[Cvetic, Heckman, Rochais, Torres, GZ: 2003.13682]

Special Holonomy: Progress and Open Problems 2020

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BHV SYSTEM

[Beasley, Heckman, Vafa '08]

Higgs bundle on a Kähler surface S

Higgs field is a complex adjoint valued (2,0)-form

Supersymmetry equations

$$\bar{\partial}_A \phi = 0$$

$$F_{(0,2)} = 0$$

Kähler form

$$J \wedge F + \frac{i}{2}\left[\phi^{\dagger}, \phi\right] = 0$$

Equations are in unitary frame

Appears in F-theory compactifications on Calabi-Yau fourfolds

Holomorphic structure makes construction of solutions feasible (even non-abelian)

PW SYSTEM

[Pantev, Wijnholt '09]

[Braun, Cizel, Hübner, Schäfer-Nameki '18]

[Barbosa, Cvetic, Heckman, Lawrie, Torres, GZ '19]

[Hübner '20]

Higgs bundle on a three-manifold Q

Higgs field is an adjoint valued 1-form

Supersymmetry equations are $$D_A\phi = 0 \qquad D_A \star \phi = 0$$
$$F = [\phi, \phi]$$

Can be written in terms of complexified connection $\mathscr{A} = A + i\phi$ as

$$\mathscr{F} = 0 \qquad D_A \star \phi = 0$$

Appears in M-theory compactifications on a G_2-manifold

SPIN(7) SYSTEM

[Heckman, Lawrie, Ling, GZ '18]

Higgs bundle on a four-manifold M

Higgs field is an adjoint valued self-dual two form

Supersymmetry equations are

$$D_A \phi = 0$$

$$F_{SD} + \phi \times \phi = 0$$

F_{SD} is the self-dual part of the curvature of the bundle

Cross product is $$(\phi \times \phi)_{ij} = \frac{1}{4}[\phi_{ik}, \phi_{jl}]g^{kl}$$

This system is very similar to the Vafa-Witten system [Vafa, Witten '94]

Spin(7) system includes other Higgs bundles as solutions

PW TO SPIN(7)

To relate the two consider the case of Spin(7) on a four-manifold $M = Q \times S^1$

Then write the SD forms as $\phi = \hat{\phi} \wedge dt + \star_3 \hat{\phi}$ $\hat{\phi} \in \Omega^1(Q)$

The Spin(7) supersymmetry equations become

$$F - [\hat{\phi}, \hat{\phi}] + \underline{\star_3 (D_t A - d_3 A_t)} = 0 \qquad D_A \hat{\phi} + \underline{\star_3 D_t \hat{\phi}} = 0$$

$$D_A \star_3 \hat{\phi} = 0$$

One recovers PW if $A_t = \partial_t A = \partial_t \hat{\phi} = 0$

PW is the dimensional reduction of Spin(7) along the additional direction

BHV TO SPIN(7)

We take the four manifold to be a Kähler manifold S

In a Kähler manifold SD two forms admit a Hodge decomposition

$$\Omega^2_+(S) \simeq H^{(2,0)}(S) \oplus H^{(0,2)}(S) \oplus H^{(1,1)}_{n.p.}(S)$$

Non-primitive

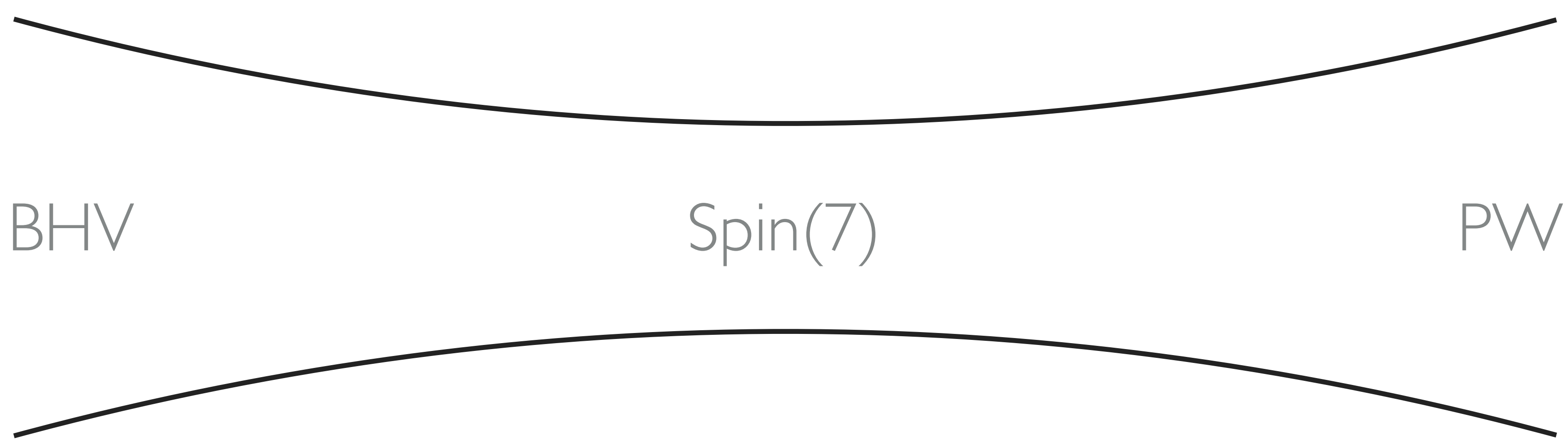
The Spin(7) equations become

$$F_{(0,2)} - \frac{i}{2}\phi_{(1,1)} \times \phi^\dagger_{(0,2)} = 0 \qquad \bar{\partial}_A \phi_{(2,0)} - \frac{i}{2}\partial_A \phi_{(1,1)} = 0$$

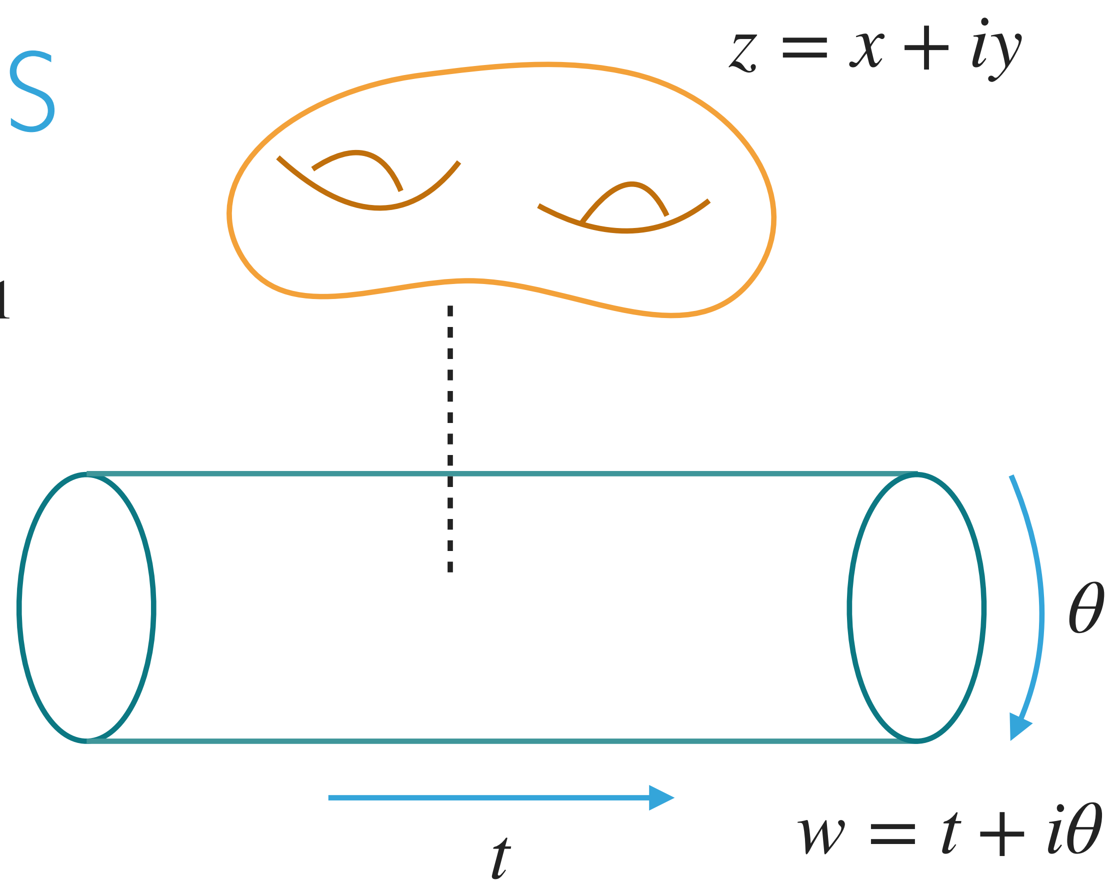
$$J \wedge F = \frac{i}{2}[\phi_{(2,0)}, \phi^\dagger_{(0,2)}]$$

BHV is recovered for configurations with the (1,1)-component set to zero

$$BHV \subset Spin(7) \supset PW$$



PW TO BHV INTERPOLATIONS



We work with a four manifold $M = \Sigma \times \mathbb{R} \times S^1$

Calling t the coordinate on $\mathbb{R}$ we have

$$|\phi_{(1,1)}| \sim e^{\lambda t} \qquad t \to -\infty$$

$$|\partial_\theta \psi| \sim e^{-\lambda_2 t} \quad |A_\theta| \sim e^{-\lambda_1 t} \quad t \to +\infty$$

For $t \to -\infty$ solution approaches BHV

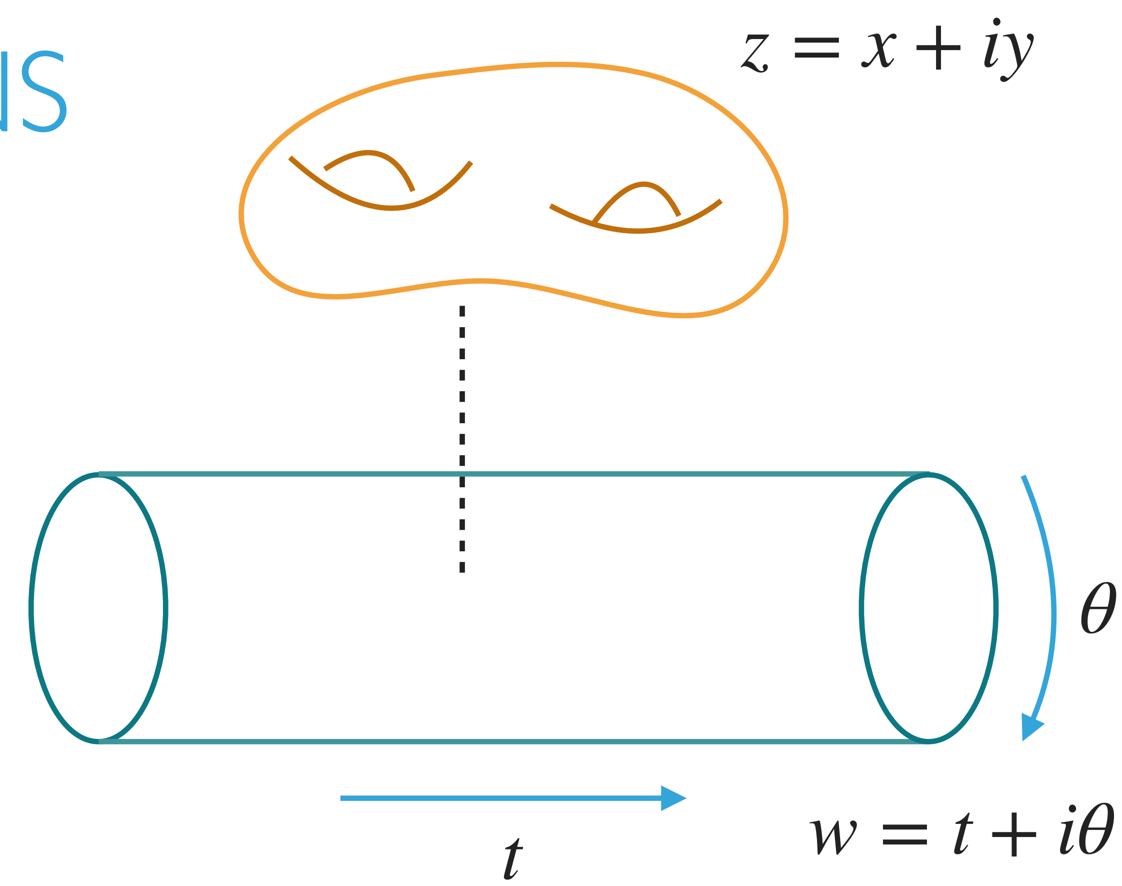
For $t \to +\infty$ solution approaches PW

In the middle there is a Spin(7) solution

This is the Higgs bundle version of GCS construction of Spin(7) manifolds

[Braun, Schäfer-Nameki '18]

PW TO BHV INTERPOLATIONS



To give a concrete example, take $\Sigma \simeq \mathbf{T}^2$

Take abelian solutions $\phi \times \phi = 0$ with no flux

$$\phi = \phi_{BHV} + \phi_{PW}$$

$$\phi_{BHV} = \omega_\Sigma \wedge \rho(w) + h.c.$$

holomorphic on Σ

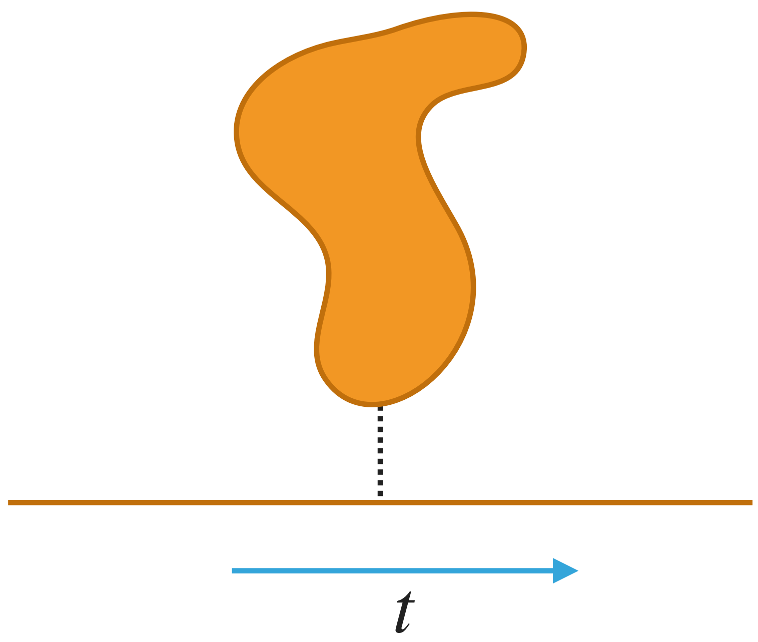
three poles on the cylinder

$$\phi_{PW} = \partial_z f dz \wedge dw + \partial_{\bar{z}} f d\bar{z} \wedge d\bar{w} + \frac{i}{2}\partial_t f (dz \wedge d\bar{z} + dw \wedge d\bar{w})$$

$$\partial_u f = \text{Re}\left[f_1(u)\frac{\tanh(u)+1}{2}\right] \quad \partial_v f = \text{Re}\left[f_2(v)\frac{\coth(v)+1}{2}\right]$$

$$u = t + ix$$
$$v = t + iy$$

PW TO PW INTERPOLATIONS



Take the four manifold $M = Q \times \mathbb{R}$

System will interpolate between different PW solutions (physically we build an interface)

Take $Q = \mathbf{T}^3$

To build solution: take a pair of coordinates (x, y) inside the $\mathbf{T}^3$

Build a PW solution on this $\mathbf{T}^2_{(x,y)} \times \mathbb{R}$

$$\phi_{(x,y)} = \mathrm{Re}\left[f_1(t+ix)\frac{\tanh(t+ix)+1}{2} + f_2(t+iy)\frac{\tanh(t+iy)+1}{2}\right]$$

Build a different PW solution with another $\mathbf{T}^2 \subset \mathbf{T}^3$, and combine them

ZERO MODES OF SPIN(7)

Zero modes equations come from taking infinitesimal variations around a solution

$$A = \langle A \rangle + a \qquad \phi = \langle \phi \rangle + \varphi$$

Zero mode equations for Spin(7) are

$$D_A a + \star D_A a + \phi \times \varphi = 0$$
$$D_A \varphi - [\phi, a] = 0$$

Solutions are identified via gauge transformations

$$a \simeq a + D_A \xi$$
$$\varphi \sim \varphi + [\phi, \xi]$$

This can be characterised in terms of a complex

$$0 \to \Omega^0(adE) \xrightarrow{\delta_0} \Omega^1(adE) \oplus \Omega^2_+(adE) \xrightarrow{\delta_1} \Omega^2_+(adE) \oplus \Omega^3(adE) \to 0$$

Space of zero modes is $\quad T\mathscr{M}_{Spin(7)} \simeq \dfrac{\ker \delta_1}{\text{im } \delta_0}$

ZERO MODES OF SPIN(7)

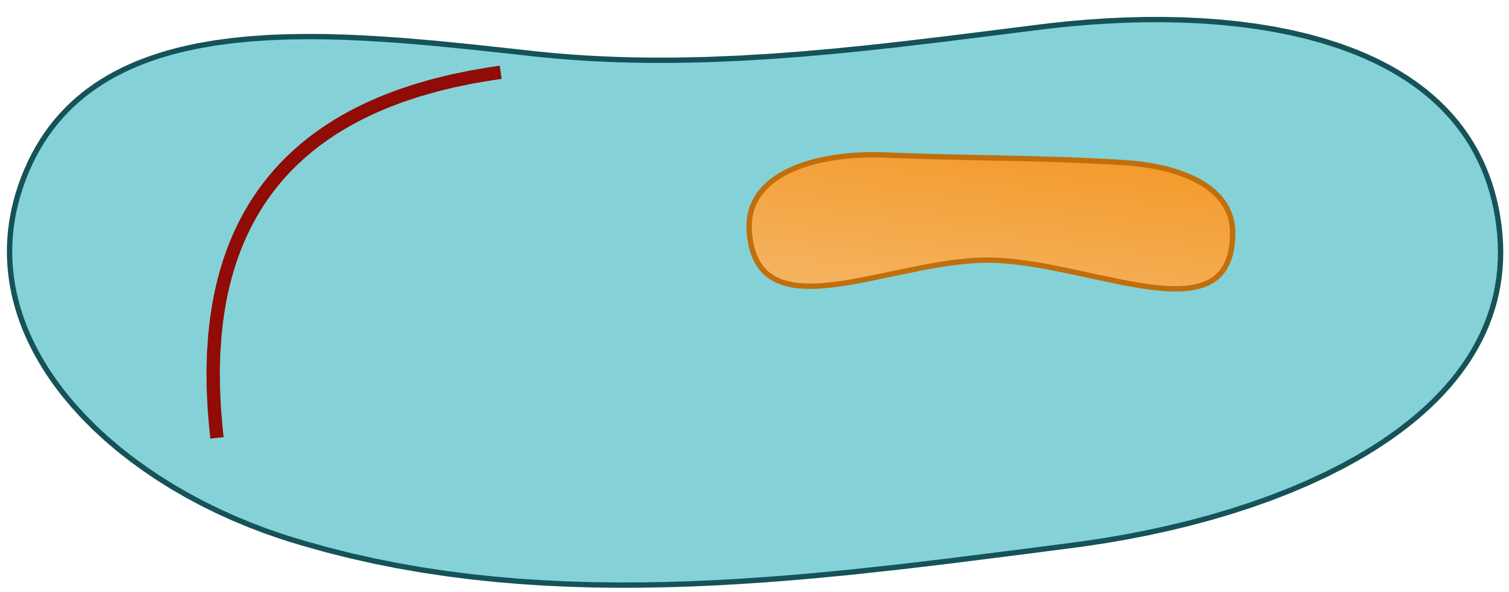
Where are zero modes localised?

Take abelian solutions (with no flux)

Use spectral cover methods: for $\mathfrak{a}_{n+1}$ in the fundamental rep

$$\det(\phi - v\mathbb{I}_n) = 0 \qquad v \in \Omega^2_+(M)$$

Modes are localised where sheets intersect: this happens in codimensions 2 and 3



CONCLUSIONS

We studied Higgs bundles that appear in Spin(7) compactifications of M-theory

This system provides a unification of other known Higgs bundles

It is possible to build interpolations between BHV and PW solutions

Moreover it is possible to build interfaces between PW solutions

Thank you!