G_2, Spin(7) BPS Equations and T-branes

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G_2 + T-branes { 1906.02212 Barbosa, Cvetic, Heckman, CL, Torres, Zoccarato

Spin(7) { 1811.01959 Heckman, CL, Lin, Zoccarato
2003.13682 Cvetic, Heckman, Rochais, Torres, Zoccarato

Special holonomy $\xrightarrow{\text{important}}$ supersymmetric compactifications of string theory

10D string thy $X_{1,9}$ (oriented spin)

holonomy is $SO(6)$

want $X_{1,9} = \mathbb{R}^{1,3} \times Y_6$

oriented compact spin manifold

"We are here!"

String thy is supersymmetric

$$\delta_s \text{ (boson) } = \text{ fermion}$$
$$\delta_s \text{ (fermion) } = \text{ boson}$$

$\rightarrow$ symmetry generated by some ε in spinor rep

of $SO(1,9)$ [rotation of $X_{1,9}$]

Want 4D spacetime to have <u>susy</u>

$$\varepsilon = \varepsilon_{4D} \otimes \varepsilon_6$$

SUSY $\Longrightarrow$ KSE:

$$D_6 \varepsilon_6 + (\text{flux})\, \varepsilon_6 = 0$$

If $(\text{flux}) = 0$ then

$$D_6 \varepsilon_6 = 0$$

"covariantly constant spinor".

$$SO(1,9) \longrightarrow SO(1,3) \times SO(6)$$

$$\varepsilon \quad \underline{16} \longrightarrow (2,1,4) \oplus (1,2,\bar{4})$$

transform non-trivially under SO(6)

If Y_6 is CY_3

$$SO(6) \longrightarrow SU(3)$$
$$4 \longrightarrow 3 \oplus \underline{\underline{1}}$$

Similar story for G_2, $Spin(7)$

$$SO(7) \longrightarrow G_2 \qquad SO(8) \longrightarrow Spin(7)$$
$$8 \longrightarrow 7 \oplus \underline{\underline{1}} \qquad 8 \longrightarrow 7 \oplus \underline{\underline{1}}$$

What if $(\text{flux}) \neq 0$?

Y_6 to be Kähler (hol $U(3)$) but turn on (flux).

$$D_6 \varepsilon_6 + (\text{flux})\, \varepsilon_6 = 0$$

"$SL(2,\mathbb{Z})$ flux" $\longrightarrow$ elliptically fibred Calabi-Yau fourfold

"F-theory" $\longrightarrow$ 4d $\mathcal{N}=1$

$\pi : Y_4 \longrightarrow B_3$

Kähler

Interesting 4D $\mathcal{N}=1$ physics

non-abelian gauge algebre $\mathfrak{g}$

matter fields in rep R of $\mathfrak{g}$.

$$SO(1,3) \times G$$

		$SO(1,3) \times G$	
	gauge field	$(2,2\,;\ \mathrm{adj})$	
matter	left fermions	$(2,1\,;\ R)$	# left and right different
	right fermions	$(1,2\,;\ R')$	$\to$ chiral spectrum.

What do ~~physicist~~ I want from G_2-compactifications?

1) supersymmetry ✓

2) non-ab. gauge group.

3) chiral matter.

M-thy on a smooth G_2-manifold:

abelian gauge gp
no charged matter

not the physics we want for the real world.

Singularities:

$\left\{\begin{array}{l}\text{codim } 4 \longrightarrow \text{non-ab. } \mathfrak{g}. \\ \text{codim } 6 \longrightarrow \text{non-chiral matter}\end{array}\right.$

$\{$ codim 7 $\longrightarrow$ chiral matter

$\to$ TCS $\longleftarrow$ singularities from CY_3 "building blocks"

$\to$ not in TCS construction??

Maxim: (DRM)

When one encounters an obstacle, what to do?

1) Mathematicians: attack the obstacle until it is gone.

2) Physicists: slip past the obstacle and get to their destination

Instead of building global geometry

→ use dual gauge thy description.

Pantev - Wijnholt : partial topological twist of $\underbrace{\text{7D super-Yang-Mills}}$

on M_3 associative inside G_2

$\longrightarrow$ BPS equations (PW system)

$\longrightarrow$ vector multiplet: A gauge field

ϕ scalar

ψ fermion.

$$SO(1,6) \times SU(2)_R \longrightarrow SO(1,3) \times \underbrace{SU(2)_{M_3} \times SU(2)_R}_{\text{diagonal } SU(2)}$$

$$A : \quad (7,1) \qquad (2,2;1,1) \oplus \underbrace{(1,1,3,1)}_{A}$$

$$\phi \quad (1,3) \qquad \underbrace{(1,1,1,3)}_{\phi}$$

A, ϕ are adjoint-valued one-forms on M_3.

combine them $\mathcal{A} = A + i\phi$.

Introduce cx connections:

$$\mathcal{D}_{\mathcal{A}} = d + \mathcal{A} \qquad \mathcal{F} = [\mathcal{D}_{\mathcal{A}}, \mathcal{D}_{\bar{\mathcal{A}}}]$$

$$D_A = d + A \qquad F = [D_A, D_A].$$

In terms of the original fields (A, ϕ).

$$\operatorname{Im} \mathcal{F}_{ij} = F_{ij} - [\phi_i, \phi_j] \qquad [i, j = 1, \dots, 3].$$

SUSY requires F-, D-terms

$$\mathcal{F}_{ij} = 0 \quad , \quad g^{ij} \mathcal{D}_{ij} = 0$$

moduli space of vacua

$$\{ \mathcal{F}_{ij} = 0 \} / G_{\mathbb{C}}$$

$$\cong$$

$$\{ \mathcal{F}_{ij} = 0 , g^{ij} \mathcal{D}_{ij} = 0 \} / G_u$$

unitary gauge transf.

Spin(7)

7D sYM on 4-manifold:

$$SO(1,3) \times \overbrace{SO(4)}^{M_4} \times SU(2)_R$$

$$SO(4) \cong SU(2)_L \times SU(2)_r$$

diagonal of these.

ϕ_{SD} self-dual two-forms

BPS equations: $D_A \phi = 0 \qquad F_{SD} + \phi \times \phi = 0$

$$\begin{tikzcd}
& \begin{array}{c}\text{PW}\\ M_3 \subset G_2\end{array} \arrow[dl] & \\
\begin{array}{c}\text{Hitchin}\\ C \subset CY_3\end{array} & & \begin{array}{c}\text{Spin}(7)\\ M_4 \subset \text{Spin}(7)\end{array} \arrow[ul] \arrow[dl] \\
& \begin{array}{c}\text{BHV}\\ S \subset CY_4\end{array} \arrow[ul] &
\end{tikzcd}$$

$$M_4 = M_3 \times S^1$$

$$\phi_{SD} = \phi_{PW} \wedge dt + *_3 \phi_{PW}$$

$$F - [\phi_{PW}, \phi_{PW}] + *_3 (D_t A - d_3 A_t) = 0$$

$$D_A \phi_{PW} + *_3 D_t \phi_{PW} = 0$$

$$D_A *_3 \phi_{PW} = 0$$

$$A_t = \partial_t A = \partial_t \phi = 0$$

$\Rightarrow$ PW equations.

Consider <u>fluxed</u> solutions of PW system.

($\underline{F \neq 0}$) [fluxless solutions

[Braun, Cizel, Hubner, Schafer-Nameki].

Consider M_3 locally

$$
\begin{tikzcd}
\Sigma \arrow[r, hook] & M_3 \arrow[d] \\
 & I
\end{tikzcd}
$$

local patch $M_3 = \Sigma \times I$.

t be a coordinate on I

$x_a = (x_1, x_2)$ coordinates on Σ.

$$\left\{\begin{array}{l} F_{ab} - [\phi_a, \phi_b] = 0 \\ D_a \phi_b - D_b \phi_a = 0 \\ g^{ab} D_a \phi_b + g^{tt} D_t \phi_t = 0 \end{array}\right.$$

$$\left.\begin{array}{l} F_{ta} - [\phi_t, \phi_a] = 0 \\ D_t \phi_a - D_a \phi_t = 0 \end{array}\right\} \text{flow}$$

→ "almost" the Hitchin equations.

Solutions:

Power series expansion in t.

$$A_i(t, x_a) = \sum_{j=0}^{\infty} A_i^{(j)}(x_a)\, t^j$$

Pick a gauge

$$A_t^{(j)} = 0.$$

$$\sum_{j=0}^{\infty} G_{ab}^{(j)} t^j = 0$$

$$\sum_{j=0}^{\infty} H_{ab}^{(j)} t^j = 0$$

+ 3 more equation which are linear in either $A^{(j+1)}$, $\phi^{(j+1)}$.

j = 0 : we get $F_{ab}^{(0)} - [\phi_a^{(0)}, \phi_b^{(0)}] = 0$

$\vdots$

Assume $A_a^{(0)}$, $\phi_a^{(0)}$ are such that these equation are solved

$\Longrightarrow$ solved at all orders in t.)

$\phi_t^{(0)}$ sets the trajectory of the solution

Non-abelian background (SU(3))

$$\phi = \begin{bmatrix} i/3\, dh & -vze^{-f(z,\bar{z})}dz + \varepsilon e^{f(z,\bar{z})}dz & 0 \\ vze^{-f(z,\bar{z})}dz - \varepsilon e^{f(z,\bar{z})}d\bar{z} & i/3\, dh & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad a = \frac{i}{2}\left(\partial_{\bar{z}} f\, d\bar{z} - \partial_z f\, dz\right)$$

fluxed

counting zero-modes.
$\Rightarrow$ observe chiral matter fields.

Conclude

T-brane configuration fcts soln to PW system behave like chiral matter ✓

$\longrightarrow$ bypassed difficulties w/ codim 7 singularities