A heterotic $G_2$ system is, by definition, the quadruple

\[ [(Y, \varphi), (V, A), (TY, \Theta), H] \]

where $\varphi$ is an integrable $G_2$ structure on a 7-dimensional manifold $Y$, $V$ is a gauge bundle with instanton connection $A$, $\Theta$ is a connection on the tangent bundle $TY$ of $Y$ which is also an instanton, and $H$ is a three form defined by the anomaly cancellation condition $H = DB + \frac{1}{4} (CS(A) - CS(\Theta))$. In string theory, such systems solve the Killing spinor equations and Bianchi identity of the heterotic string, provided that the three-form $H$ equals the torsion of the $G_2$ structure. As such, they provide an interesting class of effectively 3 dimensional supergravity theories, which are largely determined by the geometry of the compactification. It is a goal in physics to determine this effective field theory.

In this talk, I will discuss the mathematical structure of heterotic $G_2$ systems. We will see that the heterotic $G_2$ systems can be rephrased in terms of a differential $\mathcal{D}$ acting on a complex $\Omega^* (Y, Q)$, where $Q = T^* Y \oplus \text{End}(TY) \oplus \text{End}(V)$ and $\mathcal{D}$ is an appropriate projection of an exterior covariant derivative $\mathcal{D}$ which satisfies an instanton condition. The infinitesimal moduli are further parametrised by the first cohomology $H_1^\mathcal{D}(Y, Q)$. Finally, I will present a superpotential whose critical loci correspond to heterotic $G_2$ systems.

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