Higgs Bundles in String Compactifications Part I

Caveats:

- Mixed audience but (many experts!)
- Aim to be introductory
- Feel free to ask questions

Key Questions

- Physics frequently leads to compactifications on singular spaces
  - E.g.s CY/ F-theory (geometrization of 7-brane physics)
    G_2/ M-theory (Co-dim 4 + Co-dim 7)

Questions:

- What singularities are allowed?
  i.e. How degenerate can we go?
  (Physically want finite distances in field space usually, not always easy to decide geometrically)

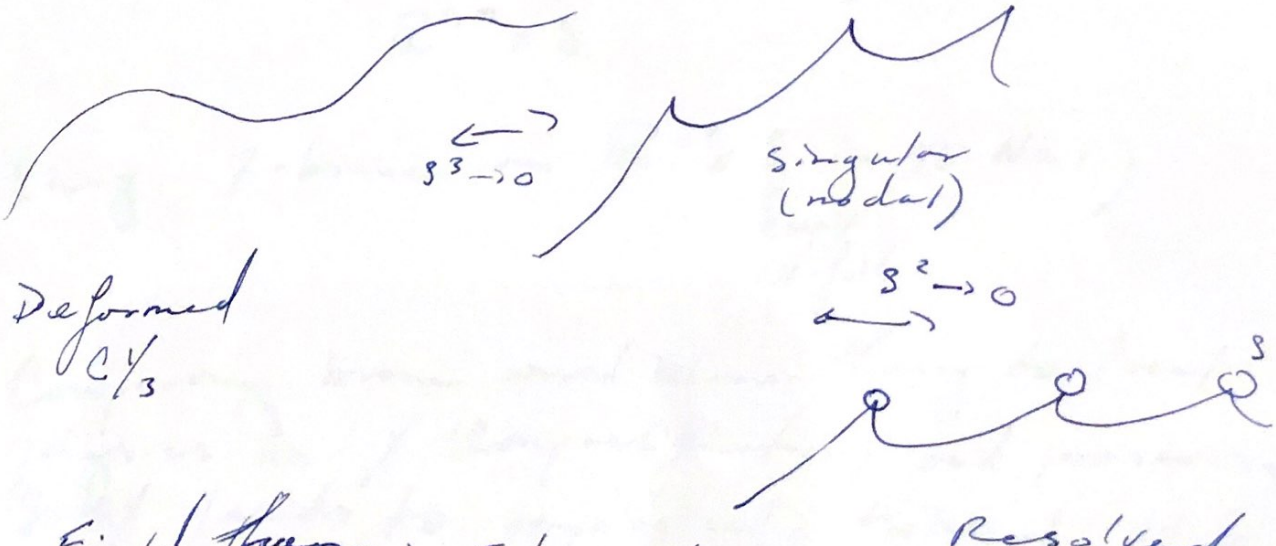
Q, Cont'd

- What features of the singular geometry describe the physical theory?
  i.e. what counts massless states couplings, etc?

(Lots of work on this) Halverson, Grassi, ... Grassi + Weigand ...

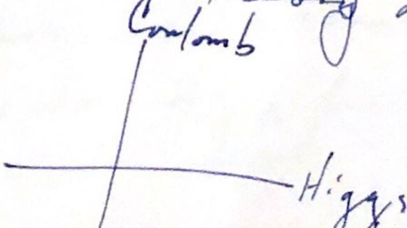
In some corners of string theory, this is fairly well understood...

[e.g. Calabi-Yau Conifold transitions in Type IIA (Greene, Morrison, Strominger) etc.



Field theory: Enhanced U(1) gauge theory at singular pt.

Branch Change:



Features:

- New light d.o.f. arising from branes wrapping collapsing cycles

- i.e. singular geometries have <u>more</u> d.o.f. than smooth counterparts

Brane World Volume Theories
(hold key to some structure) $\rightarrow$ "Local Models"

One approach to local physics:
Brane world volume theory on

$$\mathbb{R}^{1,m} \times S$$

(e.g. 7-brane on $\mathbb{R}^{1,4} \times S$ in 4D $N=1$)
S: cmplx 2-fold

Comparing brane world volume theory to local physics on X (compactification) and preserving SUSY leads to <u>topologically twisted</u> theories

- Leads to <u>Higgs Bundles</u> on S
(i.e. Hitchin-like systems)
w/ particular <u>Higgs fields</u>

Local Higgs Bundles (4D, N=1)

CY 4-folds (Singular)

$\bar{\partial}_A \phi_{BHV} = 0$

$F_{(0,2)} = 0$

$J_S \wedge F + \frac{i}{2}[\phi^{\dagger}_{BHV}, \phi_{BHV}] = 0$

J_S - Kahler form on Compl. 2-fold wrapped by brane S

Complx Surface of singularities.

$\phi_{BHV} \in H^1(K_S \times \mathrm{End}(V))$

G_2 / M-th

3-fold of ADE sing. in X_7

Local model:

$D_A \phi_{PW} = 0$ $\quad \phi_{PW}$ - Adj. Val. 1-form

$D_A * \phi_{PW} = 0$

$F = [\phi_{PW}, \phi_{PW}]$ $\quad (H^1(\mathrm{End}(V) \times T Q_3^{\vee}))$

Geometry of the local theory

E.g F-th in 6D

Higgs bundle on a complex curve

CY_3 :

e.g. Curve of ADE Singularities

In the case that C is smooth:

$$(*) \begin{cases} F + [\Phi, \Phi^\dagger] = 0 \\ \bar{\partial}_A \Phi = 0 \end{cases}$$

"Hitchin System" — Hitchin self-duality eqns on a Riemann surface

$\Phi \in H^1(\mathrm{End}(V) \times K_C)$

Higgs Bundle: A pair (V, Φ) Satisfying $(*)$

More generally, the curve can be Singular

- C possibly non-reduced, reducible, singular, etc.
- Likewise (A_μ, Φ) can develop Poles

$\Rightarrow$ "Wild/Irregular" Hitchin systems

(also, of course parabolic Higgs bundles)

More generally
Hitchin system with defects

$$F + [\Phi, \Phi^\dagger] = \sum_p \delta_{(p)} \mu_{IR}^{(p)}$$

Localized (1,1) current

$$\bar{\partial}_A \Phi = \sum_p \delta_{(p)} \mu_{\mathbb{C}}^{(p)}$$

$\mu_{IR}^{(p)}, \mu_{\mathbb{C}}^{(p)}$ form a triplet of adjoint-valued moment maps.

Back to smooth/easy case:
E.gs. V = A line bundle
Φ = Holom. 1-form

Non-abelian:
e.g. $g(C) > 2$, $V = K_C^{1/2} + K_C^{-1/2}$, $\Phi = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Solves (*)

Higgs bundles admit a notion of stability
(i.e. Mumford/slope stable for ϕ-invariant subsheaves)

form a good moduli space
→ Noncompact, Hyperkahler manifold, M

Hitchin Fibration: $H: M \to \bigoplus_{d=2}^{n} H^0(C, K_C^d)$

(Hyperkahler: admits Kahler metric on M
w/ a trio (I, J, K) of complex structures)

ϕ Endomorphism Valued, so can compute

$$\det(\phi - \lambda I) = 0$$

"Spectral Equation" or Curve

| Algebraically Complete integrable system

Fibers of Hitchin Map = Jacobian of the Spectral Curve.

(Abelian Variety or Prym for gen. groups G)

Nilpotent solns: E.g. $\phi = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow$ Singular fibers of the Hitchin fibration

| In physics "T-brane" Solutions

Link to physics:

Φ - parameterizes transverse motion of 7-brane inside singular CY geometry

E.g. $\det(\Phi - \lambda I) = 0$ Reproduces local geometry. E.g. If $y^2 = x^3 + z^5$ (local E_8 @ $z=0$)

Then $su(2)$-valued Φ

$$\Phi = \begin{bmatrix} \phi & 0 \\ 0 & \phi \end{bmatrix} \Rightarrow y^2 = x^3 + \phi^2 z^3 x + z^5$$

breaks to E_7

$\phi^2 \sim$ Quadratic Casimir of $su(2)$-Valued ϕ

(in general Casimir's of $\phi \Rightarrow$ Complex Structure of CY_3)

Taking a step back:...

Local theory is useful, but not good enough.

Need to understand rules of how to consistently embed this data into compactification manifold

- Need to understand "transition functions" to map local moduli space to global one of <u>singular</u> manifold.
- Understand limiting behavior

$$X_t \to X_0 \;(\leftarrow \text{singular})$$

Big ~~Hint~~ step-forward on necessary technology in CY n-fold case:

Donagi, Diaconescu + Pantev

Result:

<u>Consider</u>

Non-compact CY 3-fold

ADE singularities

ADE Hitchin system on Σ

CY moduli space:

"Calabi-Yau Integrable system"

$\mathcal{M}_{CY} \to \mathcal{M}_{comp.\ struc}$

Fibers = "Intermediate Jacobian"
= Abelian Variety

$J(X) = (H^{3,0}(X) + H^{2,1}(X))^* / H^3(X, \mathbb{Z})$

(smooth case)

or simply

$J^3(X) = \dfrac{H^3(X, \mathbb{R})}{H^3(X, \mathbb{Z})}$

In physics, halves of hypermultiplets ($N=2$)

e.g. IIA = $\begin{cases} \text{C.S. structure moduli} \\ \text{RR Moduli} \end{cases}$

DDP Result (roughly)

CY Integrable System $\simeq$ Hitchin Ism. Int. Sys.

Need a more refined notion of CY moduli space to deal w/ singularities

"Limiting Mixed Hodge Structures" LMHS

Aside: used recently by Grimm, Valenzuela for distance conj

Rough Idea:

- Hodge structure: A finitely generated Abelian group w/ decreasing filtration

  e.g. for a smooth Kahler manifold, Hodge Filtration

$$F^p H^k(X, \mathbb{C}) = \bigoplus_{p' \geq p} H^{p', k-p'}(X) \qquad H^{q,p} = F^p \cap \bar{F}^p$$

- Mixed Hodge structure adds another filtration ("weight") to track singular spaces/forms.

L.A. w/ Heckman, Katz, (Schaposnik)

Extended idea of DDP to compact, limiting setting:

Our Result:
In singular limit, notion of "emergent" Hitchin sys. in CY moduli space

$$\begin{tikzcd}
 & & \mathcal{M} \arrow[d] \\
\pi^* H \arrow[urr, "\text{inclusion}"] \arrow[rr] \arrow[d] & & \tilde{\mathcal{M}}_{cplx} \arrow[d, "\pi"] \\
H \arrow[rr] & & \mathcal{M}_{loc}
\end{tikzcd}$$

H = Hitchin Moduli Space

$\tilde{M}_{cplex}$ = C.S. moduli space of resolved geometry

M = CY moduli Space

M_{loc} = Moduli of local theory (i.e. Hitchin base)
Moduli of 7-brane

LMHS analysis:
Identifies fibers of Hitchin sys
with (part) of limits of Int. Jac. $J(X_t)$
of 1-parameter smoothings, X_t

Open Questions

- What characterizes the allowed singularities (@ higher co-dim) in CY_n, G_2 7-fold, etc?

  Even in simplest, best understood case: CY_3

  T-brane solutions → Non-crepant singul.

- Limiting Moduli Spaces for
  - CY 4-folds

    (Here flux, truly new solutions Vacua of theory)
  - G_2 Manifolds + PW Sys. ?

    Sing.?

    What techniques like LMHS for real forms?

    (Pontev + Barbarosa, ...)
  - Dualities of many kinds

    (see Andreas' talk)