

Discussion Session For Simons
Collaboration On Special Holonomy:
Questions For Future Research.

January 15, 2021

Tom Bridgeland's Talk

What is the physical interpretation of the Joyce function?

Is his τ -function related to superpotentials of surface defects?

What is the physical interpretation of the hypercomplex manifold he constructs?

Is it related to hypermultiplet geometry? Or is it something else?

Ivan Smith's Talk

The generalization to A_1 -type with $n=1$ puncture should be possible, and the technical difficulties Bridgeland and Smith encounter

However, the generalization to $n=0$ punctures (Hitchin systems on \mathbb{C} without punctures) should also make sense yet should be qualitatively different.

What can we say about $\Omega(\gamma)$? Asymptotics?

And the physics papers suggest that the higher rank generalizations should be quite interesting.

Marcos Marino's Talk

A striking relation between Stokes factors for a complex Chern-Simons theory and the superconformal index for the 3d SQFT was presented.

What is the conceptual explanation? The 3d/3d correspondence naturally suggests itself.

Marcos Marino's Talk - 2/2

Remark: 6d (2,0) on $M_3 \times \mathbb{R} \times Cigar$ is equivalent to a 2d (gauged) LG model with target space the Space of complex gauge connections on M_3 and superpotential given by the Chern-Simons functional.

For compact gauge groups G it can be very fruitful to study the special case of Chern-Simons-Witten theory where G is a finite group. (“Dijkgraaf-Witten theory”). When G is noncompact the theory is qualitatively different (as Marcos demonstrated). Is there a useful solvable analog of DW theory for infinite, but discrete groups?

Ioana Coman's Talk

Some related ideas came up 30 years ago in the context of matrix models of 2d quantum gravity:

1. Quantization of spectral curves
2. Isomonodromic deformation and τ -functions
(related to Painlevé !)
3. Equivalent free-fermion theories on the spectral curve.

Is there a relation of her results to the old 2d matrix model results? KP hierarchy in the topological string?

Three Other Questions – 1/3

RH/TBA/Spectral network techniques raise the possibility that we can write explicit formulae for $K3$ metrics. [GMN unpublished; Kachru, Tripathy, Zimet]

What would we want to do with an explicit formula for $K3$ metrics?

Three Other Questions – 2/3

BPS states have more physical/mathematical content than just the DT invariants. DT invariants should be Euler characters of some complexes, and the homotopy type of those complexes has further information.

There should be a categorical analog of wall-crossing theory, and in particular Stokes phenomena. Some success:
Gaiotto, GM, Witten;
Kontsevich, Kapranov, Soibelman;
A. Khan & GM;
Kapranov, Soibelman, Soukhanov.

So what? Applications?

Three Other Questions – 3/3

Given a 4-manifold there is a measure over the base of a Hitchin system and the integral is a key ingredient to deriving invariants of four-manifolds generalizing Donaldson invariants. Can any of the results at this conference be used here?

Gravitational couplings involve automorphic forms on moduli space of curves, and variation of this conformal structure of C played an important role in Tom Bridgeland's talk.

