

# Branes in moduli spaces of Higgs bundles, II

1. Brane in space time  
 $\rightarrow$  D-brane

Type II A/B string on  $CY_3 \times \mathbb{R}^4$

D2, D3, D4  
 $\omega$     $\Omega$     $\omega^2$   
 Kähler form   hol. 3-form

special D-brane on  $\mathcal{Y} \times \mathbb{R}^4 \rightsquigarrow$  particles  
 calibrated  
 central charge  $[\mathcal{Y}] \in H_*(CY_3) \leq$  mass  $\sim$  volume  
 BPS bound

2. Brane in target space

2d (2,2) sigma model w/ target  $CY_3$

BPS (supersymmetric) boundary conditions  $\leftarrow$  special D-branes  
 A-brane & B-branes      A, B-type boundary condition

Also in topologically twisted 2d A-model / B-model

Example of A-brane: Lagrangian submanifold + local system

B-brane: complex submanifold + hol. vector bundle

3. Branes in moduli space of Higgs bundles

Physics realization:  $N$  M2-brane on  $\Sigma \times S^1 \times S^1 \times \mathbb{R}^2$  "world sheet"

Given a brane (boundary cond.) one can ask whether it can be lifted to higher-dim.

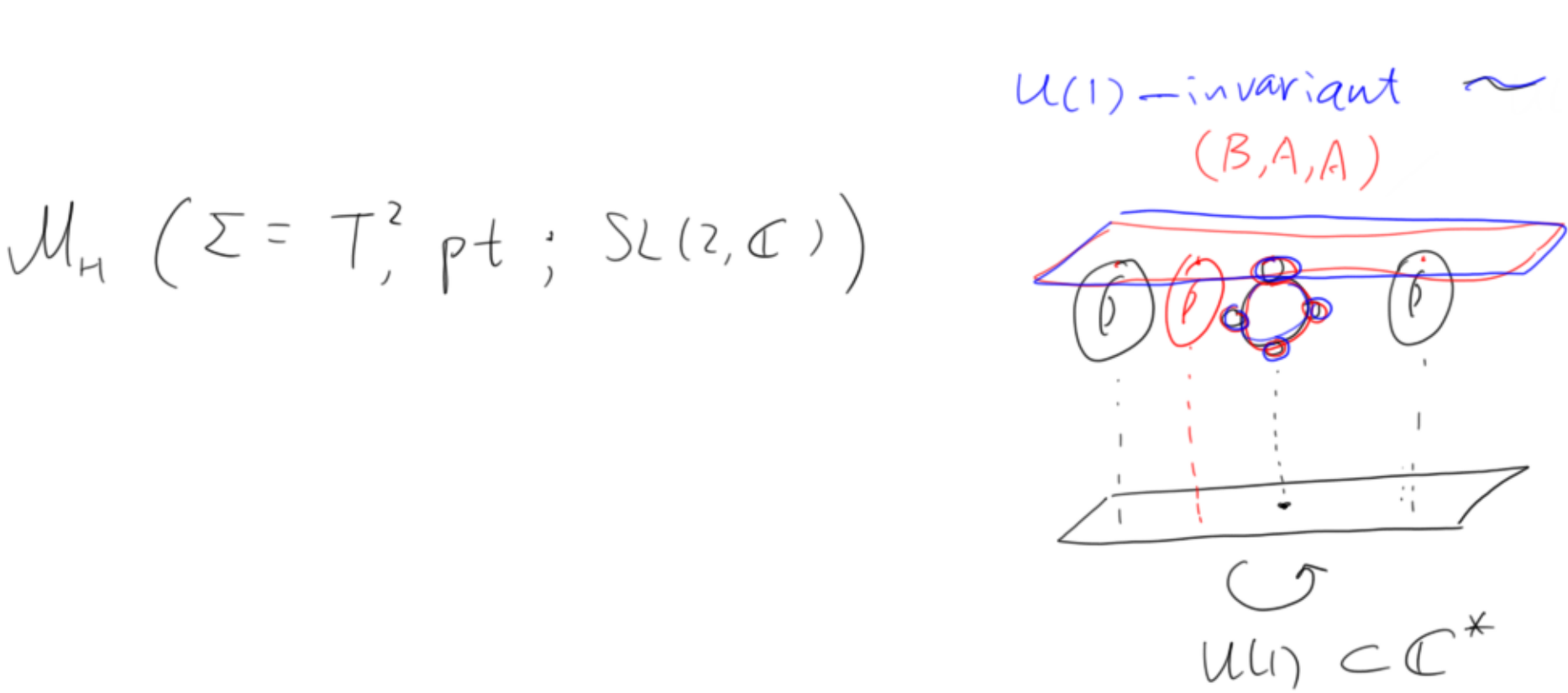
4d  $\mathcal{N}=4$  theory  $\xrightarrow{\text{on } T^2}$  sigma model to  $\mathcal{M}_H(\Sigma, G_{\mathbb{C}})$   
 4d  $\mathcal{N}=2$  theory  $\xrightarrow{\text{on } T^2}$   $(G_{\mathbb{C}} = GL(N, \mathbb{C}))$

Sigma model  $(4,4)$  supersymmetry  $\leftrightarrow$  very special branes ( $\frac{1}{2}$  BPS)  
 hyper-Kähler target      A/B-branes preserve 2 supersymms ( $\frac{1}{4}$  BPS)

$\omega_I$     $\omega_J$     $\omega_K$   
 $I$     $J$     $K=IJ$

Examples:  $(B, B, B)$ : pt,  $\mathcal{M}_H(G_{\mathbb{C}})$  or in between  $\mathcal{M}_H(H_{\mathbb{C}}) \rightarrow \mathcal{M}_H(G_{\mathbb{C}})$   
 $(B, A, A)$ : fiber of Hitchin map, Hitchin section,  $\mathcal{M}_H(G_{\mathbb{R}}) \rightarrow \mathcal{M}_H(G_{\mathbb{C}})$   
 $(A, B, A)$ :  $\partial M_3 = \Sigma, \{\pi_1(M_3) \rightarrow G_{\mathbb{C}}\} / G_{\mathbb{C}} \subset \mathcal{M}_H(\Sigma, G_{\mathbb{C}})$   
 $(A, A, B)$ : ...

$H_{\mathbb{C}} \subset G_{\mathbb{C}}$  complex subgroup  
 $G_{\mathbb{R}} \subset G_{\mathbb{C}}$  real subgroup  
 [Nadler, Gaiotto-Witten, Baraglie-Schaposnik]  
 Hitchin, Hausel-Mecheril, P...



4. Mirror symmetry

in 6d  $MCG(T^2)$   
 in 4d S-duality of  $\mathcal{N}=4$  theory  
 Langlands dual of  $G_{\mathbb{C}}$

same sigma-model  $\mathcal{M}_H(\Sigma, G_{\mathbb{C}}) \leftrightarrow \mathcal{M}_H(\Sigma, G_{\mathbb{C}}^V)$

$\sigma(\mathcal{M}_H(G_{\mathbb{C}})) \simeq \sigma(\mathcal{M}_H(G_{\mathbb{C}}^V))$   
 $B \xrightarrow{\dots} \tilde{T} \xrightarrow{\dots} B_{\text{mirror}}$   
 not as (4,4) theory  
 only as (2,2) theory  $\rightarrow$  types of branes can change

$(B, B, B) \leftrightarrow (B, A, A)$   
 $(A, B, A) \leftrightarrow (A, B, A)$

- Questions:
1. Mirror symmetry for mod. space of wild Higgs bundles.
  2. application to F-theory