Special Holonomy Metrics, Degenerate Limits and Intersecting Branes

Nipol Chaemjumrus and CH:

[arXiv:1907.04040] **Degenerations of K3, Orientifolds and Exotic Branes**

[arXiv:1908.04623] Special Holonomy Manifolds, Domain Walls, Intersecting Branes and T-folds

- 1. Degenerate Limit of K3
- 2. Gibbons-Hawking Metrics
- 3. Ooguri-Vafa Metrics
- 4. Special Holonomy Generalisations
- 5. Intersecting Brane Duals
- 6. Applications to String Theory and Dualities: Type I' string, D8-branes and orientifolds

Degenerate Limit of K3

Hein, Sun, Viaclovsky and Zhang [HSVZ]

- Family of K3 Metrics g(t), limit t=0 is line interval
- Long Neck Region at small t
- Segment of neck is nilfold fibred over a line.
- Nilfold is S¹ bundle over T², with degree (Chern number) m. Different values of m in different segments.
- Jump in m: insertion of gravitational instanton (Kaluza Klein monopole)
- Ends of neck capped by Tian-Yau spaces: complete non-compact hyperkahler manifolds asymptotic to nilfold fibred over a line

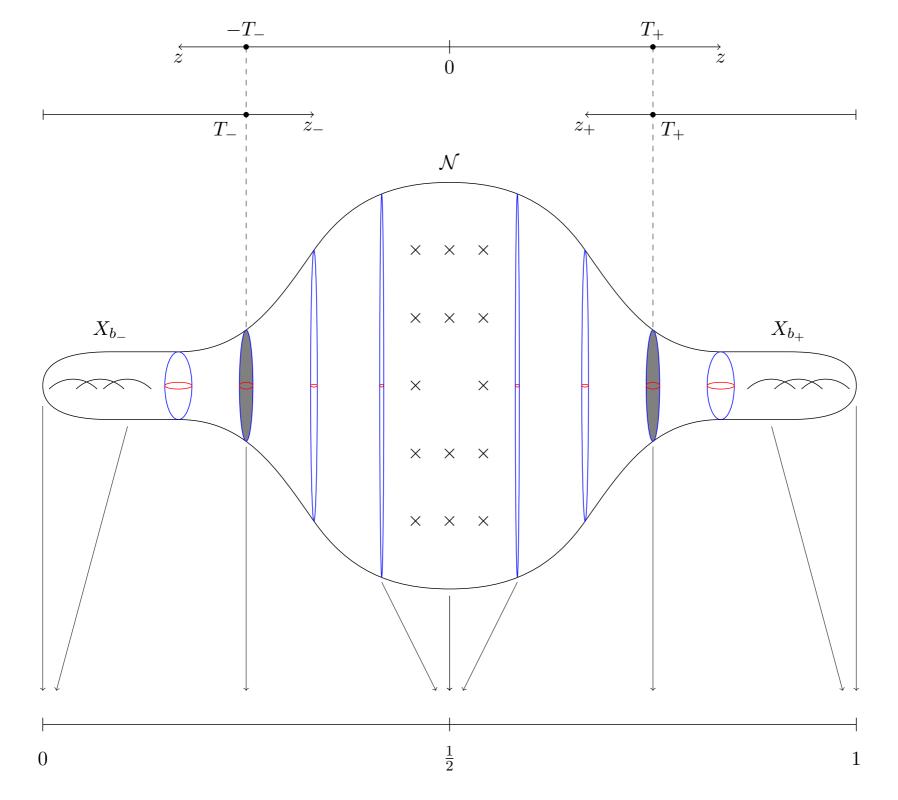


FIGURE 1. The vertical arrows represent collapsing to a one-dimensional interval. The red circles represent the S^1 fibers and the blue curves represent the base \mathbb{T}^2 s of the nilmanifolds. The \times s are the monopole points in the neck region \mathcal{N} . The gray regions are in the "damage zones".

Gibbons-Hawking Metric

Hyperkahler metric with S¹ symmetry

$$g = V(d\tau^2 + dx^2 + dz^2) + V^{-1}(dy + \omega)^2$$

 $V(\tau, x, z)$ a harmonic function on \mathbb{R}^3

$$\vec{\nabla}\times\vec{\omega}=\vec{\nabla}V$$

Delta-function sources at points (m an integer)

$$V = a + \sum_{i} \frac{m}{|\vec{r} - \vec{r_i}|}$$

S¹ Bundle on \mathbb{R}^3 - {points}

Regular at sources if m=1: multi-Taub-NUT

Orbifold singularities for m>1

Smeared GH Metrics

 $V(\tau, x, z)$ a harmonic function on \mathbb{R}^3

"Smeared" solutions: V independent of one or more coordinates

Can then take those coordinates to be periodic Metric typically singular

Smear on x,y: $V(\tau) = m\tau + c$

or $V(\tau) = \begin{cases} c + m'\tau, & \tau \le 0\\ c + m\tau, & \tau > 0. \end{cases}$

Singular at kink at $\tau = 0$

Domain wall: 2-plane dividing space into 2 parts

N=m-m': energy density (tension) of domain wall (2-brane)

Piecewise linear:

multi-wall solution with domain walls at $\tau = \tau_1, \tau_2, \dots, \tau_n$

$$V(\tau) = \begin{cases} c_1 + m_1 \tau, & \tau \le \tau_1 \\ c_2 + m_2 \tau, & \tau_1 < \tau \le \tau_2 \\ \vdots \\ c_n + m_n \tau, & \tau_{n-1} < \tau \le \tau_n \\ c_{n+1} + m_{n+1} \tau, & \tau > \tau_n . \end{cases}$$

The charge of the domain wall at τ_r is the integer

$$N_r = m_{r+1} - m_r$$

$$ds^{2} = V(\tau)(d\tau^{2} + dx^{2} + dz^{2}) + \frac{1}{V(\tau)}(dy + M(\tau)xdz)^{2}$$

 $M(\tau) \equiv V'(\tau)$

Can take x,y,z periodic

Single-sided domain wall

$$V = c + m \left| \tau \right|$$

Quotient by reflection $\tau \rightarrow -\tau$ gives "single-sided" wall at $\tau = 0$

$$V(\tau) = m\tau + c$$
$$ds^2 = V(\tau)(d\tau^2 + dx^2 + dz^2) + \frac{1}{V(\tau)}(dy + mxdz)^2$$

Take x,y,z periodic

Fixed τ : **nilfold**

$$ds_{\mathcal{N}}^2 = dx^2 + (dy + mxdz)^2 + dz^2$$

 S^1 Bundle over T^2

$$F = mdx \wedge dz$$

Quotient of the group manifold of the Heisenberg group by a cocompact discrete subgroup

4-d space: nilfold fibred over a line

1st approximation to HSVZ K3

Interval $\tau \in [0,\pi]$ Multi-domain wall solution with domain walls at $\tau = \tau_1, \tau_2, \dots \tau_n$ Single-sided domain walls at $\tau = 0,\pi$

$$ds^{2} = V(\tau)(d\tau^{2} + dx^{2} + dz^{2}) + \frac{1}{V(\tau)}(dy + M(\tau)xdz)^{2}$$

$$V(\tau) = \begin{cases} c_{1} + m_{1}\tau, & 0 \le \tau \le \tau_{1} \\ c_{2} + m_{2}\tau, & \tau_{1} < \tau \le \tau_{2} \\ \vdots \\ c_{n} + m_{n}\tau, & \tau_{n-1} < \tau \le \tau_{n} \\ c_{n+1} + m_{n+1}\tau, & \tau_{n} < \tau \le \pi \end{cases} M(\tau) \equiv V'(\tau)$$

HSVZ resolve singularities:

- Resolve domain walls with Ooguri-Vafa construction
- Resolve single-sided domain walls with Tian-Yau spaces

Ooguri-Vafa Metric

Want Gibbons-Hawking metric, \mathbb{R}^3 replaced with $\mathbb{R} \times T^2$ 1st approximation: smear over T^2

<u>Ooguri-Vafa:</u>

- On \mathbb{R}^3 , take periodic array of sources in (x,z) plane
- Regularised sum of potentials gives harmonic function
- Can now periodically identify x,z directions, to get single source on $\mathbb{R} \times T^2$.
- Near source, non-singular, looks like Taub-NUT
- Can then take superpositions to get multiple sources on $\mathbb{R} \times T^2$.
- Solutions regular on finite interval in $\mathbb R$

Resolve GH metric with

$$V(\tau) = \begin{cases} c + m'\tau, & \tau \le 0\\ c + m\tau, & \tau > 0. \end{cases}$$
 Charge N=m-m'

by OV metric with V harmonic on $\mathbb{R} \times T^2$

Monopole charge N

Near sources, N-centre multi Taub-NUT, or one source of charge N, orbifold singularity: bubbling limit to Taub-NUT

For N sources, regular hyperkahler metric for some interval $-T < \tau < T'$

Far enough away from $\tau = 0$, tends to GH with

$$V(\tau) = \begin{cases} c + m'\tau, & \tau \le 0\\ c + m\tau, & \tau > 0. \end{cases}$$

Tian-Yau Spaces

- Complete non-singular non-compact hyperkahler space
- Asymptotic to a nilfold bundle over a line.
- Of the form M \ D, where M is a del Pezzo surface, D ⊂ M is a smooth anticanonical divisor
- Del Pezzo surfaces are complex surfaces classified by their degree b, where b = 1, 2, ..., 9
- The del Pezzo surface of degree nine is CP²
- A degree b del Pezzo surface can be constructed from blowing up 9 b points in CP²
- A 2nd del Pezzo surface of degree 8 is $CP^1 \times CP^1$
- The TY space M_b of degree b is constructed from del Pezzo of degree b
- M_b is asymptotic to GH metric on $N_b \times \mathbb{R}$ where N_b is nilfold of degree b
- Degree zero: Take M to be rational elliptic surface, $N_0 = T^3$, M_0 is ALH, asymptotic to cylinder given by $T^3 \times \mathbb{R}$

1st approximation to HSVZ K3

Interval $\tau \in [0,\pi]$

Multi-domain wall solution with domain walls at $\tau = \tau_1, \tau_2, \dots \tau_n$ Single-sided domain walls at $\tau = 0, \pi$

$$ds^{2} = V(\tau)(d\tau^{2} + dx^{2} + dz^{2}) + \frac{1}{V(\tau)}(dy + M(\tau)xdz)^{2}$$

$$V(\tau) = \begin{cases} c_{1} + m_{1}\tau, & 0 \leq \tau \leq \tau_{1} \\ c_{2} + m_{2}\tau, & \tau_{1} < \tau \leq \tau_{2} \\ \vdots & & M(\tau) \equiv V'(\tau) \\ \vdots & & c_{n} + m_{n}\tau, & \tau_{n-1} < \tau \leq \tau_{n} \\ c_{n+1} + m_{n+1}\tau, & \tau_{n} < \tau \leq \pi \end{cases}$$

HSVZ resolve singularities:

Glue together Ooguri-Vafa spaces, Tian-Yau spaces to get complete K3 metric

Tian-Yau spaces of degree b_-, b_+ $b_- = -m_1, b_+ = m_{n+1}$ $0 \le b_{\pm} \le 9$ $N_i = m_{i+1} - m_i$ $\sum_{i=1}^n N_i = b_- + b_+ \le 18$

Special Holonomy Generalisations

- Replace 3-d nilfold with higher dim nilmanifold
- Quotient of nilpotent Lie group by discrete subgroup
- T^n bundle over T^m
- Special holonomy metrics on nilmanifold fibred over a line Gibbons, Lu, Pope and Stelle [GLPS]

Dimension	Nilmanifold: torus bundle over torus	Holonomy
4	S ¹ over T ²	SU(2)
6	S ¹ over T ⁴	SU(3)
6	T ² over T ³	SU(3)
7	T ² over T ⁴	G2
7	T ³ over T ³	G2
8	S ¹ over T ⁶	SU(4)
8	T ³ over T ⁴	Spin(7)

Nilmanifold: S¹ bundle over T⁴

5-dimensional nilpotent Lie algebra T_i Only non-vanishing commutators are

 $[T_2, T_3] = mT_1, \qquad [T_4, T_5] = mT_1$

Metric

$$ds^{2} = \left(dz^{1} + m(z^{3}dz^{2} + z^{5}dz^{4})\right)^{2} + (dz^{2})^{2} + (dz^{3})^{2} + (dz^{4})^{2} + (dz^{5})^{2}.$$

Fibre coord: z¹

Nilmanifold fibred over line: SU(3) holonomy

$$ds^{2} = V^{2}(\tau)(d\tau)^{2} + V(\tau)\left((dz^{2})^{2} + (dz^{3})^{2} + (dz^{4})^{2} + (dz^{5})^{2}\right)$$
$$+ V^{-2}(\tau)\left(dz^{1} + M(\tau)(z^{3}dz^{2} + z^{5}dz^{4})\right)^{2}$$
$$M(\tau) \equiv V'(\tau)$$

V Piecewise linear

Nilmanifold: T³ bundle over T⁴

7-dimensional nilpotent Lie algebra Ti

Only non-vanishing commutators are

$$[T_4, T_5] = mT_1,$$
 $[T_6, T_7] = mT_1,$ $[T_4, T_6] = mT_2$
 $[T_5, T_7] = -mT_2,$ $[T_4, T_7] = mT_3,$ $[T_5, T_6] = mT_3$

Metric

$$ds^{2} = \left(dz^{1} + m(z^{5}dz^{4} + z^{7}dz^{6})\right)^{2} + \left(dz^{2} + m(z^{6}dz^{4} - z^{7}dz^{5})\right)^{2} + \left(dz^{3} + m(z^{7}dz^{4} + z^{6}dz^{5})\right)^{2} + (dz^{4})^{2} + (dz^{5})^{2} + (dz^{6})^{2} + (dz^{7})^{2}$$

Fibre coords: $z^1 z^2 z^3$

Nilmanifold fibred over line: Spin(7) holonomy

$$ds^{2} = V^{6}(\tau)(d\tau)^{2} + V^{3}(\tau) \left((dz^{4})^{2} + (dz^{5})^{2} + (dz^{6})^{2} + (dz^{7})^{2}) \right) + V^{-2}(\tau) \left(dz^{1} + M(z^{5}dz^{4} + z^{7}dz^{6}) \right)^{2} + V^{-2}(\tau) \left(dz^{2} + M(z^{6}dz^{4} - z^{7}dz^{5}) \right)^{2} + V^{-2}(\tau) \left(dz^{3} + M(z^{7}dz^{4} + z^{6}dz^{5}) \right)^{2} M(\tau) \equiv V'(\tau)$$

Further Nilmanifolds

Extends to further examples with more general nilmanifolds.

The previous case are from 2-step nilpotent Lie groups giving a torus bundle over a torus

p-step nilpotent Lie groups $[X_1, [X_2, [\cdots [X_p, Y] \cdots]] = 0$

Torus bundle over a torus bundle over a torus.....

Chiossi and Salamon:

For any 6-dimensional nilmanifold with half-flat SU(3) structure, fibration over a line interval gives a G₂ metric 24 cases.

SU(3) structure: almost complex 6-manifold, with (1,1) form ω , (3,0) form Ω Half-flat: $\omega \wedge d\omega$ and $Re(\Omega)$ closed

T-duality

Taub-NUT

NS

NS 5-brane

Multi 5-brane

[CH+Townsend]

ALF Multi-instanton



GLPS Special Holonomy



Intersecting 5-brane solution with one function

[Chaemjumrus and CH]

Special Holonomy with several functions Intersecting 5-brane solution with several functions

Semi-local solution

Nilmanifold: S¹ bundle over T⁴

Nilmanifold fibred over line: SU(3) holonomy

$$ds^{2} = V^{2}(\tau)(d\tau)^{2} + V(\tau)\left((dz^{2})^{2} + (dz^{3})^{2} + (dz^{4})^{2} + (dz^{5})^{2}\right)$$
$$+ V^{-2}(\tau)\left(dz^{1} + M(\tau)(z^{3}dz^{2} + z^{5}dz^{4})\right)^{2}$$
$$M(\tau) \equiv V'(\tau)$$

2-function generalisation, SU(3) holonomy

$$ds^{2} = V_{1}(\tau)V_{2}(\tau)d\tau^{2} + \frac{1}{V_{1}(\tau)V_{2}(\tau)} \left(dz^{1} + M_{1}(\tau)z^{3}dz^{2} + M_{2}(\tau)z^{5}dz^{4}\right)^{2}$$
$$+ V_{1}(\tau)\left((dz^{2})^{2} + (dz^{3})^{2}\right) + V_{2}(\tau)\left((dz^{4})^{2} + (dz^{5})^{2}\right)$$

Functions $V_1(\tau),\,V_2(\tau)$ piecewise linear, $M_1=V_1,\,M_2=V_2'$

[Chaemjumrus and CH]

Nilmanifold: T³ bundle over T⁴

Nilmanifold fibred over line: Spin(7) holonomy

$$ds^{2} = V^{6}(\tau)(d\tau)^{2} + V^{3}(\tau)\left((dz^{4})^{2} + (dz^{5})^{2} + (dz^{6})^{2} + (dz^{7})^{2})\right)$$
$$+ V^{-2}(\tau)\left(dz^{1} + M(z^{5}dz^{4} + z^{7}dz^{6})\right)^{2}$$
$$+ V^{-2}(\tau)\left(dz^{2} + M(z^{6}dz^{4} - z^{7}dz^{5})\right)^{2} + V^{-2}(\tau)\left(dz^{3} + M(z^{7}dz^{4} + z^{6}dz^{5})\right)^{2}$$

6-function generalisation, Spin(7) holonomy

$$ds^{2} = V_{1}(\tau)V_{2}(\tau)V_{3}(\tau)V_{4}(\tau)V_{5}(\tau)V_{6}(\tau)d\tau^{6} + \frac{1}{V_{1}(\tau)V_{2}(\tau)}\left(dz^{1} + M_{1}z^{5}dz^{4} + M_{2}z^{7}dz^{6}\right)^{2}$$
$$+ \frac{1}{V_{3}(\tau)V_{4}(\tau)}\left(dz^{2} + M_{3}z^{6}dz^{4} - M_{4}z^{7}dz^{5}\right)^{2} + \frac{1}{V_{5}(\tau)V_{6}(\tau)}\left(dz^{3} + M_{5}z^{7}dz^{4} + M_{6}z^{6}dz^{5}\right)^{2}$$

 $+ V_1(\tau) V_3(\tau) V_5(\tau) (dz^4)^2 + V_1(\tau) V_4(\tau) V_6(\tau) (dz^5)^2$

 $+ V_2(\tau) V_3(\tau) V_6(\tau) (dz^6)^2 + V_2(\tau) V_4(\tau) V_5(\tau) (dz^7)^2$

Functions $V_1(\tau),\,\cdots,\,V_6(\tau)$ piecewise linear, $M_1=V_1,\,M_2=V_2',\ldots$

[Chaemjumrus and CH]

Nilmanifold: S¹ bundle over T⁴

2-function solution, SU(3) holonomy

$$ds^{2} = V_{1}(\tau)V_{2}(\tau)d\tau^{2} + \frac{1}{V_{1}(\tau)V_{2}(\tau)} \left(dz^{1} + M_{1}(\tau)z^{3}dz^{2} + M_{2}(\tau)z^{5}dz^{4}\right)^{2}$$
$$+ V_{1}(\tau)\left((dz^{2})^{2} + (dz^{3})^{2}\right) + V_{2}(\tau)\left((dz^{4})^{2} + (dz^{5})^{2}\right)$$

If $V_2 = 1, M_2 = 0$, (Gibbons-Hawking)x \mathbb{R}^2 GH: $\tau, z^1, z^2, z^3 \qquad \mathbb{R}^2: z^4, z^5$

If $V_1 = 1, M_1 = 0$, (Gibbons-Hawking)x \mathbb{R}^2 GH: $\tau, z^1, z^4, z^5 \qquad \mathbb{R}^2: z^2, z^3$

"Overlap" or "intersection" of smeared Kaluza-Klein monopole solutions Singular at kinks in $V_1(\tau)$, $V_2(\tau)$ Can this be generalised to overlap of **localised** Kaluza-Klein monopole s

Can this be generalised to overlap of **localised** Kaluza-Klein monopole solutions? Solution with functions on \mathbb{R}^3 :

$$V_1(\tau, z^2, z^3)$$
 $V_2(\tau, z^4, z^5)$

Localised Solutions?

$$ds^{2} = V_{1}V_{2}d\tau^{2} + \frac{1}{V_{1}V_{2}}\left(dz + \omega\right)^{2}$$
$$+ V_{1}\left((dx^{1})^{2} + (dx^{2})^{2}\right) + V_{2}\left((dy^{1})^{2} + (dy^{2})^{2}\right)$$

Solution with local functions?

$$V_1(\tau, x^1, x^2), V_2(\tau, y^1, y^2)$$

T-dualise semi-local brane intersection solutions OR Special case of equations of Pederesen and Poon, and Zharkov

$$\partial_{\tau}^{2} V_{1} + V_{2} (\partial_{x^{1}}^{2} + \partial_{x^{2}}^{2}) V_{1} = 0$$

$$\partial_{\tau}^{2} V_{2} + V_{1} (\partial_{y^{1}}^{2} + \partial_{y^{2}}^{2}) V_{2} = 0$$

Localised Solutions?

$$ds^{2} = V_{1}V_{2}d\tau^{2} + \frac{1}{V_{1}V_{2}}\left(dz + \omega\right)^{2}$$
$$+ V_{1}\left((dx^{1})^{2} + (dx^{2})^{2}\right) + V_{2}\left((dy^{1})^{2} + (dy^{2})^{2}\right)$$

Solution with local functions?

$$V_1(\tau, x^1, x^2), V_2(\tau, y^1, y^2)$$

T-dualise semi-local brane intersection solutions OR Special case of equations of Pederesen and Poon, and Zharkov

$$\partial_{\tau}^{2} V_{1} + V_{2} (\partial_{x^{1}}^{2} + \partial_{x^{2}}^{2}) V_{1} = 0$$
$$\partial_{\tau}^{2} V_{2} + V_{1} (\partial_{y^{1}}^{2} + \partial_{y^{2}}^{2}) V_{2} = 0$$

Unfortunately, also

$$\partial_{x^i} V_1 \, \partial_{y^i} V_2 = 0$$

Either V_1 independent of x^i or V_2 independent of y^i

Semi-Localised Solutions

$$ds^{2} = V_{1}V_{2}d\tau^{2} + \frac{1}{V_{1}V_{2}}\left(dz + \omega\right)^{2} + V_{1}\left((dx^{1})^{2} + (dx^{2})^{2}\right) + V_{2}\left((dy^{1})^{2} + (dy^{2})^{2}\right)$$

Solution with semi-local functions

 $V_1(\tau, x^1, x^2), V_2(\tau)$

$$\partial_{\tau}^2 V_1 + V_2 (\partial_{x^1}^2 + \partial_{x^2}^2) V_1 = 0 \qquad \qquad \partial_{\tau}^2 V_2 = 0$$

Solution

$$V_{2} = a + m\tau \qquad V_{1} = b + \frac{q}{\left(3m^{2}\mathbf{x}^{2} + (a + m\tau)^{3}\right)^{2/3}}$$

 $\mathbf{x}^2 = (x^1)^2 + (x^2)^2$

 V_1 localised in τ, x^1, x^2 Near origin: Cone over squashed sphere SU(3) holonomy Can add to V_1 : linear $f(\tau)$ or harmonic $h(x^1, x^2)$ Generalise to piecewise linear V_2 , superposition of sources for V_1

Localised Solutions and Degenerate Limits

- Are there fully localised non-singular solutions?
- Use duality to intersecting branes, brane webs to motivate ansatz
- Are these part of neck region of some compact special holonomy space, just as the hyperkahler metrics were model metrics for part of degenerate limit of K3?
- Relation to other configurations via string dualities?

T-Duality

Nilmanifold: T^n bundle over T^m Curvature 2-forms F_a , a=1,...,n

T-duality untwists bundle to T^{n+m}

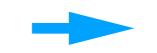
3-form H, dH=0

$$H = dy^a \wedge F_a$$

Nilfold: S^1 Bundle over $T^2 \longrightarrow T^3$ with H-flux

$$ds_{T^3}^2 = dx^2 + dy^2 + dz^2$$
 $H = mdx \wedge dy \wedge dz$

GH Metric: Nilfold fibred over a line



$$ds^{2} = V(\tau)(d\tau^{2} + dx^{2} + dy^{2} + dz^{2}) \qquad H = M(\tau)dx \wedge dy \wedge dz$$

Product with 6-d Minkowski space: **NS5-brane** smeared over 3 directions and wrapped on T^3

Kahler — Generalised Kahler Gates, CH, Rocek

2 complex structures J^{\pm} , g bihermitian

Connections with torsion $\nabla^{\pm} = \nabla_{Levi-Civita} \pm g^{-1}H$

Local Product structure: complex coordinates z^a, w^i

$$ds^2 = g_{a\bar{b}}dz^a d\bar{z}^{\bar{b}} + g_{i\bar{j}}dw^i d\bar{w}^{\bar{j}}$$

Generalised Kahler potential $K(z, \overline{z}, w, \overline{w})$

$$g_{a\bar{b}} = K,_{a\bar{b}}, \quad g_{i\bar{j}} = -K,_{i\bar{j}}$$

Holonomy $Hol(\nabla^{\pm}) \subseteq U(r)$

CY, Monge Ampere — Generalised Kahler

Generalised Monge Ampere

 $det(K,_{a\bar{b}}) = det(-K,_{i\bar{j}})$ Buscher; CH; Rocek

Holonomy
$$Hol(\nabla^{\pm}) \subseteq SU(r)$$
 CH

T-dualising Generalised Monge Ampere gives equations of

Pederesen and Poon, and Zharkov

Further Dualities

Smeared KK Monopole

NS5-brane Smeared on T^3

D8-brane Wrapped on T^3

D8-brane: domain wall in 9+1 dimensions

Type I' String Theory

Interval x $\mathbb{R}^{1,8}$

16 D8-branes of charge 1: N_i branes at points τ_i on interval

Orientifold 8-planes of charge -8 at end-points $\tau = 0, \pi$

$$ds^{2} = V^{-1/2}ds^{2}(\mathbb{R}^{1,8}) + V^{1/2}d\tau^{2} \qquad V(\tau) = \begin{cases} c_{1} + m_{1}\tau, & 0 \le \tau \le \tau_{1} \\ c_{2} + m_{2}\tau, & \tau_{1} < \tau \le \tau_{2} \\ \vdots \\ c_{n} + m_{n}\tau, & \tau_{n-1} < \tau \le \tau_{n} \\ c_{n+1} + m_{n+1}\tau, & \tau_{n} < \tau \le \pi \end{cases}$$
$$N_{i} = m_{i+1} - m_{i} \qquad \sum_{i=1}^{n} N_{i} = 16$$

Or, if at $\tau = 0$ there are N_{-} branes giving charge $b_{-} = -8 + N_{-}$ and at $\tau = \pi$ there are N_{+} branes giving charge $b_{+} = -8 + N_{+}$

$$b_{-} = -m_{1}, b_{+} = m_{n+1}$$
 $0 \le b_{\pm} \le 8$
 $\sum_{i=1}^{n} N_{i} = b_{-} + b_{+} \le 16$ Like K3 story, but 16, not 18

This is correct story for *perturbative* type I' theory At strong coupling, O8 plane can emit one D8 brane to leave

O8* plane of charge -9

Morrison and Seiberg

Then O8* planes at either end and 18 D8-branes on interval If at $\tau = 0$ there are N_{-} branes giving charge $b_{-} = -9 + N_{-}$

and at $\tau = \pi$ there are N_+ branes giving charge $b_+ = -9 + N_+$ $b_- = -m_1, b_+ = m_{n+1}$ $\sum_{i=1}^n N_i = b_- + b_+ \le 18$

Same equations as for degenerate K3

String Dualities

IIA string on K3 dual to Heterotic string on T^4

[CH+Townsend]

IIA string on K3 dual to Type I' string on (Interval) x T^3

- Strong coupling in type I' mapped to weak coupling in type IIA, so dual gives insight into mysterious I' strong coupling regime
- KK monopoles dual to D8-branes wrapped on T^3
- Tian-Yau end caps dual to orientifold planes
- Moduli space of type I' mapped to subspace of moduli space of IIA on K3

- D8 branes and O8 planes on interval give fully consistent background for type I' string theory.
- Singularities of metric reflect presence of physical objects (branes)
- Dual configurations also fully consistent
- Duality taking D8-branes wrapped on T^4 to KK monopoles takes type I' on T^3 to type IIA on K3.
- Predicts(!) region of K3 moduli space in which K3 looks like HSVZ
- Dualities take intersecting branes to special holonomy metrics
- Intersecting branes part of consistent string backgrounds
- Predicts(!?) consistent background incorporating the special holonomy solutions as part of a compact complete special holonomy space.