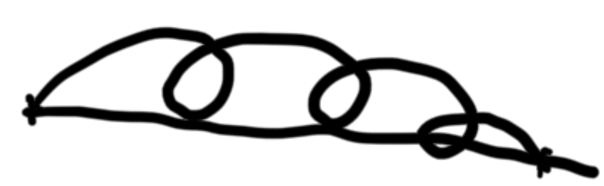
Every finite graph arises as the singular set of 3-d calibrated area-minimizing surface

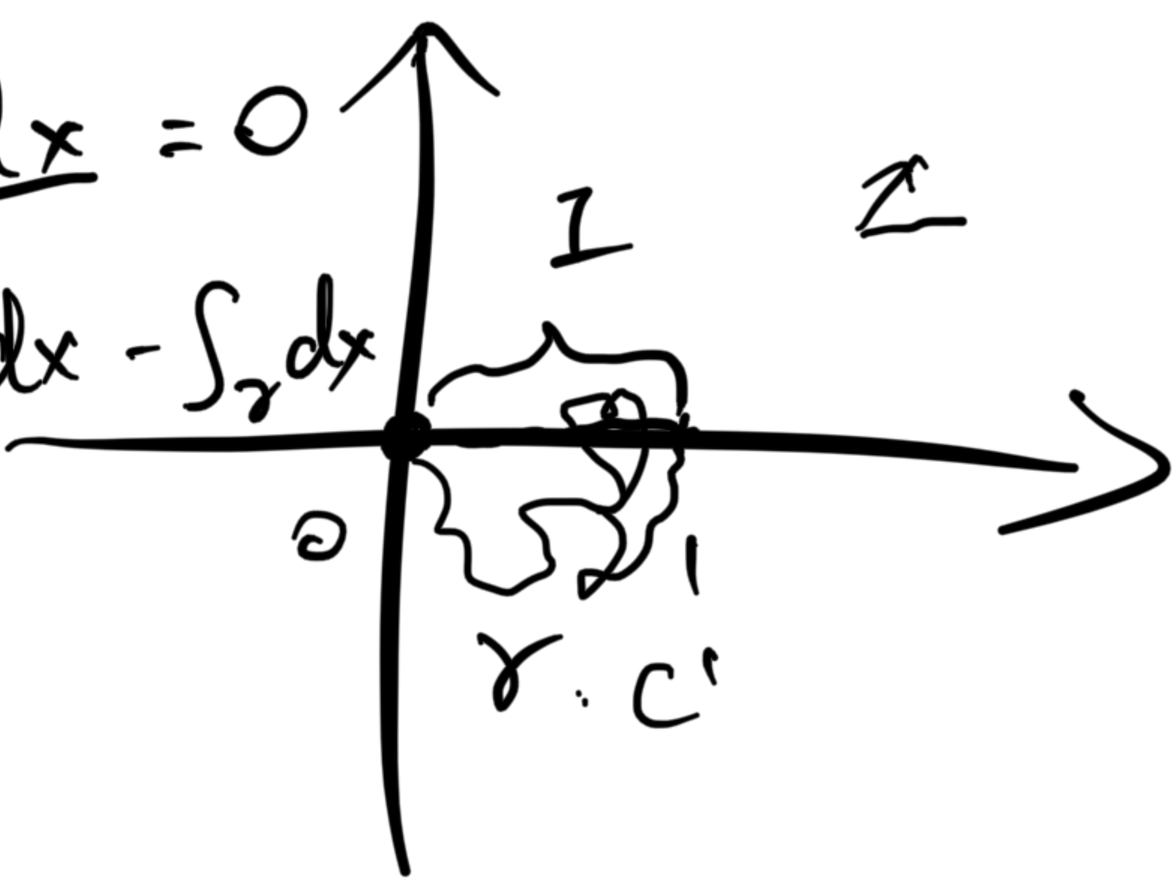
I. Calibration

To prove that straight lines realizes the min dist between points

E-L : stationary 

up to rotations, etc.

$$\int_\Omega d\,dx = 0$$
$$= \int_I dx - \int_\gamma dx$$



$$\partial[I] = \partial[\gamma]$$
$$[I] - [\gamma] = [\Omega]$$
$$|I| = \int_I dx$$
$$= \int_\gamma dx$$
x-comp of unit length tang
$$= \int_\gamma dx(T_p\gamma)\, d|\gamma|$$
$$\leq |\gamma|$$

Harvey - Lawson   Calibrated geo

① k-dim diff form ϕ

closed (Seebo...)

② $-1 \le \phi(\xi) \le 1$ $\quad \xi \in \Lambda_k(\mathbb{R}^n)$ simple,
$\|\xi\| = 1$,

Call ξ that $\phi(\xi) = 1$ the calibrated plane

For surfaces Σ whose oriented tangent planes are the maximum of ϕ, $\phi(T_p\Sigma) = 1$,

Σ minimizes area among in the same homology class

ϕ : $dx_1 dy_1 + dx_2 dy_2 = \omega$

$\omega \in \mathbb{C}^2$

$|\omega(\xi)|^2 + |dz_1 \wedge dz_2(\xi)|^2 = |\xi|^2$

$|\xi|^2 = \sum_{\text{all } \pi} |\pi(\xi)|^2$

ω calibrates holomorphic.

$z_1 = z_2^3 + t$ 

Almgren - De Lellis - Spadaro

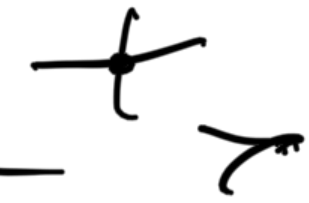
Chang

for oriented area-mini surf Σ^k of dim k, dim sing $\Sigma^k \le k-2$

Hausdorff dim

$k=3$ dim 1

2-dim Σ

$\Sigma \overset{\text{imm}}{\hookrightarrow} M$ 

current

k=2 all integral area-mini ~~integral~~ are minimal branched surf in interior

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$k=3$ by example

$\underline{\Phi} = \mathrm{Re}\, dz_1 \wedge \cdots \wedge dz_n$

calibration

$dx_1dx_2dx_3 - dx_1dy_2dy_3 - dy_1dx_2dy_3 - dy_1dy_2dx_3$

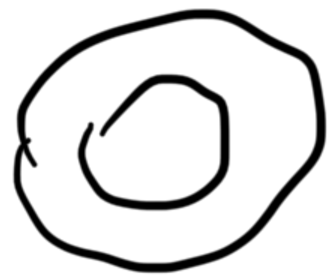
$\mathbb{C}^3$

$\Phi = \mathrm{Re}\, dz_1 \wedge dz_2 \wedge dz_3$ special Lagrangian form

Harvey-Lawson cone

cone over L^2 in S^5

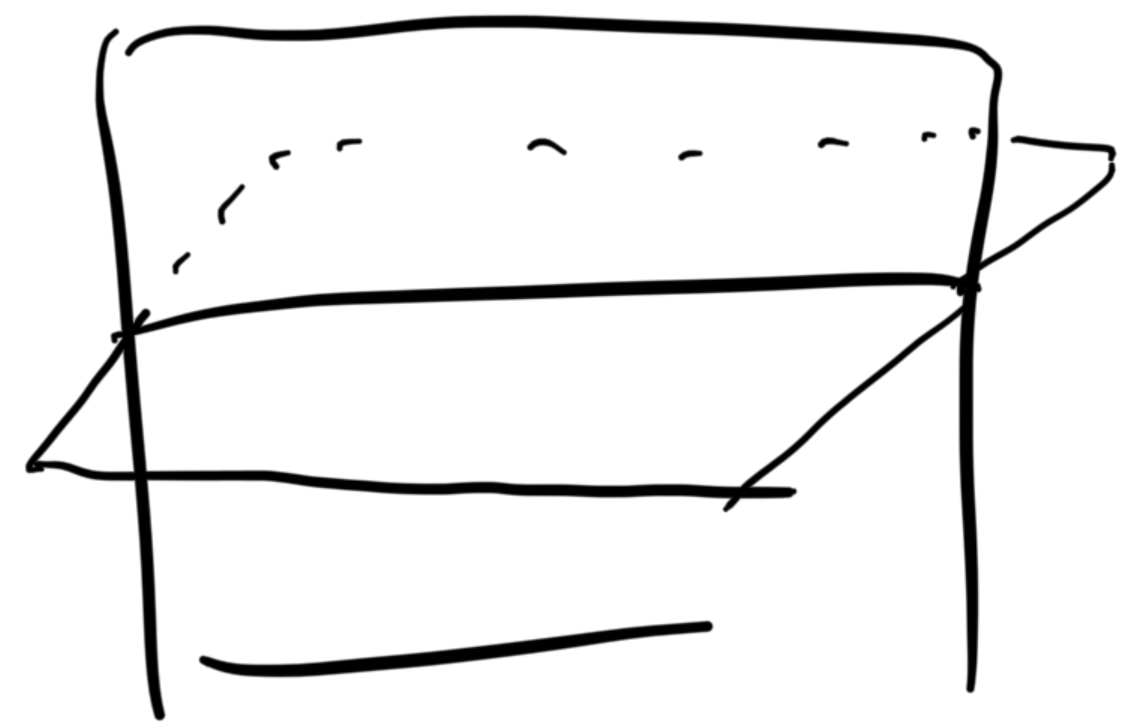
L^2: $\begin{cases} |z_1| = |z_2| = |z_3| = \frac{1}{\sqrt{3}} \\ z_1 z_2 z_3 = \frac{1}{3\sqrt{3}} \end{cases}$



Take cone C over L^2

C is a special Lagrangian

isolated sing



$[x_1\, x_2\, y_3] + [-y_1\, y_2\, x_3]$

lines

holomorphic curve are 2-d special Lag up to change of coor

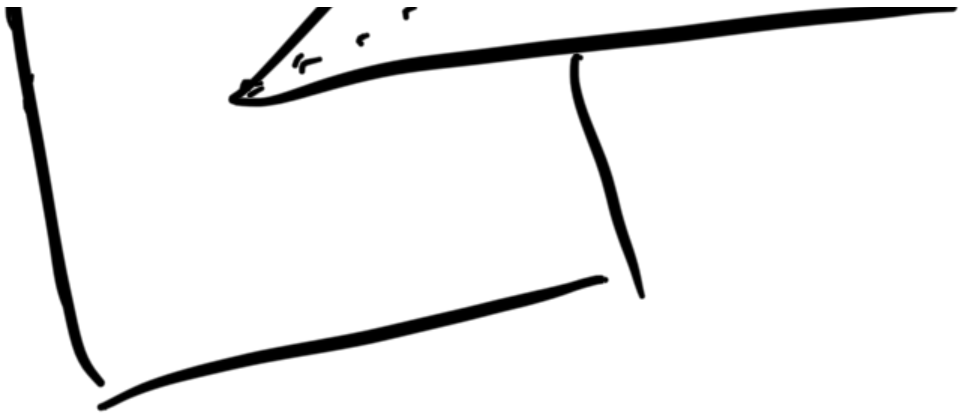
holo curve x_3-axis

line

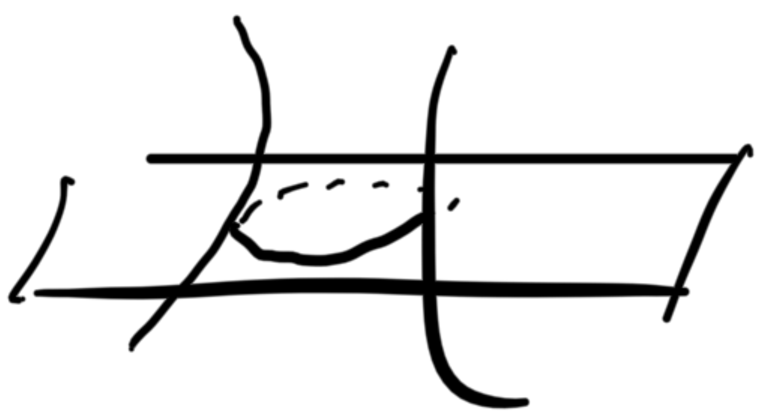
P C

$[P] + [C]$

ray

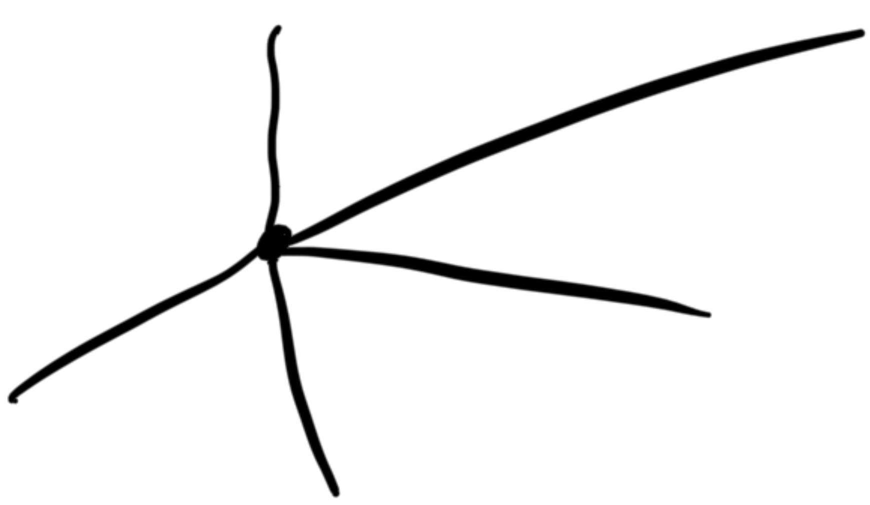


Robert Bryant $\forall$ analytic complete curve γ in a plane. $\exists$ a special with γ as singular

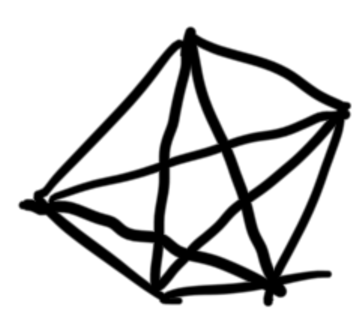


emb

Every finite graph arises as sing set of 3-d



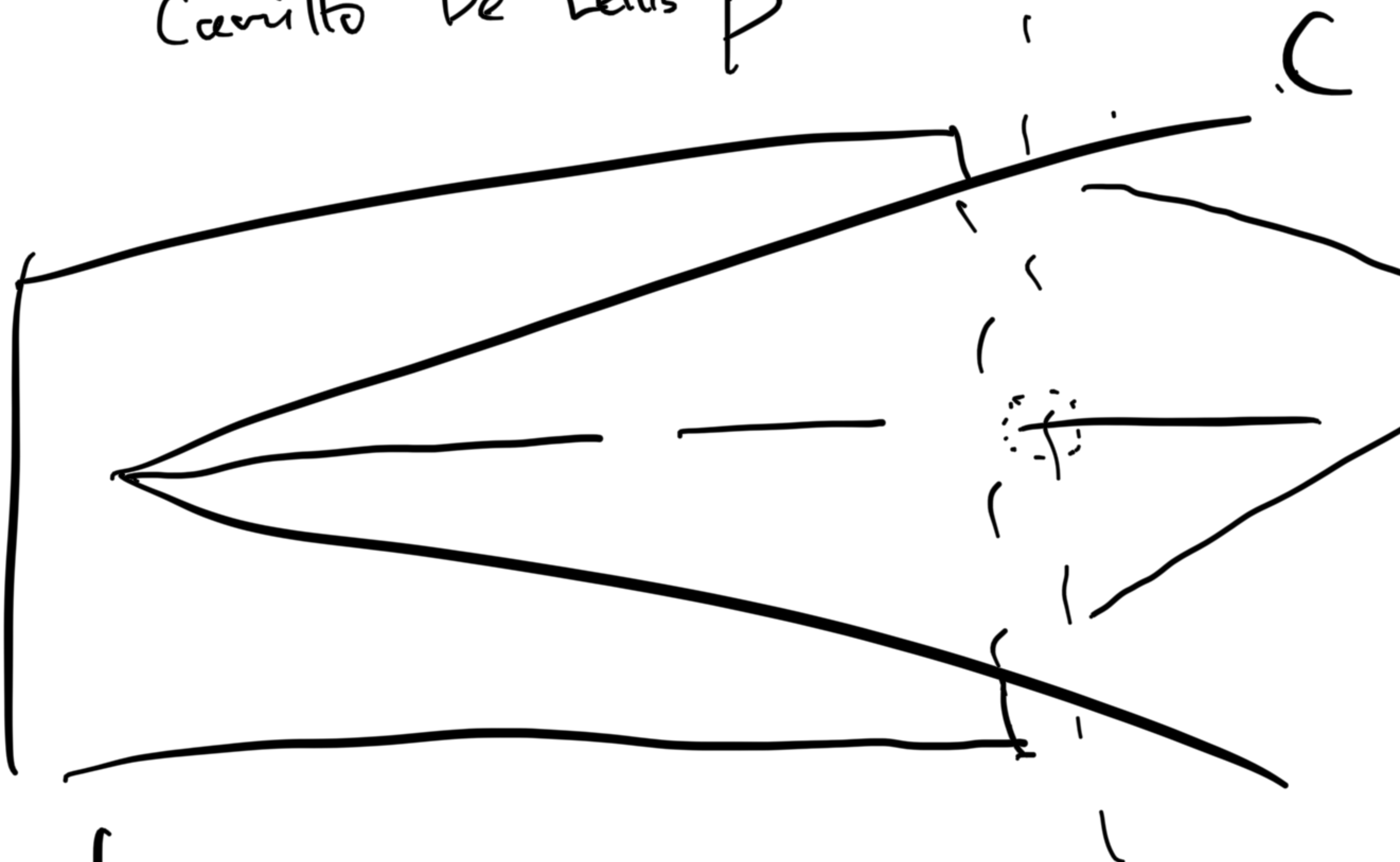
$\exists$ finite graph cannot be embed into $\mathbb{R}^2$



Leon Simon for codim 1 stationary stable hypersurface. Cantor sets can arise on smooth manifold as sing

sing

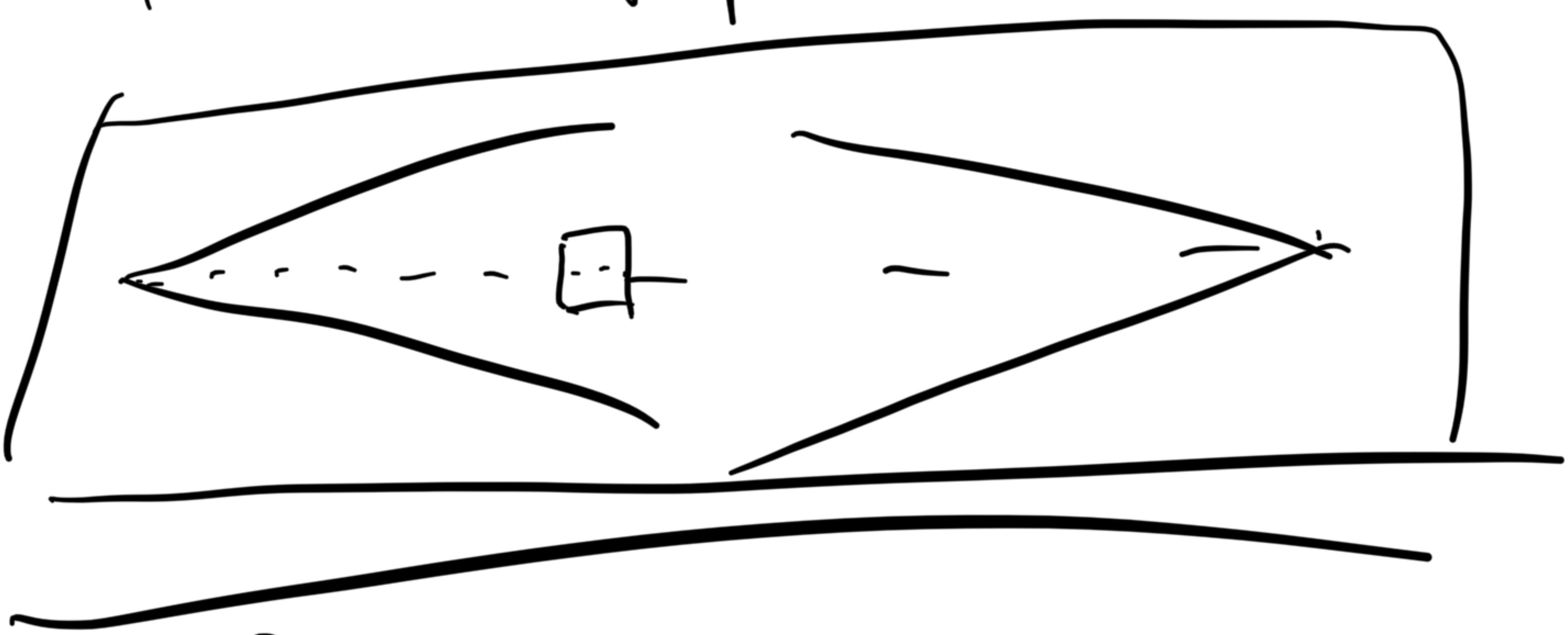
Camillo De Lellis



(Bryant assoc. cones)

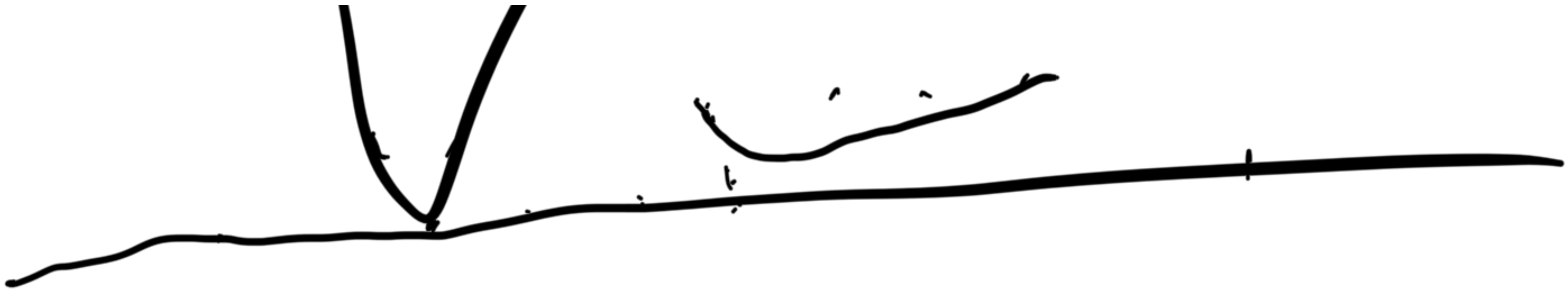
ϕ is <u>generic</u>.

ψ near ϕ. ψ is $GL(\mathbb{R}^6)$ pull back of ϕ



$\mathbb{R}^8$ Simons

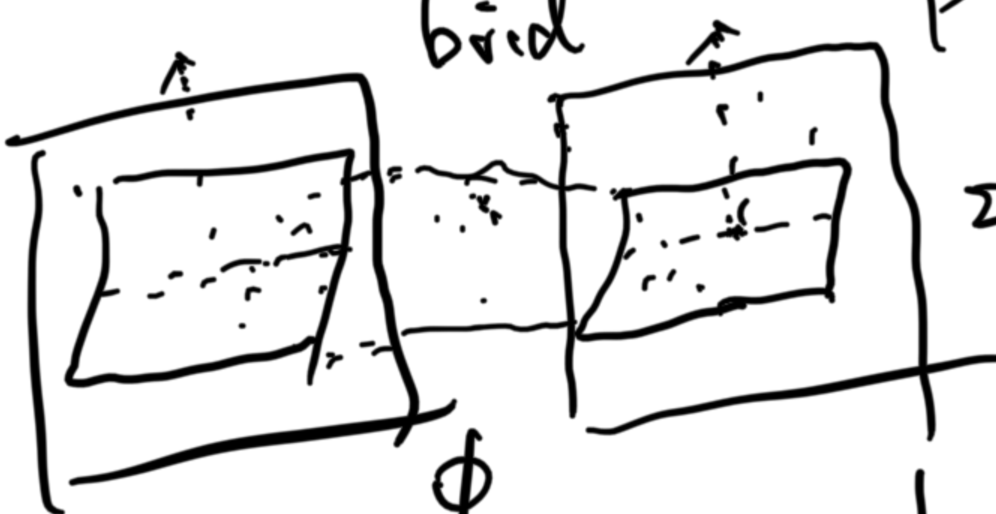
<u>$\mathbb{R}^8$</u>



8

dim sing ≤ 1 sing closed

cpt of line



ϕ'

$= (h_P^{-1})^* \phi$

h_P is $GL(\mathbb{R}^6)$

sends $x_1 x_2 x_3$ to T_P brid

$\overline{\phi'} \equiv 0$ on Σ, brid and P

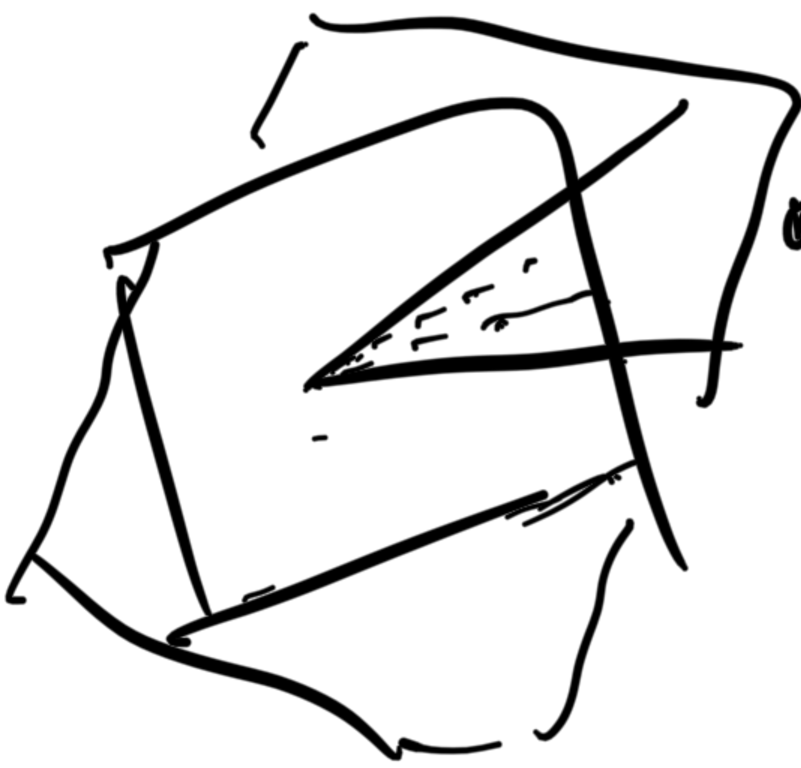
$d\overline{\phi'} = d\phi'$

$\psi = \phi' - \overline{\phi'}$

$GL(\mathbb{R}^6)$

closed

$\forall$ deg of vertex can be realized $P_1, \ldots, P_d$



d-ray that pairwise intersect at origin

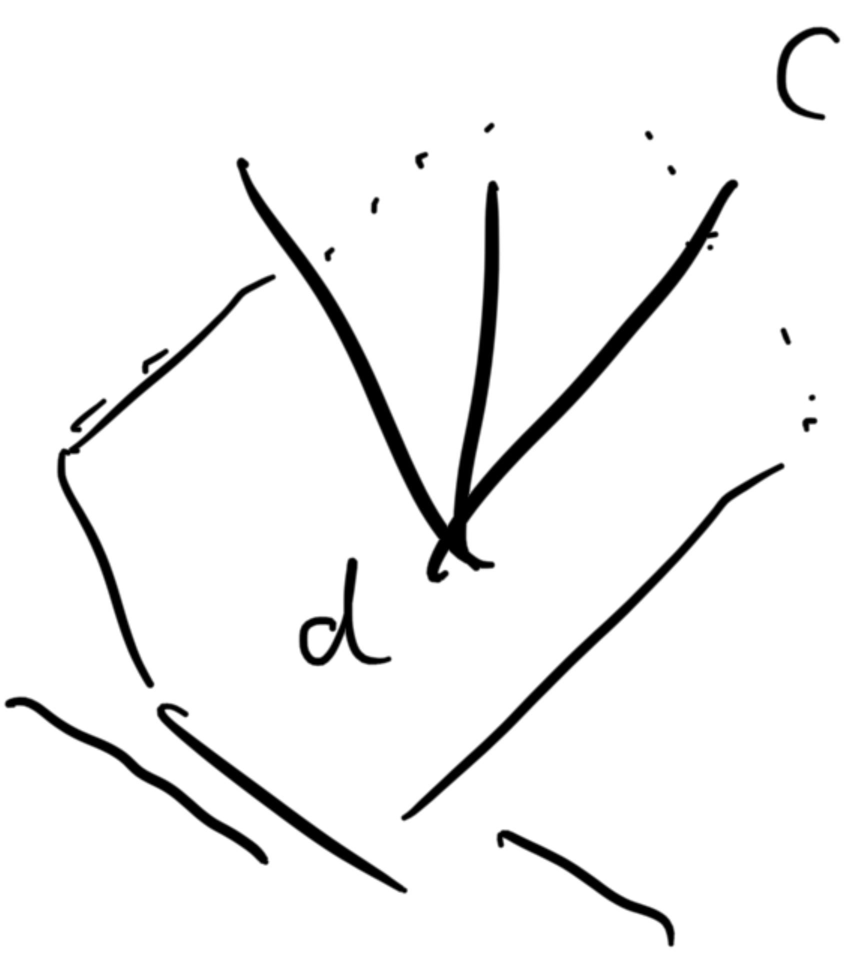
(?+? = b)

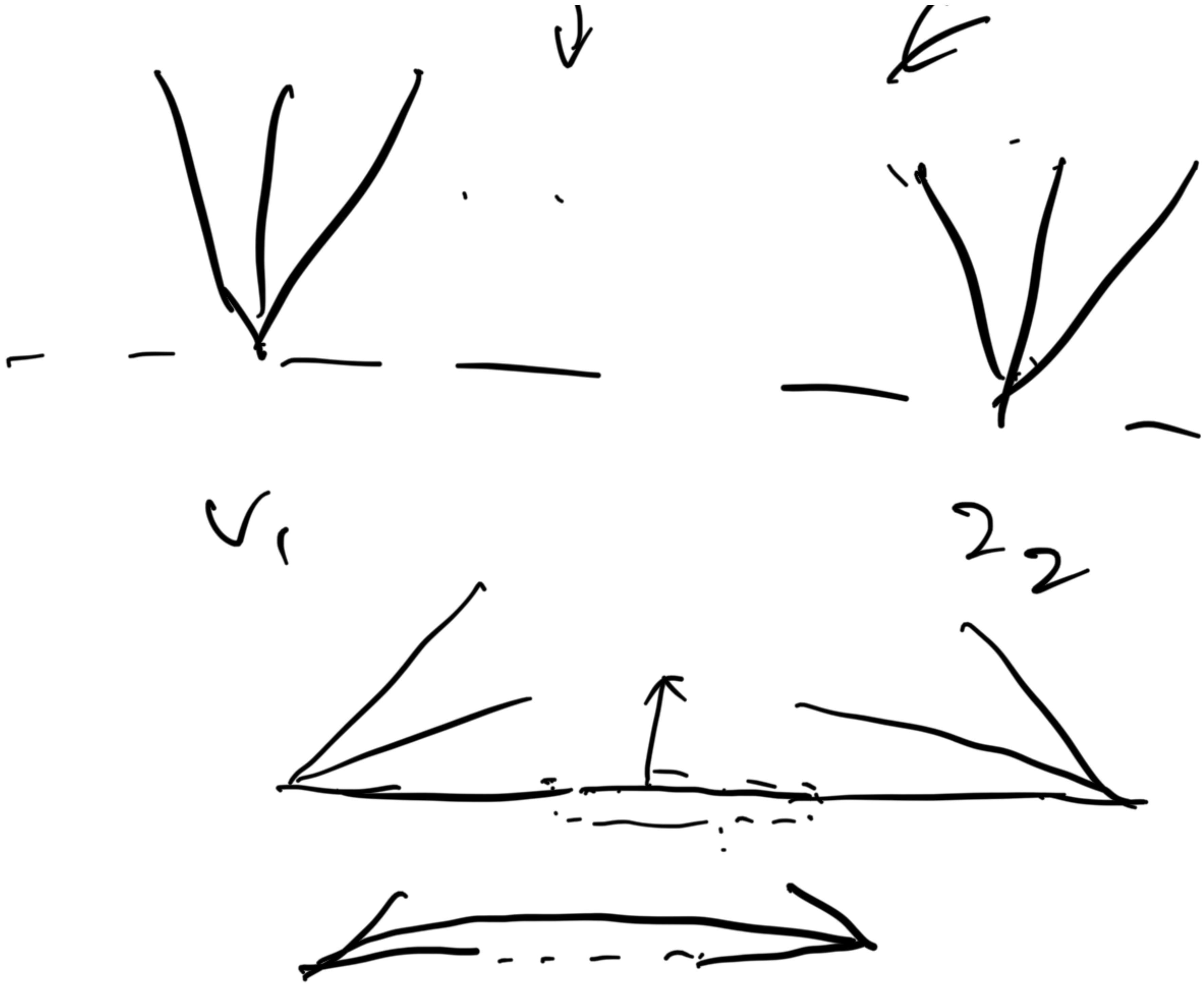
$P_i \cap C =$

L^2 of C $\quad \begin{bmatrix} e^{ia} & & \\ & e^{ib} & \\ & & e^{-i(a+b)} \end{bmatrix}$

rotate P and C

d



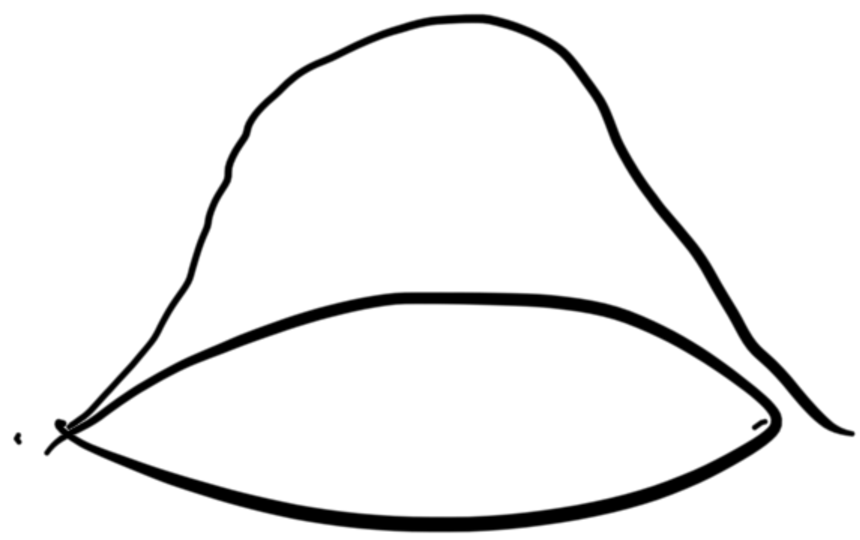


Yongsheng Zhang

framework for gluing

calib & producing

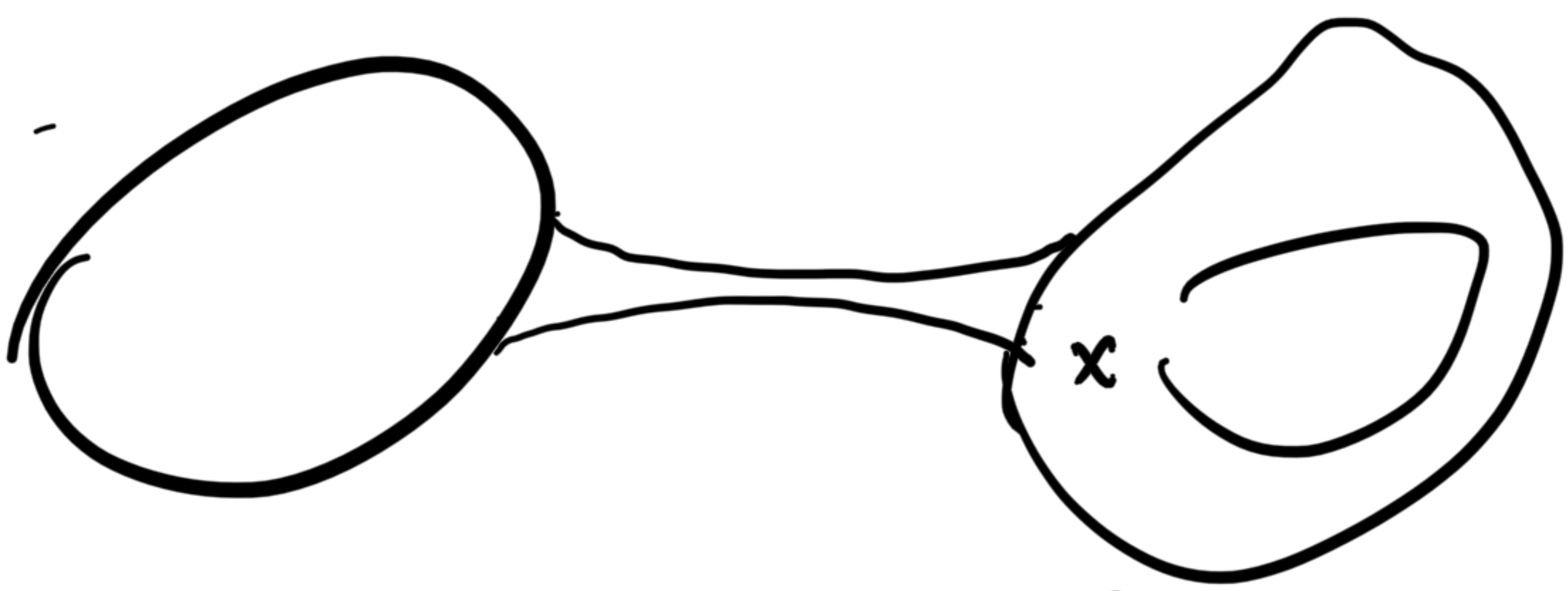
Zhang



$\mathbb{R}^7$

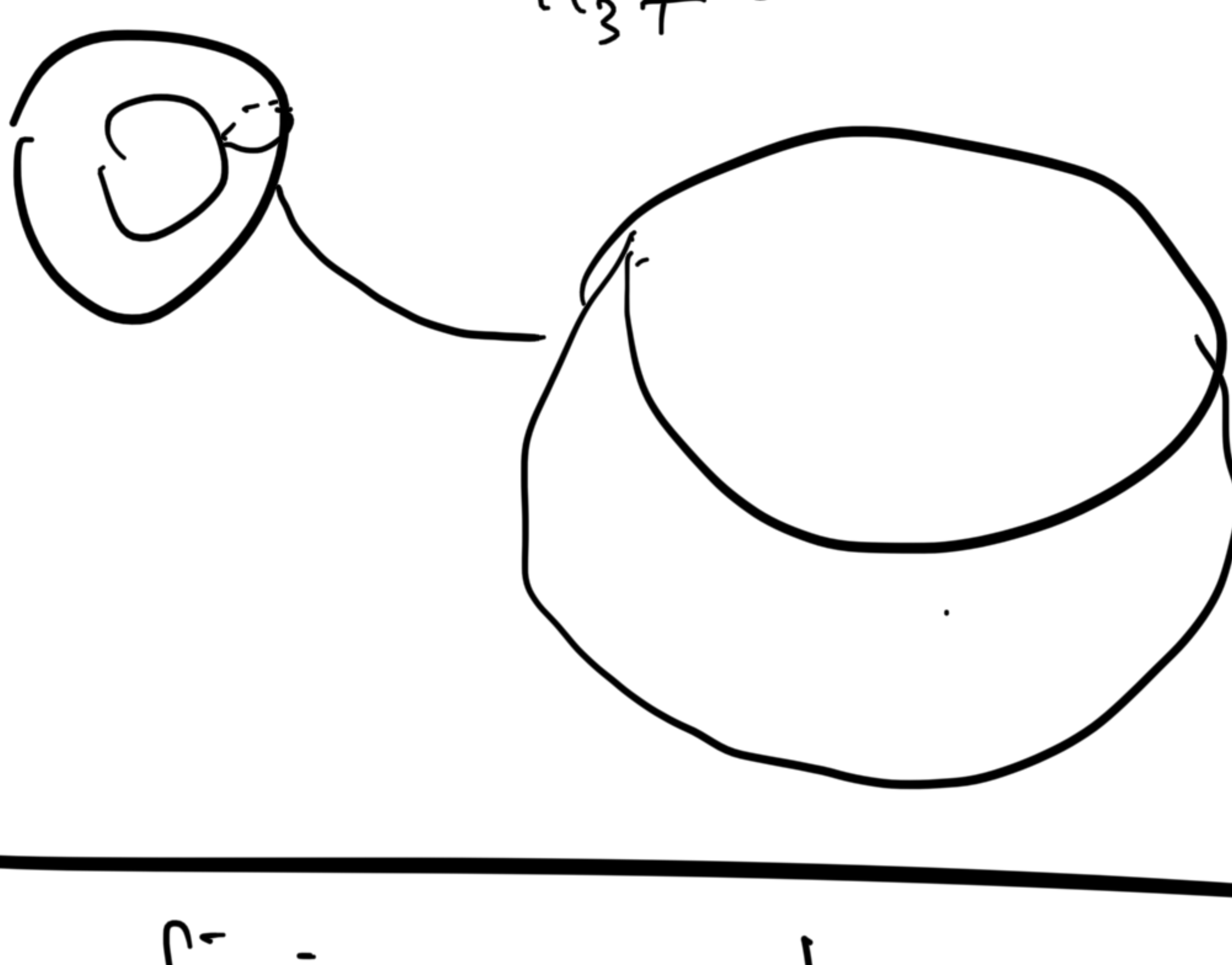
$\mathbb{R}^6$

several complete surf
conical



connects everything

M^7 with $H^7_3 \neq 0$



$\mathbb{R}^7$

Y

$\forall$ finite graph.

$\exists$ 7 dim M^7. (g, φ)

a homo area-min $\bar{\Sigma}$

$\operatorname{sing} \bar{\Sigma} = G$

$\Sigma \xhookrightarrow{\text{imm}} M$

conical p+s

self int and conical form G graph $\bar{\Sigma}$

can be perturbed away

not introducing cusp sing

metric structure of G

em $\mathbb{R}^7$