_1, k_2, \ldots) \in Z$ satisfy $\sum_i k_i^2 \frac{n_i - 1}{2n_i} \in \mathbb{Z}$.

- Note: $Z(\tilde{G}) \cong (R^*)^*/R \cong \mathbb{Z}_{n_1} \times \mathbb{Z}_{n_2} \times \ldots$

$\sum_i k_i^2 \frac{n_i - 1}{2n_i} \in \mathbb{Z}$ as an anomaly condition…

- $\tilde{G}/Z$ has *fractional* instantons ['t Hooft '81]:
  $\int_{\Sigma_4} \frac{1}{4}\mathrm{Tr}(F^2) \equiv \int_{\Sigma_4} c_2(F) \in \frac{1}{2n}\mathbb{Z}$ for $SU(n)/\mathbb{Z}_n$-bundle with curvature F.

- Equivalently: $c_2(F) \equiv \alpha_{\tilde{G}} A^{(2)} \cup A^{(2)} \equiv \alpha_{\tilde{G}} (A^{(2)})^2 \mod \mathbb{Z}$, where $A^{(2)}$ gauge field for Z; $(A^{(2)})^2$ integer, but $\alpha_{\tilde{G}}$ fractional; e.g.; $\alpha_{SU(n)} = \frac{n-1}{2n}$ [Kapustin/Seiberg '13]

- Gauging Z 1-form: sum over $\tilde{G}/Z$ bundles in path integral for partition function.
  $\Leftrightarrow$ sum over all configurations of $A^{(2)}$.

- Only possible if partition function remains invariant under all (other) gauge symmetries!

$\sum_i k_i^2 \frac{n_i - 1}{2n_i} \in \mathbb{Z}$ as an anomaly condition in 8d

- In 8d $\mathcal{N} = 1$ SYM (only vector multiplet): no restrictions on Z 1-form.
- But with gravity multiplet: $S \supset \int \sum_i c_2(F_i) \wedge B_4$ [Awada/Townsend '85];
  with gauge field for Z 1-form: $\Delta S \equiv \int \sum_i \alpha_{\tilde{G}_i} (A_i^{(2)})^2 \wedge B_4 \mod \mathbb{Z}$.
- For SUSY: B_4 enjoys $U(1)$ *gauge* symmetry w/ large gauge transformations $B_4 \to B_4 + b_4$.
- In $A_i^{(2)}$ background, partition function $\int \mathscr{D}[B_4, A^{(2)}, \ldots] \exp(2\pi i S[B_4, A^{(2)}, \ldots])$
  acquires phase $\exp(2\pi i \int \sum_i \alpha_{\tilde{G}_i} (A_i^{(2)})^2 \wedge b_4) \implies$ anomaly!

$\sum_i k_i^2 \frac{n_i - 1}{2n_i} \in \mathbb{Z}$ as an anomaly condition in 8d

- Phase $\exp(2\pi i \int \sum_i \alpha_{\tilde{G}_i} (A_i^{(2)})^2 \wedge b_4)$ in general non-trivial, so gauge group $[\prod_i \tilde{G}_i]/Z$ inconsistent for generic $Z \subset \prod_i Z(\tilde{G}_i)$.

- Necessary consistency condition: no anomaly generated by $(k_1, k_2, \ldots) \in Z$.

- Take $\tilde{G}_i = SU(n_i)$, with $Z(\tilde{G}_i) = \mathbb{Z}_{n_i}$ and $\alpha_i = (n_i - 1)/2n_i$. Then, for $\mathbb{Z}_\ell \subset \prod_i \mathbb{Z}_{n_i}$ with generator $(k_1, k_2, \ldots)$, background field sets $A_i^{(2)} = k_i A^{(2)}$ [Cordova/Freed/Lam/Seiberg '19].

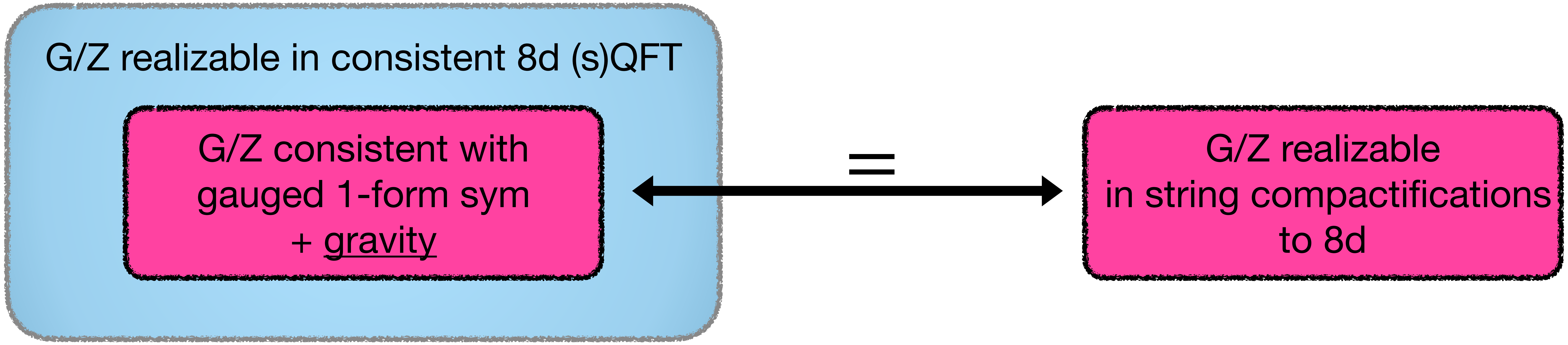
- Phase becomes:

$$2\pi \int \sum_i \alpha_{\tilde{G}_i} (A_i^{(2)})^2 \wedge b_4 = 2\pi \left(\sum_i k_i^2 \frac{n_i - 1}{2n_i} \right) \times \int (A^{(2)})^2 \wedge b_4$$

Allowed gauge groups in 8d $\mathcal{N} = 1$ supergravity

- "Anomalies" of non-SU groups is integer sum of SU groups [Cordova/Freed/Lam/Seiberg '19].
- Solutions to $\sum_i k_i^2 \frac{n_i - 1}{2n_i} \in \mathbb{Z}$, subject to $\sum_i n_i - 1 = 18$ [Montero/Vafa '20], limited.
- E.g.: no $\tilde{G}/\mathbb{Z}_\ell$ with $\ell > 8$ anomaly-free; unique solutions for $\ell = 7{,}8$:

  $SU(7)^3/\mathbb{Z}_7$ and $[SU(8)^2 \times SU(4) \times SU(2)]/\mathbb{Z}_8$.
- With slight modifications: also makes predictions for rank 10 and 2 theories. Confirmed in string compactifications [Cvetič/Dierigl/LL/Zhang '21, '22].



Anomaly constraint for gauging 1-form symmetry (due to gravity!) $\cong$ Geometric constraints in compactification

Compactifications to 6d / arithmetics of elliptic CY3

- In 6d: $c_2(F_i)$ couples to $B_2 \Leftrightarrow$ BPS strings, mechanism also present in non-gravitational theories / SCFTs [Apruzzi/Dierigl/LL '20] $\Rightarrow$ constrains global symmetry *group* [Heckman/Lawrie/LL/Zhang/Zoccarato '22].

- In compact F-theory models / 6d $\mathcal{N} = (1,0)$ supergravity, there is a similar arithmetic constraint for Tor(MW):

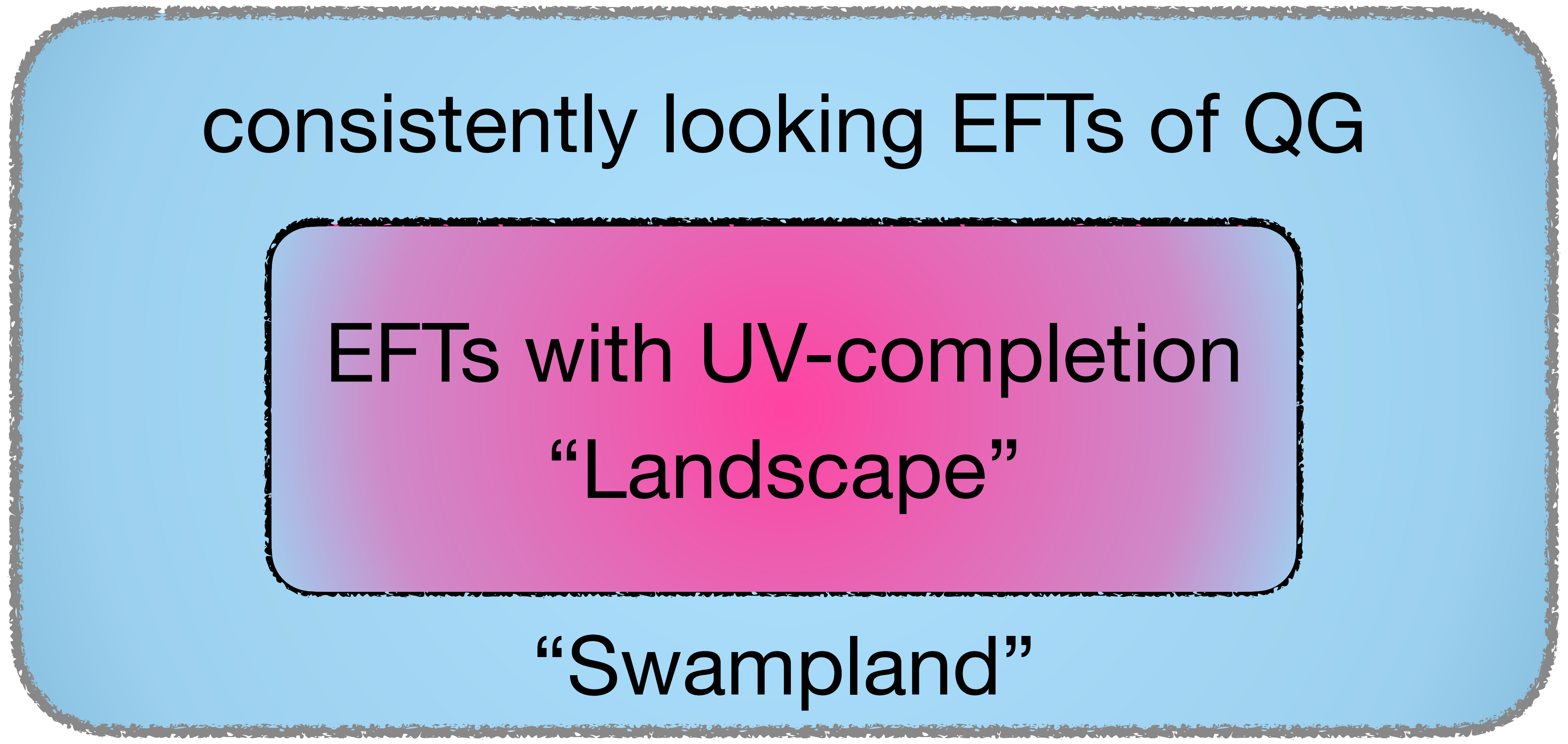
Let $\pi : X \to B$ be smooth, flat elliptically fibered CY3, with I_{n_i} fibers over $W_i \in H_2(B, \mathbb{Z})$. Let $s \in \mathrm{Tor(MW)}$ meet the k_i-th exceptional component of I_{n_i}.

Then, for *any* $D \in H_2(B, \mathbb{Z})$, one must have

$$\sum_i \frac{n_i - 1}{2n_i} k_i^2 \times (W_i \cdot_{B_2} D) \in \mathbb{Z}.$$

Conclusion

- Geometric constraints in string compactifications are features, not bugs: reflect UV-constraints on EFT description (“Swampland”).
- Can be quantified and sharpened — independent of string theory — using generalized symmetries.



Outlook

- Can generalized symmetries inspire new geometric insights?
- Interplay between geometry and further generalizations of symmetries (higher-groups, non-invertible, …) in quantum gravity or non-perturbative QFT?

Thank you!