

**Simons conference 11-15 January 2021, "Donaldson-Thomas invariants and Resurgence".**

**Timetable**

Time, time zone	Monday 11 January	Tuesday 12 January	Wednesday 13 January	Thursday 14 January	Friday 15 January
4.0-5.0pm GMT (London, UK) 11am-12noon EST (New York, East USA) 5.0-6.0pm CET (Paris, much of Europe) 8.0-9.0am PST (California, West USA)	4.0-4.05 Introduction Robert Bryant	Maxim Kontsevich "Analyticity and resurgence"	Tom Bridgeland "From Donaldson- Thomas invariants to complex hyperkahler structures II"	Tom Bridgeland "From Donaldson- Thomas invariants to complex hyperkahler structures III"	Ioana-Alexandra Coman "Geometric description of topological string partition functions from quantum curves and integrability"
	4.05 - 5.05 Brent Pym "Introduction to Stokes phenomena and resurgence"				
5.0-5.30 GMT 12.0-12.30 EST	5.05-5.30 Break	Questions/Break	Questions/Break	Questions/Break	Questions/Break
5.30-6.30 GMT 12.30-1.30 EST 6.30-7.30pm CET 9.30-10.30am PST	Tom Bridgeland "From Donaldson- Thomas invariants to complex hyperkahler structures I"	Marcos Marino "From resurgence to topological strings"	Fei Yan: "Line defects, UV-IR map and exact WKB"	Simon Donaldson: "Deformations of singular sets and Nash-Moser theory I"	Simon Donaldson: "Deformations of singular sets and Nash-Moser theory II"
6.45-8.0 GMT 1.45-3.0 EST	Break	Break	Break	Break	Break
8.0-9.0 GMT 3.0-4.0 EST 9.0-10.0pm CET 12 noon-1.0pm PST	Ivan Smith "Quadratic differentials as stability conditions"	Andy Neitzke "Riemann-Hilbert problems, Hitchin systems and the conformal limit"	8.0 onwards Discussion, led by Maxim Kontsevich and Richard Thomas	8.0 onwards Discussion on complex hyperkahler manifolds.	8.0 onwards Discussion on "DT invariants and resurgence: Good questions for the future?", led by Joerg Teschner
9.0-9.30 GMT 4.0-4.30 EST	Questions/ Discussion	Questions/ Discussion			

## **Tom Bridgeland**

**Title:** From Donaldson-Thomas invariants to complex hyperkahler structures (3 lecture series).

**Abstract:** I will report on an ongoing project which aims to use the DT invariants of a CY3 triangulated category to encode a geometric structure on its stability space. The basic idea is to interpret DT invariants as defining non-linear Stokes factors, as in the work of Gaiotto, Moore and Neitzke. Lecture 1 will be mostly background material: I will discuss stability conditions, the wall-crossing formula for DT invariants, and Stokes data. Lecture 2 will be about the particular type of complex hyperkahler structure we expect to find on stability space: I will give a local description involving Plebanski's second heavenly equation and discuss a (partly conjectural) class of examples relating to moduli spaces of holomorphic connections on rank 2 vector bundles over Riemann surfaces. Lecture 3 will be about attempting to construct the complex hyperkahler structure on stability space from the DT invariants: this involves a class of Riemann-Hilbert problems for maps from the complex plane into a group of symplectic automorphisms; I will discuss their solutions in some simple examples.

## **Ioana-Alexandra Coman**

**Title:** Geometric description of topological string partition functions from quantum curves and integrability.

**Abstract:** I will give a progress update on work relating topological string partition functions  $Z_{\text{top}}$  for a class of supersymmetric gauge theories to quantum Seiberg-Witten curves through integrability. In particular, I will discuss a geometric characterisation of the  $Z_{\text{top}}$  functions in terms of a line bundle over the moduli space of quantum curves, providing evidence for this picture through examples. Part of this discussion will review earlier results which show how the  $Z_{\text{top}}$  functions enter certain series expansions of isomonodromic tau functions associated to quantised SW curves. New insight then concerns the existence of certain preferred coordinates on the moduli space of quantum curves, which are defined from the curves via exact WKB analysis and which enter theta-series expansions of appropriately normalised tau functions, in a way that allows to extract the functions  $Z_{\text{top}}$ . Understanding these coordinates, how they are related on different patches as a consequence of Stokes phenomena, leads to the proposed geometric characterisation of the tau functions and  $Z_{\text{top}}$ .

## **Simon Donaldson**

**Title:** Deformations of singular sets and Nash-Moser theory I, II

**Abstract.** We consider "multivalued" solutions of certain elliptic PDE, with codimension 2 singular sets. The PDE of primary concern are the Laplace equation on a Riemannian manifold and the nonlinear "maximal submanifold" equation. The multivalued nature is expressed more precisely by saying that the solutions take values in a flat bundle over the complement of the singular set. Solutions of these kinds are relevant in various ways to manifolds of special holonomy, as we will review in the lectures. In particular, in dimension 2, when the singular set is a finite set of points, the square of the derivative of such a multivalued harmonic function is a holomorphic quadratic differential and there are relations to work of Bridgeland and Smith discussed in this meeting.

## **Maxim Kontsevich**

**Title:** Analyticity and resurgence.

**Abstract:** I will talk on my recent work with Yan Soibelman on analytic wall-crossing structures, and a hypothetical relation to theory of resurgent series by Jean Ecalle. In particular, our considerations imply the resurgence property of WKB series.

**Marcos Marino**

**Title:** From resurgence to topological strings.

**Abstract:** The theory of resurgence suggests that the perturbative series that we often calculate in physics and mathematics are the tip of the iceberg in an extended structure, involving generalized formal power series (also called trans-series), and relations between them, encoded in Stokes constants. In topological field and strings theories, these additional sectors potentially provide new topological invariants for geometric objects. In this talk, after introducing some basic tools of the theory of resurgence, I will discuss the example of complex Chern-Simons theory, where Stokes constants provide an infinite number of integer invariants of hyperbolic knots.

I will also discuss what is known in the case of topological strings and enumerative invariants of Calabi-Yau threefolds, and present some open problems.

**Andy Neitzke**

**Title:** Riemann-Hilbert problems, Hitchin systems and the conformal limit

**Abstract:** Given a Riemann surface  $C$  equipped with a meromorphic quadratic differential, one can define two natural families of flat  $sl(2)$ -connections on  $C$ . One of these families consists of  $sl(2)$ -opers (Schrodinger equations); the other is determined by a solution of Hitchin's equations on  $C$ . For either of these families, the monodromy data is expected to be the solution of a Riemann-Hilbert problem over  $\mathbf{CP}^1$ , with Stokes phenomena determined by generalized Donaldson-Thomas invariants. In the case of  $sl(2)$ -opers, this Riemann-Hilbert problem should be identified with a special case of the one described by Tom Bridgeland in his lecture series. I will describe these two Riemann-Hilbert problems and the expected relation between them.

**Brent Pym**

**Title:** Introduction to Stokes phenomena and resurgence

**Ivan Smith**

**Title:** Quadratic differentials as stability conditions.

**Abstract:** Consider a quasi-projective Calabi-Yau 3-fold which is an affine conic fibration over a two-dimensional surface. I will explain why the space of stability conditions on (a subcategory of) its Fukaya category can be understood in terms of meromorphic quadratic differentials on the surface. This talk reports on old joint work with Tom Bridgeland.

**Fei Yan**

**Title:** Line defects, UV-IR map and exact WKB

**Abstract:** In this talk I'll give an overview of the relations between class  $S$  theories and Hitchin systems, focusing on roles played by line defects in class  $S$  theories. Deforming onto the Coulomb branch triggers a UV-IR map for line defects, corresponding to a trace map for certain flat nonabelian connections over a Riemann surface. The UV-IR map admits a  $q$ -deformation, which corresponds to a quantum trace map embedding certain skein algebra into a quantum torus algebra. I'll also briefly describe connections to exact WKB and a potential  $q$ -deformation thereof.