Special Holonomy Metrics, Degenerate Limits and Intersecting Branes

Nipol Chaemjumrus and CH:

[arXiv:1907.04040] Degenerations of K3, Orientifolds and Exotic Branes

1. Degenerate Limit of K3

2. Gibbons-Hawking Metrics

3. Ooguri-Vafa Metrics

4. Special Holonomy Generalisations

5. Intersecting Brane Duals

6. Applications to String Theory and Dualities: Type I’ string, D8-branes and orientifolds
Degenerate Limit of K3

Hein, Sun, Viaclovsky and Zhang [HSVZ]

- Family of K3 Metrics $g(t)$, limit $t=0$ is line interval
- Long Neck Region at small $t$
- Segment of neck is nilfold fibred over a line.
- Nilfold is $S^1$ bundle over $T^2$, with degree (Chern number) $m$. Different values of $m$ in different segments.
- Jump in $m$: insertion of gravitational instanton (Kaluza Klein monopole)
- Ends of neck capped by Tian-Yau spaces: complete non-compact hyperkahler manifolds asymptotic to nilfold fibred over a line
Figure 1. The vertical arrows represent collapsing to a one-dimensional interval. The red circles represent the $S^1$ fibers and the blue curves represent the base $T^2$'s of the nilmanifolds. The $\times$'s are the monopole points in the neck region $\mathcal{N}$. The gray regions are in the “damage zones”.
Gibbons-Hawking Metric

Hyperkahler metric with $S^1$ symmetry

$$ g = V (d\tau^2 + dx^2 + dz^2) + V^{-1} (dy + \omega)^2 $$

$V(\tau, x, z)$ a harmonic function on $\mathbb{R}^3$

$$ \vec{\nabla} \times \vec{\omega} = \vec{\nabla} V $$

Delta-function sources at points (m an integer)

$$ V = a + \sum_i \frac{m}{|\vec{r} - \vec{r}_i|} $$

$S^1$ Bundle on $\mathbb{R}^3 - \{\text{points}\}$

Regular at sources if $m=1$: multi-Taub-NUT

Orbifold singularities for $m>1$
Smeared GH Metrics

\[ V(\tau, x, z) \text{ a harmonic function on } \mathbb{R}^3 \]

“Smeared” solutions: \( V \) independent of one or more coordinates

Can then take those coordinates to be periodic
Metric typically singular

Smear on \( x, y \):
\[ V(\tau) = m\tau + c \]

or
\[ V(\tau) = \begin{cases} 
  c + m'\tau, & \tau \leq 0 \\
  c + m\tau, & \tau > 0.
\end{cases} \]

Singular at kink at \( \tau = 0 \)

Domain wall: 2-plane dividing space into 2 parts

\( N=m-m' \): energy density (tension) of domain wall (2-brane)
Piecewise linear:
**multi-wall solution** with domain walls at $\tau = \tau_1, \tau_2, \ldots \tau_n$

$$V(\tau) = \begin{cases} 
  c_1 + m_1 \tau, & \tau \leq \tau_1 \\
  c_2 + m_2 \tau, & \tau_1 < \tau \leq \tau_2 \\
  \vdots \\
  c_n + m_n \tau, & \tau_{n-1} < \tau \leq \tau_n \\
  c_{n+1} + m_{n+1} \tau, & \tau > \tau_n 
\end{cases}$$

The charge of the domain wall at $\tau_r$ is the integer

$$N_r = m_{r+1} - m_r$$

$$ds^2 = V(\tau)(d\tau^2 + dx^2 + dz^2) + \frac{1}{V(\tau)}(dy + M(\tau)x dz)^2$$

$$M(\tau) \equiv V'(\tau)$$

**Can take** x,y,z **periodic**

**Single-sided domain wall**

$$V = c + m |\tau|$$

Quotient by reflection $\tau \rightarrow -\tau$ gives

“single-sided” wall at $\tau = 0$
\[ V(\tau) = m\tau + c \]
\[ ds^2 = V(\tau)(d\tau^2 + dx^2 + dz^2) + \frac{1}{V(\tau)}(dy + mxdz)^2 \]

Take \(x, y, z\) periodic

Fixed \(\tau\): nilfold

\[ ds^2_{\mathcal{N}} = dx^2 + (dy + mxdz)^2 + dz^2 \]

\(S^1\) Bundle over \(T^2\)

\[ F = mdx \wedge dz \]

Quotient of the group manifold of the Heisenberg group by a cocompact discrete subgroup

4-d space: nilfold fibred over a line
1st approximation to HSVZ K3

Interval $\tau \in [0, \pi]$

Multi-domain wall solution with domain walls at $\tau = \tau_1, \tau_2, \ldots \tau_n$

Single-sided domain walls at $\tau = 0, \pi$

$$ds^2 = V(\tau)(d\tau^2 + dx^2 + dz^2) + \frac{1}{V(\tau)}(dy + M(\tau)xdz)^2$$

$$V(\tau) = \begin{cases} 
  c_1 + m_1\tau, & 0 \leq \tau \leq \tau_1 \\
  c_2 + m_2\tau, & \tau_1 < \tau \leq \tau_2 \\
  \vdots \\
  c_n + m_n\tau, & \tau_{n-1} < \tau \leq \tau_n \\
  c_{n+1} + m_{n+1}\tau, & \tau_n < \tau \leq \pi 
\end{cases}$$

HSVZ resolve singularities:

- Resolve domain walls with Ooguri-Vafa construction
- Resolve single-sided domain walls with Tian-Yau spaces
Ooguri-Vafa Metric

Want Gibbons-Hawking metric, $\mathbb{R}^3$ replaced with $\mathbb{R} \times T^2$

1st approximation: smear over $T^2$

**Ooguri-Vafa:**

- On $\mathbb{R}^3$, take periodic array of sources in $(x,z)$ plane
- Regularised sum of potentials gives harmonic function
- Can now periodically identify $x,z$ directions, to get single source on $\mathbb{R} \times T^2$
- Near source, non-singular, looks like Taub-NUT
- Can then take superpositions to get multiple sources on $\mathbb{R} \times T^2$
- Solutions regular on finite interval in $\mathbb{R}$
Resolve GH metric with

\[ V(\tau) = \begin{cases} 
  c + m'\tau, & \tau \leq 0 \\
  c + m\tau, & \tau > 0.
\end{cases} \]

Charge \( N = m - m' \)

by OV metric with \( V \) harmonic on \( \mathbb{R} \times T^2 \)

Monopole charge \( N \)

Near sources, \( N \)-centre multi Taub-NUT, or one source of charge \( N \), orbifold singularity: bubbling limit to Taub-NUT

For \( N \) sources, regular hyperkahler metric for some interval

\[ -T < \tau < T' \]

Far enough away from \( \tau = 0 \), tends to GH with

\[ V(\tau) = \begin{cases} 
  c + m'\tau, & \tau \leq 0 \\
  c + m\tau, & \tau > 0.
\end{cases} \]
Tian-Yau Spaces

• Complete non-singular non-compact hyperkähler space
• Asymptotic to a nilfold bundle over a line.
• Of the form \( M \setminus D \), where \( M \) is a del Pezzo surface, \( D \subset M \) is a smooth anticanonical divisor
• Del Pezzo surfaces are complex surfaces classified by their degree \( b \), where \( b = 1, 2, \ldots, 9 \)
• The del Pezzo surface of degree nine is \( \mathbb{CP}^2 \)
• A degree \( b \) del Pezzo surface can be constructed from blowing up \( 9 - b \) points in \( \mathbb{CP}^2 \)
• A 2nd del Pezzo surface of degree 8 is \( \mathbb{CP}^1 \times \mathbb{CP}^1 \)
• The TY space \( M_b \) of degree \( b \) is constructed from del Pezzo of degree \( b \)
• \( M_b \) is asymptotic to GH metric on \( N_b \times \mathbb{R} \) where \( N_b \) is nilfold of degree \( b \)
• Degree zero: Take \( M \) to be rational elliptic surface, \( N_0 = T^3 \), \( M_0 \) is ALH, asymptotic to cylinder given by \( T^3 \times \mathbb{R} \)
1st approximation to HSVZ K3

Interval $\tau \in [0,\pi]$

Multi-domain wall solution with domain walls at $\tau = \tau_1, \tau_2, \ldots \tau_n$

Single-sided domain walls at $\tau = 0, \pi$

$$ds^2 = V(\tau)(d\tau^2 + dx^2 + dz^2) + \frac{1}{V(\tau)}(dy + M(\tau)xdz)^2$$

$$V(\tau) = \begin{cases} 
  c_1 + m_1\tau, & 0 \leq \tau \leq \tau_1 \\
  c_2 + m_2\tau, & \tau_1 < \tau \leq \tau_2 \\
  \vdots \\
  c_n + m_n\tau, & \tau_{n-1} < \tau \leq \tau_n \\
  c_{n+1} + m_{n+1}\tau, & \tau_n < \tau \leq \pi 
\end{cases}$$

$M(\tau) \equiv V'(\tau)$

HSVZ resolve singularities:
Glue together Ooguri-Vafa spaces, Tian-Yau spaces to get complete K3 metric

Tian-Yau spaces of degree $b_-, b_+$

$$b_- = -m_1, b_+ = m_{n+1} \quad 0 \leq b_\pm \leq 9$$

$$N_i = m_{i+1} - m_i$$

$$\sum_{i=1}^{n} N_i = b_- + b_+ \leq 18$$
Special Holonomy Generalisations

- Replace 3-d nilfold with higher dim nilmanifold
- Quotient of nilpotent Lie group by discrete subgroup
- $T^n$ bundle over $T^m$
- Special holonomy metrics on nilmanifold fibred over a line
  Gibbons, Lu, Pope and Stelle [GLPS]
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Nilmanifold: $S^1$ bundle over $T^4$

5-dimensional nilpotent Lie algebra $T_i$

Only non-vanishing commutators are

$$[T_2, T_3] = mT_1, \quad [T_4, T_5] = mT_1$$

Metric

$$ds^2 = \left( dz^1 + m(z^3dz^2 + z^5dz^4) \right)^2 + (dz^2)^2 + (dz^3)^2 + (dz_4)^2 + (dz^5)^2.$$ 

Fibre coord: $z^1$

**Nilmanifold** fibred over line: SU(3) holonomy

$$ds^2 = V^2(\tau)(d\tau)^2 + V(\tau)\left( (dz^2)^2 + (dz^3)^2 + (dz^4)^2 + (dz^5)^2 \right)$$

$$+ V^{-2}(\tau) \left( dz^1 + M(\tau)(z^3dz^2 + z^5dz^4) \right)^2$$

$$M(\tau) \equiv V'(\tau)$$

V Piecewise linear
**Nilmanifold**: $T^3$ bundle over $T^4$

7-dimensional nilpotent Lie algebra $T_i$

Only non-vanishing commutators are

$$[T_4, T_5] = mT_1, \quad [T_6, T_7] = mT_1, \quad [T_4, T_6] = mT_2$$

$$[T_5, T_7] = -mT_2, \quad [T_4, T_7] = mT_3, \quad [T_5, T_6] = mT_3$$

Metric

$$ds^2 = \left( dz^1 + m(z^5 dz^4 + z^7 dz^6) \right)^2 + \left( dz^2 + m(z^6 dz^4 - z^7 dz^5) \right)^2$$

$$+ \left( dz^3 + m(z^7 dz^4 + z^6 dz^5) \right)^2 + (dz^4)^2 + (dz^5)^2 + (dz^6)^2 + (dz^7)^2$$

Fibre coords: $z^1 z^2 z^3$

**Nilmanifold** fibred over line: Spin(7) holonomy

$$ds^2 = V_6^2(\tau)(d\tau)^2 + V_3^2(\tau)\left( (dz^4)^2 + (dz^5)^2 + (dz^6)^2 + (dz^7)^2 \right)$$

$$+ V^{-2}(\tau) \left( dz^1 + M(z^5 dz^4 + z^7 dz^6) \right)^2$$

$$+ V^{-2}(\tau) \left( dz^2 + M(z^6 dz^4 - z^7 dz^5) \right)^2 + V^{-2}(\tau) \left( dz^3 + M(z^7 dz^4 + z^6 dz^5) \right)^2$$

$$M(\tau) \equiv V'(\tau)$$
Further Nilmanifolds

Extends to further examples with more general nilmanifolds. The previous case are from 2-step nilpotent Lie groups giving a torus bundle over a torus.

$p$-step nilpotent Lie groups
\[[X_1, [X_2, \cdots [X_p, Y] \cdots]] = 0\]
Torus bundle over a torus bundle over a torus…..

Chiossi and Salamon:
For any 6-dimensional nilmanifold with half-flat SU(3) structure, fibration over a line interval gives a $G_2$ metric
24 cases.

SU(3) structure: almost complex 6-manifold, with $(1,1)$ form $\omega$, $(3,0)$ form $\Omega$
Half-flat: $\omega \wedge d\omega$ and $Re(\Omega)$ closed
T-duality

Taub-NUT \rightarrow NS 5-brane [CH+Townsend]

ALF Multi-instanton \rightarrow Multi 5-brane

GLPS Special Holonomy \rightarrow Intersecting 5-brane solution with one function [Chaemjumrus and CH]

Special Holonomy with several functions \rightarrow Intersecting 5-brane solution with several functions

Semi-local solution
**Nilmanifold**: $S^1$ bundle over $T^4$

**Nilmanifold** fibred over line: SU(3) holonomy

$$ds^2 = V^2(\tau)(d\tau)^2 + V(\tau)\left((dz^2)^2 + (dz^3)^2 + (dz^4)^2 + (dz^5)^2\right)$$

$$+ V^{-2}(\tau)\left(dz^1 + M(\tau)(z^3dz^2 + z^5dz^4)\right)^2 \quad M(\tau) \equiv V'(\tau)$$

2-function generalisation, SU(3) holonomy

$$ds^2 = V_1(\tau)V_2(\tau)d\tau^2 + \frac{1}{V_1(\tau)V_2(\tau)}\left(dz^1 + M_1(\tau)z^3dz^2 + M_2(\tau)z^5dz^4\right)^2$$

$$+ V_1(\tau)\left((dz^2)^2 + (dz^3)^2\right) + V_2(\tau)\left((dz^4)^2 + (dz^5)^2\right)$$

Functions $V_1(\tau), V_2(\tau)$ piecewise linear,

$M_1 = V_1, M_2 = V'_2$

[Chaemjumrus and CH]
Nilmanifold: $T^3$ bundle over $T^4$

Nilmanifold fibred over line: Spin(7) holonomy

$$ds^2 = V^6(\tau)(d\tau)^2 + V^3(\tau)\left((dz^4)^2 + (dz^5)^2 + (dz^6)^2 + (dz^7)^2\right)$$

$$+ V^{-2}(\tau)\left(dz^1 + M(z^5dz^4 + z^7dz^6)\right)^2$$

$$+ V^{-2}(\tau)\left(dz^2 + M(z^6dz^4 - z^7dz^5)\right)^2 + V^{-2}(\tau)\left(dz^3 + M(z^7dz^4 + z^6dz^5)\right)^2$$

6-function generalisation, Spin(7) holonomy

$$ds^2 = V_1(\tau)V_2(\tau)V_3(\tau)V_4(\tau)V_5(\tau)V_6(\tau)d\tau^6 + \frac{1}{V_1(\tau)V_2(\tau)}\left(dz^1 + M_1z^5dz^4 + M_2z^7dz^6\right)^2$$

$$+ \frac{1}{V_3(\tau)V_4(\tau)}\left(dz^2 + M_3z^6dz^4 - M_4z^7dz^5\right)^2 + \frac{1}{V_5(\tau)V_6(\tau)}\left(dz^3 + M_5z^7dz^4 + M_6z^6dz^5\right)^2$$

$$+ V_1(\tau)V_3(\tau)V_5(\tau)(dz^4)^2 + V_1(\tau)V_4(\tau)V_6(\tau)(dz^5)^2$$

$$+ V_2(\tau)V_3(\tau)V_6(\tau)(dz^6)^2 + V_2(\tau)V_4(\tau)V_5(\tau)(dz^7)^2$$

Functions $V_1(\tau), \ldots, V_6(\tau)$ piecewise linear,
$M_1 = V_1, M_2 = V_2, \ldots$
Nilmanifold: $S^1$ bundle over $T^4$

2-function solution, SU(3) holonomy

\[
\begin{align*}
    ds^2 &= V_1(\tau)V_2(\tau)d\tau^2 + \frac{1}{V_1(\tau)V_2(\tau)}\left(d\tau^1 + M_1(\tau)\tau^3 dz^2 + M_2(\tau)\tau^5 dz^4\right)^2 \\
    &+ V_1(\tau)\left((dz^2)^2 + (dz^3)^2\right) + V_2(\tau)\left((dz^4)^2 + (dz^5)^2\right)
\end{align*}
\]

If $V_2 = 1, M_2 = 0$, (Gibbons-Hawking)$\times \mathbb{R}^2$

GH: $\tau, \tau, \tau^2, \tau^3$  \quad $\mathbb{R}^2: \tau^4, \tau^5$

If $V_1 = 1, M_1 = 0$, (Gibbons-Hawking)$\times \mathbb{R}^2$

GH: $\tau, \tau^1, \tau^4, \tau^5$  \quad $\mathbb{R}^2: \tau^2, \tau^3$

"Overlap" or "intersection" of smeared Kaluza-Klein monopole solutions

Singular at kinks in $V_1(\tau), V_2(\tau)$

Can this be generalised to overlap of localised Kaluza-Klein monopole solutions?

Solution with functions on $\mathbb{R}^3$: $V_1(\tau, \tau^2, \tau^3), V_2(\tau, \tau^4, \tau^5)$
Localised Solutions?

\[ ds^2 = V_1 V_2 d\tau^2 + \frac{1}{V_1 V_2} (dz + \omega)^2 \]

\[ + V_1 \left( (dx^1)^2 + (dx^2)^2 \right) + V_2 \left( (dy^1)^2 + (dy^2)^2 \right) \]

Solution with local functions?

\[ V_1(\tau, x^1, x^2), \ V_2(\tau, y^1, y^2) \]

T-dualise semi-local brane intersection solutions

OR Special case of equations of Pederesen and Poon, and Zharkov

\[ \partial_\tau^2 V_1 + V_2 (\partial_{x^1}^2 + \partial_{x^2}^2) V_1 = 0 \]

\[ \partial_\tau^2 V_2 + V_1 (\partial_{y^1}^2 + \partial_{y^2}^2) V_2 = 0 \]
Localised Solutions?

\[ ds^2 = V_1 V_2 d\tau^2 + \frac{1}{V_1 V_2} \left( dz + \omega \right)^2 \]

\[ + V_1 \left( (dx^1)^2 + (dx^2)^2 \right) + V_2 \left( (dy^1)^2 + (dy^2)^2 \right) \]

Solution with local functions?

\[ V_1(\tau, x^1, x^2), \; V_2(\tau, y^1, y^2) \]

T-dualise semi-local brane intersection solutions
OR Special case of equations of Pederesen and Poon, and Zharkov

\[ \partial_i^2 V_1 + V_2 \left( \partial_{x^1}^2 + \partial_{x^2}^2 \right) V_1 = 0 \]

\[ \partial_i^2 V_2 + V_1 \left( \partial_{y^1}^2 + \partial_{y^2}^2 \right) V_2 = 0 \]

Unfortunately, also

\[ \partial_{x^i} V_1 \partial_{y^j} V_2 = 0 \]

Either \( V_1 \) independent of \( x^i \) or \( V_2 \) independent of \( y^i \)
Semi-Localised Solutions

\[ ds^2 = V_1 V_2 d\tau^2 + \frac{1}{V_1 V_2} (dz + \omega)^2 + V_1 \left( (dx^1)^2 + (dx^2)^2 \right) + V_2 \left( (dy^1)^2 + (dy^2)^2 \right) \]

Solution with semi-local functions \( V_1(\tau, x^1, x^2), V_2(\tau) \)

\[ \partial^2 \tau V_1 + V_2 (\partial^2_{x^1} + \partial^2_{x^2}) V_1 = 0 \]

\[ \partial^2 \tau V_2 = 0 \]

Solution

\[ V_2 = a + m\tau \quad V_1 = b + \frac{q}{\left( 3m^2 x^2 + (a + m\tau)^3 \right)^{2/3}} \]

\[ x^2 = (x^1)^2 + (x^2)^2 \]

\( V_1 \) localised in \( \tau, x^1, x^2 \)
Near origin: Cone over squashed sphere
SU(3) holonomy
Can add to \( V_1 \): linear \( f(\tau) \) or harmonic \( h(x^1, x^2) \)
Generalise to piecewise linear \( V_2 \), superposition of sources for \( V_1 \)
Localised Solutions and Degenerate Limits

- Are there fully localised non-singular solutions?
- Use duality to intersecting branes, brane webs to motivate ansatz
- Are these part of neck region of some compact special holonomy space, just as the hyperkahler metrics were model metrics for part of degenerate limit of K3?
- Relation to other configurations via string dualities?
T-Duality

Nilmanifold: $T^n$ bundle over $T^m$  
Curvature 2-forms $F_a$, $a=1,\ldots,n$

T-duality untwists bundle to $T^{n+m}$

3-form $H$, $dH=0$

$$H = dy^a \wedge F_a$$

Nilfold: $S^1$ Bundle over $T^2$  $\longrightarrow$  $T^3$ with H-flux

$$ds^2_{T^3} = dx^2 + dy^2 + dz^2 \quad H = m dx \wedge dy \wedge dz$$

GH Metric: Nilfold fibred over a line

$$ds^2 = V(\tau)(d\tau^2 + dx^2 + dy^2 + dz^2) \quad H = M(\tau)dx \wedge dy \wedge dz$$

Product with 6-d Minkowski space: **NS5-brane**

smeared over 3 directions and wrapped on $T^3$
2 complex structures \( J^\pm \), \( g \) bihermitian

Connections with torsion \( \nabla^\pm = \nabla_{\text{Levi–Civita}} \pm g^{-1}H \)

Local Product structure: complex coordinates \( z^a, w^i \)

\[
ds^2 = g_{a\bar{b}} dz^a d\bar{z}^\bar{b} + g_{ij} dw^i d\bar{w}^j
\]

Generalised Kahler potential \( K(z, \bar{z}, w, \bar{w}) \)

\[
g_{a\bar{b}} = K_{,a\bar{b}}, \quad g_{ij} = -K_{,ij}
\]

Holonomy \( \text{Hol}(\nabla^\pm) \subseteq U(r) \)
$\det(K_{ab}) = \det(-K_{ij})$

Holonomy $Hol(\nabla^\pm) \subseteq SU(r)$

T-dualising Generalised Monge Ampere gives equations of Pederesen and Poon, and Zharkov
Further Dualities

Smeared KK Monopole

NS5-brane Smeared on $T^3$

D8-brane Wrapped on $T^3$  

D8-brane: domain wall in 9+1 dimensions
Type I’ String Theory

Interval $\times \mathbb{R}^{1,8}$

16 D8-branes of charge 1: $N_i$ branes at points $\tau_i$ on interval

Orientifold 8-planes of charge -8 at end-points $\tau = 0, \pi$

$$ds^2 = V^{-1/2}d\tau^2(\mathbb{R}^{1,8}) + V^{1/2}d\tau^2$$

$$V(\tau) = \begin{cases} 
    c_1 + m_1\tau, & 0 \leq \tau \leq \tau_1 \\
    c_2 + m_2\tau, & \tau_1 < \tau \leq \tau_2 \\
    \vdots \\
    c_n + m_n\tau, & \tau_{n-1} < \tau \leq \tau_n \\
    c_{n+1} + m_{n+1}\tau, & \tau_n < \tau \leq \pi 
\end{cases}$$

$$N_i = m_{i+1} - m_i \quad \sum_{i=1}^{n} N_i = 16$$

Or, if at $\tau = 0$ there are $N_-$ branes giving charge $b_- = -8 + N_-$

and at $\tau = \pi$ there are $N_+$ branes giving charge $b_+ = -8 + N_+$

$$b_- = -m_1, b_+ = m_{n+1} \quad 0 \leq b_\pm \leq 8$$

$$\sum_{i=1}^{n} N_i = b_- + b_+ \leq 16$$

Like K3 story, but 16, not 18!
This is correct story for *perturbative* type I’ theory

At *strong coupling*, O8 plane can emit one D8 brane to leave

O8* plane of charge -9

Then O8* planes at either end and 18 D8-branes on interval

If at $\tau = 0$ there are $N_-$ branes giving charge $b_- = -9 + N_-$

and at $\tau = \pi$ there are $N_+$ branes giving charge $b_+ = -9 + N_+$

$$b_- = -m_1, \quad b_+ = m_{n+1}$$

$$\sum_{i=1}^{n} N_i = b_- + b_+ \leq 18$$

Same equations as for degenerate K3
String Dualities

IIA string on K3 dual to Heterotic string on $T^4$  

IIA string on K3 dual to Type I’ string on (Interval) $\times T^3$

• Strong coupling in type I’ mapped to weak coupling in type IIA, so dual gives insight into mysterious I’ strong coupling regime

• KK monopoles dual to D8-branes wrapped on $T^3$

• Tian-Yau end caps dual to orientifold planes

• Moduli space of type I’ mapped to *subspace* of moduli space of IIA on K3

[CH+Townsend]
• D8 branes and O8 planes on interval give fully consistent background for type I’ string theory.

• Singularities of metric reflect presence of physical objects (branes)

• Dual configurations also fully consistent

• Duality taking D8-branes wrapped on $T^4$ to KK monopoles takes type I’ on $T^3$ to type IIA on K3.

• *Predicts(!)* region of K3 moduli space in which K3 looks like HSVZ

• Dualities take intersecting branes to special holonomy metrics

• Intersecting branes part of consistent string backgrounds

• *Predicts(!?)* consistent background incorporating the special holonomy solutions as part of a compact complete special holonomy space.