

Heterotic string theory and G2 structure manifolds

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Special Holonomy: Progress and Open Problems 2019

X. de la Ossa, ML, E. Svanes (1607.03473, 1704.08717, 1709.06974)

X. de la Ossa, ML, M. Magill, E. Svanes (1904.01027)

Motivation and summary

Compactifications of heterotic string on 7-dim manifolds with G_2 structure.

Physics motivation

Assuming minimal supersymmetry, what is the 3D effective field theory?

In particular: topology/geometry \rightsquigarrow spectrum, couplings, ...

Also: study dualities in string theory.

Math motivation

New perspective on geometry, deformations, invariants,...

In particular: understand coupled moduli space of vector bundle and geometry.

Motivation and summary

Heterotic string to $\mathcal{O}(\alpha')$

Green, Schwarz:84, Gross *et.al.*:85, Bergshoeff, deRoo:89

Physics description: 10D supergravity Lagrangian for

- Bosonic fields: Metric G , B-field B , dilaton ϕ , gauge field A for $G \subset E_8 \times E_8$
- Fermionic fields: Gravitino, dilatino, gaugino

Look for compactifications $\mathcal{M}_{10} = \mathcal{M}_3 \times Y$ that preserve SUSY

- SUSY $\iff \exists$ spinor λ on Y , nowhere vanishing, $\nabla_H \lambda = 0$
 \exists connection A on $V \rightarrow X$, such that $\gamma^{mn} F_{mn}(A) = 0$

Motivation and summary

Heterotic compactifications $\mathcal{M}_{10} = \mathcal{M}_3 \times Y$

We have the following mathematical objects

- 7-dim Riemannian manifold Y with metric g_{mn}
- Vector bundle $V \rightarrow Y$ with connection A and structure group $G \subset E_8 \times E_8$
- Scalar ϕ
- 3-form "flux" defined by $H = dB + \frac{\alpha'}{4}(CS(A) - CS(\Theta))$
where Θ is a connection of TY

SUSY constrains the geometry:

- Y has G_2 structure φ with torsion $T = H$.
- the connections A and Θ are G_2 instantons

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

Motivation and summary

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Key ideas:

- Reformulate heterotic system as nilpotent differential \check{D}
 - \check{D} acts on \mathcal{Q} -valued forms, where topologically $\mathcal{Q} = T^*Y \oplus \text{End}TY \oplus \text{End}V$.
- Infinitesimal moduli counted by \mathcal{Q} -valued canonical G_2 cohomology $H_{\check{D}}^1(\mathcal{Q})$.
- Comparison with 4D $\mathcal{N} = 1$ Strominger–Hull system.
- Superpotential.
- Conclusions and outlook.

G_2 structures

Bonan:66, Fernandez–Gray:82, Bryant:87,03, Hitchin:00, Joyce:00, Chiossi–Salamon:02

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

- (Y, φ) has G_2 structure specified by non-degenerate associative 3-form φ

Comment: true whenever Y is orientable and spin.

- $\varphi \rightarrow$ Riemannian metric g on Y , and a coassociative 4-form $\psi = *\varphi$

- Heterotic compactifications:

SUSY constrains $d\varphi$ and $d\psi$, i.e. the torsion of the G_2 structure

G_2 structures

Fernandez-Ugarte:98, Friedrich-Ivanov:01, Gauntlett et.al.:01, ...

- (Y, φ) has G_2 structure specified by non-degenerate associative 3-form φ
- Torsion: decompose into *torsion classes* \sim irreps of G_2 :

$$d\varphi = \tau_0 \psi + 3\tau_1 \wedge \varphi + *\tau_3 ,$$

$$d\psi = 4\tau_1 \wedge \psi + *\tau_2 ,$$

$$\Lambda^4 = \Lambda_1^4 \oplus \Lambda_7^4 \oplus \Lambda_{27}^4 ,$$

$$\Lambda^5 = \Lambda_7^5 \oplus \Lambda_{14}^5 .$$

- Heterotic compactifications:

$$\text{SUSY} \iff \tau_0 \text{ constant, } 2\tau_1 = d\phi, \boxed{\tau_2 = 0}$$

$\Lambda^k(Y)$ decomposes into $\Lambda_p^k(Y)$, p denotes G_2 irrep. Find these using φ :

Example: $\Lambda^1 = \Lambda_7^1 = T^*Y \cong TY$

\implies any $\beta \in \Lambda^2$ decomposes as $\beta = \alpha \lrcorner \varphi + \gamma$, where $\alpha \in \Lambda^1$ and $\gamma \lrcorner \varphi = 0$

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G_2 structures

- (Y, φ) has G_2 structure specified by non-degenerate associative 3-form φ

- Heterotic compactifications:

SUSY \implies $\tau_2 = 0$ \rightsquigarrow integrable G_2 structure: (cf. Fernandez, Ugarte:98)

$$\boxed{d\varphi = i_H(\varphi), d\psi = i_H(\psi)} \text{ where } H = \frac{1}{6} \tau_0 \varphi - \tau_1 \lrcorner \psi - \tau_3 .$$

- No H -flux $\iff Y$ has G_2 holonomy
- H : torsion of unique G_2 compatible connection $\nabla\varphi = 0 = \nabla\psi$

Encode geometry by a differential? cf. Dolbeault differential on complex manifold

G_2 structures

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Encode geometry by a differential? cf. Dolbeault differential on complex manifold

G_2 structures: canonical G_2 cohomology

Decomposition of de Rham cohomology

Reyes-Carrion:93, Fernandez-Ugarte:98

Analogue of Dolbeault operator: project d onto G_2 irreps.

- Define differential operator \check{d} by

$$\check{d}_0 = d, \quad \check{d}_1 = \pi_7 \circ d, \quad \check{d}_2 = \pi_1 \circ d.$$

- Can show

$$\tau_2 = 0 \iff \check{d}^2 = 0$$

cf. Dolbeault differential $\bar{\partial}$
 \rightsquigarrow "integrable G_2 structure"

- Differential, elliptic complex

$$0 \rightarrow \Lambda^0(Y) \xrightarrow{\check{d}} \Lambda^1(Y) \xrightarrow{\check{d}} \Lambda^2_7(Y) \xrightarrow{\check{d}} \Lambda^3_1(Y) \rightarrow 0$$

\rightarrow canonical G_2 -cohomology $H_{\check{d}}^*(Y)$

cf. Dolbeault cohomology $H_{\bar{\partial}}^*(X)$

G_2 instanton bundle

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

SUSY constrains $F(A)$: $F(A) \wedge \psi = 0 \iff A$ is G_2 instanton

Reyes-Carrion:93, 98, Fernandez-Ugarte:98

Canonical G_2 -cohomology for instanton bundle

(cf. holomorphic bundles on complex manifolds)

- Recall: $\tau_2 = 0 \iff \check{d}^2 = 0$

$$\check{d}_0 = d, \quad \check{d}_1 = \pi_7 \circ d, \quad \check{d}_2 = \pi_1 \circ d.$$

- Generalizes to vector bundles V with instanton connection:

$$\tau_2 = 0 \text{ and } F(A) \wedge \psi = 0 \iff \check{d}_A^2 = 0$$

Anomaly cancellation condition

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

Anomaly cancellation condition

$$dB + \frac{\alpha'}{4}(CS(A) - CS(\Theta)) = H = \frac{1}{6}\tau_0\varphi - \tau_1\lrcorner\psi - \tau_3.$$

→ Bianchi identity

$$\frac{\alpha'}{4}(\text{tr}F(A) \wedge F(A) - \text{tr}R(\Theta) \wedge R(\Theta)) = dH = d\left(\frac{1}{6}\tau_0\varphi - \tau_1\lrcorner\psi - \tau_3\right).$$

Other conditions?

Hull:86, Ivanov:10, Martelli–Sparks:10

SUSY+anomaly cancellation \implies EOM $\iff \Theta$ is a G_2 instanton

$$R(\Theta) \wedge \psi = 0$$

Geometry as differential

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$:

- 1 (Y, φ) 7-manifold with integrable G_2 structure
- 2 $(V, A), (TY, \Theta)$ G_2 instanton bundles
- 3 Anomaly cancellation $dB + \frac{\alpha'}{4}(CS(A) - CS(\Theta)) = H = \frac{1}{6} \tau_0 \varphi - \tau_{1 \dashv 7} \psi - \tau_3$

Encode constraints as nilpotency $\check{D}^2 = 0$ of suitable differential \check{D}

Geometry as differential

Want: Heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H] \iff \check{D}^2 = 0$

- Start with

$$\mathcal{D} = \begin{pmatrix} d_\zeta & \mathcal{R} & -\mathcal{F} \\ \mathcal{R} & d_\Theta & 0 \\ \mathcal{F} & 0 & d_A \end{pmatrix}$$

- \mathcal{D} acts on Q -valued forms where topologically $Q = T^*Y \oplus \text{End}TY \oplus \text{End}V$
- $d_A = d + A \wedge$ etc.
- d_ζ has torsion $-H$ (cf. G_2 connection ∇_H)
- \mathcal{F}, \mathcal{R} linear maps

$$\mathcal{F}: \Omega^p(Y, T^*Y) \oplus \Omega^p(Y, \text{End}(V)) \longrightarrow \Omega^{p+1}(Y, \text{End}(V)) \oplus \Omega^{p+1}(Y, T^*Y)$$

$$\mathcal{F}(M) = (-1)^p g^{ab} M_a \wedge F_{bc} dx^c,$$

$$\mathcal{F}(\alpha)_a = (-1)^p \frac{\alpha'}{4} \text{tr}(\alpha \wedge F_{ab} dx^b).$$

Geometry as differential

Theorem: Heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H] \iff \check{D}^2 = 0$

Sketch of proof: Compute the square

$$\mathcal{D}^2 = \begin{pmatrix} d_\zeta^2 + \mathcal{R}^2 - \mathcal{F}^2 & d_\zeta \mathcal{R} + \mathcal{R} d_\Theta & -(d_\zeta \mathcal{F} + \mathcal{F} d_A) \\ \mathcal{R} d_\zeta + d_\Theta \mathcal{R} & \mathcal{R}^2 + d_\Theta^2 & -\mathcal{R} \mathcal{F} \\ \mathcal{F} d_\zeta + d_A \mathcal{F} & \mathcal{F} \mathcal{R} & -\mathcal{F}^2 + d_A^2 \end{pmatrix},$$

Next, project to get \check{D}

$$\check{D}_0 = \mathcal{D}, \quad \check{D}_1 = \pi_7 \circ \mathcal{D}, \quad \check{D}_2 = \pi_1 \circ \mathcal{D}.$$

Find: components of \mathcal{D}^2 are (combinations of) heterotic KSE and BI, q.e.d.

Infinitesimal moduli

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

Key ideas:

- Reformulate heterotic system as nilpotent differential \check{D} ✓
- Infinitesimal moduli counted by \mathcal{Q} -valued canonical G_2 cohomology $H_{\check{D}}^1(\mathcal{Q})$

Infinitesimal moduli

Consider a family

$$[(Y, \varphi(t)), (V, A(t)), (TY, \Theta(t)), H(t)]$$

which for $t = 0$ gives a heterotic G_2 system.

First guess for infinitesimal moduli

- $\delta_t \psi$ (determines $\delta_t \varphi$): geometric moduli: $H^3(Y)$ if $H = 0$
 $H = 0$: Joyce:96, Hitchin:00, Dai, Wang, Wei:03, de Boer, Naqvi, Shomer:05,...
- $\delta_t A$: Vector bundle moduli: $H^1(Y, \text{End}(V))$
 $H = 0$: Sa-Earp:09,...
- $\delta_t B$: deformations of B -field, $H = dB + \alpha'(\dots)$

SUSY and anomaly cancellation couples these objects; need more details.

Infinitesimal moduli

Geometric moduli: deformations of $[(Y, \varphi)]$

Will encode deformations in terms of 1-forms:

- Let $\delta_t \psi = i_M(\psi) = \frac{1}{3!} M_t^a \wedge \psi_{bcda} dx^{bcd}$, where $M_t \in \Lambda^1(Y, T^*Y)$.

- Preserve $\tau_2 = 0$:

$$i_{\check{d}_\zeta M_t}(\psi) = 0$$

- Diffeomorphisms:

$$\mathcal{L}_V \psi = i_{\check{d}_\zeta V}(\psi) \quad \text{where } V \in \Lambda^0(Y, T^*Y)$$

$$\implies \mathcal{TM}_Y = \{M_t : i_{\check{d}_\zeta M_t}(\psi) = 0\} / \{M_t : M_t = \check{d}_\zeta V\}$$

Infinitesimal moduli

Geometric moduli: deformations of $[(Y, \varphi)]$

$$\mathcal{T}\mathcal{M}_Y = \{M_t : i_{\check{d}_\zeta M_t}(\psi) = 0\} / \{M_t : M_t = \check{d}_\zeta V\}$$

- But $\check{d}_\zeta^2 \neq 0$: ζ is not an instanton

Dimension of infinitesimal geometric moduli space is not finite, in general.

- Exception: G_2 holonomy $\mathcal{T}\mathcal{M}_Y \cong H_d^3(Y) \cong H_{\check{d}_\zeta}^1(Y, TY)$

Infinitesimal moduli

Deformations of $[(Y, \varphi), (V, A)]$

Want deformations that preserve $F \wedge \psi = 0$

$$\implies \check{d}_A \alpha_t = d_A \alpha_t \wedge \psi = -F \wedge i_{M_t}(\psi) = \check{F}(M_t)$$

- Vector bundle moduli $d_A \alpha_t = 0 \rightsquigarrow H^1(Y, \text{End}(V))$
- Geometric moduli must lie in kernel of map \check{F}

$$\implies \boxed{\mathcal{T}\mathcal{M}_{[(Y, \varphi), (V, A)]} = H^1(Y, \text{End}(V)) \oplus \ker \check{F}, \quad \ker \check{F} \subset \mathcal{T}\mathcal{M}_Y}$$

- Not enough to prove finiteness when $H \neq 0$

Infinitesimal moduli

Infinitesimal moduli of $[(Y, \varphi), (V, A), (TY, \Theta), H]$

Consider $\mathcal{Z}_t = (M_t, \kappa_t, \alpha_t)^T \in \Lambda^1(Y, \mathcal{Q})$ and compute

$$\mathcal{D}\mathcal{Z}_t = \begin{pmatrix} d_\zeta M_t + \mathcal{R}(\kappa_t) - \mathcal{F}(\alpha_t) \\ d_\Theta \kappa_t + \mathcal{R}(M_t) \\ d_A \alpha_t + \mathcal{F}(M_t) \end{pmatrix} \quad \text{with } \mathcal{D} = \begin{pmatrix} d_\zeta & \mathcal{R} & -\mathcal{F} \\ \mathcal{R} & d_\Theta & 0 \\ \mathcal{F} & 0 & d_A \end{pmatrix}$$

- Heterotic constraints $*$ $\implies \boxed{\check{D}\mathcal{Z}_t = 0}$

*Subtlety: B -field variations do not decouple! \rightsquigarrow antisymmetric part of matrix M_t

- Diffeomorphisms and gauge symmetry $\implies \boxed{\mathcal{Z}_{triv} = \check{D}\mathcal{V}}$

$$\implies \boxed{\mathcal{TM}_{[(Y, \varphi), (V, A), (TY, \Theta), H]} \cong H_{\check{D}}^1(\mathcal{Q})}$$

- This moduli space is finite dimensional.

Infinitesimal moduli

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

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- Reformulate heterotic system as nilpotent differential \check{D} ✓
 - \check{D} acts on \mathcal{Q} -valued forms where topologically $\mathcal{Q} = T^*Y \oplus \text{End}TY \oplus \text{End}V$
- Infinitesimal moduli $\sim \mathcal{Q}$ -valued canonical G_2 cohomology $H_{\check{D}}^1(\mathcal{Q})$ ✓

Comparison of 3D and 4D heterotic $\mathcal{N} = 1$ systems

6D Strominger–Hull system

$[(X, \Omega, \omega), (V, A), (TY, \Theta), H]$

- Nilpotent differential \bar{D}

$T^*_{(1,0)}X \oplus \text{End}(TX) \oplus \text{End}(V) \oplus T_{(1,0)}X$

- SUSY \rightarrow F-term/D-term
- \bar{D} upper triangular
- Hol. Courant algebroid
 \rightsquigarrow Hitchin's generalised geometry

7D Heterotic G_2 system

$[(Y, \varphi), (V, A), (TY, \Theta), H]$

- Nilpotent differential \check{D}

$T^*Y \oplus \text{End}(TY) \oplus \text{End}(V)$

- SUSY \rightarrow F-term
- \check{D} not upper triangular
- ———
but, see Clarke *et.al.*:16

6D deformation theory *Atiyah:57, Kodaira, Spencer:58,60, Candelas, de la Ossa:91, Becker, Tseng, Yau:06, Anderson, et.al:10,11,13, Fu, Yau:11, Baraglia, Hekmati:13, Anderson, Gray, Sharpe:14, de la Ossa, Svanes:14, Garcia-Fernandez, et.al:13,15,...*

Superpotential

Alternative perspective:

Moduli space as critical locus of a superpotential on off-shell parameter space

Strategy

- Dimensional reduction \implies 3D gravitino mass $M_{3/2} = e^K W$

Remark: need Hessian K

- Check $\delta W = 0 \iff \mathcal{N} = 1$ heterotic G_2 system
- Follows that $\delta^2 W \iff$ equation for moduli

Superpotential: Dimensional reduction

Fermionic part of 10D heterotic supergravity action

Bergshoeff, de Roo:89, Gurrieri, Lukas, Micu:07

→ 3D kinetic and mass terms for gravitino

$$S_{0,f} = -\frac{1}{2\kappa_{10}^2} \int_{M^{10}} d^{10}x \sqrt{-g} e^{-2\phi} \left(\bar{\Psi}_M \Gamma^{MNP} D_N \Psi_P - \frac{1}{24} \left(\bar{\Psi}_M \Gamma^{MNPQR} \Psi_R + 6 \bar{\Psi}^N \Gamma^P \Psi^Q \right) H_{NPQ} \right)$$

$$S_{3D} \supset -\frac{1}{2\kappa_3^2} \int d^3x \sqrt{-g} \left(\bar{\psi}_\mu \Gamma^{\mu\nu\kappa} D_\nu \psi_\kappa + m \bar{\psi}_\mu \Gamma^{\mu\kappa} \psi_\kappa \right)$$

Straightforward, but

- field normalisation (correct EH term)
- conventions for gravitino mass in AdS (want SUSY $\leftrightarrow 0 = M_{3/2} \sim W$).

Superpotential: Dimensional reduction

To match 3D EH term decompose

$$\begin{aligned}g_{10} &= e^n g_3 \oplus g_7 \\ \Gamma_{(10)}^\mu &= e^{-n/2} \Gamma_{(3)}^\mu \otimes \text{Id} \otimes \sigma_2 \\ \Gamma_{(10)}^i &= \text{Id} \otimes \Gamma_{(7)}^i \otimes \sigma_1 \\ \Psi_\mu &= e^{n/2} (\rho_\mu \otimes \lambda \otimes \theta) \\ \bar{\Psi}_\mu &= (\bar{\rho}_\mu \otimes \lambda^\dagger \otimes \theta^\dagger \sigma_2),\end{aligned}$$

Then, e.g.

$$\begin{aligned}& \int d^{10} X \sqrt{-g_{10}} e^{-2\phi} \left(-\frac{1}{24} \bar{\Psi}_\mu \Gamma^{\nu\mu} \Gamma^{ijk} \Psi_\nu H_{ijk} \right) \\ &= \int d^3 x \sqrt{-g_3} (\bar{\rho}_\mu \Gamma^{\mu\nu} \rho_\nu) \left[\frac{1}{24} \int d^7 y \sqrt{g_7} e^{-2\phi+n} (-i \lambda^\dagger \Gamma^{ijk} \lambda) H_{ijk} \right] \\ \implies m &\sim \frac{1}{4} \int *_7 H \wedge \varphi \cdot e^{-2\phi+n}.\end{aligned}$$

Superpotential: Dimensional reduction

Result:

Three mass contributions

$$\tilde{M}_{3/2} = -\frac{1}{8} \int_7 e^{-2\phi+n} (d\varphi \wedge \varphi - 2 *_7 H \wedge \varphi + 2 *_7 f), \quad (1)$$

Fixing conventions so that SUSY $\implies M_{3/2} = 0$ (with $h = -\frac{2}{7}f$)

$$M_{3/2} = \frac{1}{4} \int_7 e^{-2\phi+n} \left(-\frac{1}{2} d\varphi \wedge \varphi + (H + h\varphi) \wedge \psi \right).$$

Analysing the Einstein–Hilbert term, identify $K \simeq n \implies$

$$W = \frac{1}{4} \int_Y e^{-2\phi} \left((H + h\varphi) \wedge \psi - \frac{1}{2} d\varphi \wedge \varphi \right)$$

Superpotential: Variations and SUSY

Want to show $\delta W = 0 \iff \mathcal{N} = 1$ heterotic G_2 system

$$\begin{aligned} \delta W = \int_Y e^{-2\phi} \left\{ -2 \delta\phi \left((H + h\varphi) \wedge \psi - \frac{1}{2} d\varphi \wedge \varphi \right) \right. \\ \left. - \mathcal{B} \wedge e^{2\phi} d(e^{-2\phi} \psi) + \frac{\alpha'}{2} [\text{tr}(\delta A F \wedge \psi) - \text{tr}(\delta \Theta R(\Theta) \wedge \psi)] + \right. \\ \left. + (H + h\varphi) \wedge \delta\psi + \delta\varphi \wedge (h\psi - d\varphi + d\phi \wedge \varphi) \right\} \end{aligned}$$

with $\delta H = d\mathcal{B} + \frac{\alpha'}{2} (\text{tr}(F\delta A) - \text{tr}(R(\Theta)\delta\Theta))$.

Result: critical points of $W \iff$ heterotic G_2 system with $\tau_1 = \frac{1}{2} d\phi$, $h = \frac{1}{3} \tau_0$,
and $W = 0$ ✓

Conclusions and outlook

Conclusions

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- Infinitesimal moduli $\sim \mathcal{Q}$ -valued canonical G_2 cohomology $H_{\check{D}}^1(\mathcal{Q})$
- Superpotential W s.t. $\delta W = W = 0 \iff \mathcal{N} = 1$ heterotic G_2 system

Conclusions and outlook

Outlook

- Examples?

Instantons on G2 holonomy: Walpuski:13, Sá-Earp, Menet, Nordström, Sá-Earp:15

Heterotic G2 systems (with flux):

Font:10, Fernandez, Ivanov, Ugarte, Villacampa:11, F, I, U, Vassiliev:15, Hinoue, Yasui:14

- Compute cohomologies?

- $W \rightsquigarrow$ new perspective on Donaldson–Segal invariants?

- Structure of moduli space: metric, Hessian potential, singularities, ...?

- Higher order deformations: obstructions, integrability, Maurer–Cartan eq?

- “Decompactification” and relation to $SU(3)$ structure moduli spaces?

- Dualities? Geometric transitions?

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Thank You!