Higgs Bundle Program
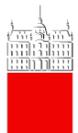## [Applications to G_2, Spin(7) Holonomy Spaces]

Mirjam Cvetič
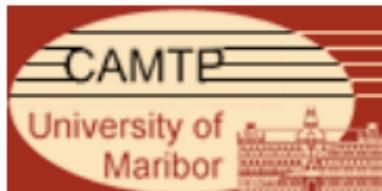
Univerza *v Ljubljani*
Fakulteta za *matematiko in fiziko*

Motivation/Goals

- Primary Motivation: M-theory/String Theory/F-theory compactified on G_2 [Spin(7)] holonomy spaces → four [three] space-time dimensions with N=1 supersymmetry.

- One Focus: gauge degrees (QFT - Standard Model?) within G_2 [Spin(7)] holonomy spaces are associated with co-dimension four (ADE) singularities → governed by a Hitchin-type system.

- One Goal: localized modes (chiral matter - quarks, leptons?) → in G_2 context add Hitchin flux (T-brane configurations).

R. Barbosa, M.C., J. Heckman, C. Lawrie, E. Torres, G. Zoccarato, ``T-branes and G_2 backgrounds,'' doi:10.1103/PhysRevD.101.026015 [arXiv:1906.02212 [hep-th]].

Hitchin system in Calabi-Yau three-fold background

- A co-dimension four (ADE) singular subspace in Calabi-Yau three-fold → Riemann surface Σ [signifies world-volume of D-brane with associated gauge dynamics].

- Two-dimensional supersymmetric gauge theory on Riemann surface Σ governed by Hitchin system, specified by:
  [one-form Higgs field ϕ_{hit}, vector bundle V connection A] - Higgs bundle
  - adjoint-valued in ADE groups -

  ϕ_{hit} - section of $End(V) \otimes (T^*\Sigma \otimes \mathbb{C})^{1,0}$

- Hitchin equations:
  - BPS eqs. -

$$\overline{\partial}_A \phi_{\text{Hit}} = 0$$

$$F_A + \frac{i}{2}[\phi_{\text{Hit}}^{\dagger}, \phi_{\text{Hit}}] = 0$$

- ϕ_{hit} characterize local deformations in the shape of the ambient Calabi-Yau space.
  A parameterizes gauge field deformations (flux), sourced by D-brane.

Hitchin-type system in G_2 background

- Gauge degrees in G_2 background realized on three-dimensional manifold M_3 associated with co-dimension four (ADE) singularities.

  Pantev, Wijnholt 0905.1968; Acharya; Barbosa 1910.10742

- Described as a partial topological twist of a six-brane wrapped on three-manifold M_3, whose supersymmetric gauge theory -Hitchin-type system- specified by (one-form Higgs field ϕ, vector bundle connection A)

  -Higgs Bundle-

  Chiral matter localized at the center of solitonic configuration of ϕ, with vanishing value there (co-dimension seven singularities).

  Braun, Cizel, Hübner, Schäfer-Nameki 1812.06072

- Extensive study further developed and extended to co-dimension six singularities (non-chiral matter). Appealing feature: they could possibly connect to building compact G_2 manifolds via twisted connected sums (TCS).

  Kovalev; Corti, Haskins, Nordström, Pacini

- Most prior analyses of localized matter have assumed one-form ϕ is diagonal and no A in M_3

R.Barbosa,M.C.,J.Heckman,C.Lawrie,E.Torres,G.Zoccarato 1906.02212

Hitchin-type system generalized to include non-diagonal one-form ϕ & non-zero flux *A*

$$\Phi \sim \begin{bmatrix} \lambda & 1 & 0 \\ z & \lambda & 0 \\ 0 & 0 & -2\lambda \end{bmatrix}$$

- One-form ϕ components will not commute
  refer to this as a ``T-brane type configuration''
  (naturally fit in the broader scheme of T-brane like phenomena).

- Local model: three-manifold M_3 as a Riemann surface Σ
  fibered over an interval S^1: The gauge theory on Σ is *a mild deformation* of a Hitchin system on a complex curve Σ.

- As Hitchin system on Σ describes a local Calabi-Yau geometry →
  obtain a local deformation of a TCS-like construction →
  Interpreted as building up a local G_2 background.

- Localized modes (in co-dim seven) obtained for these backgrounds
  → explicit solutions; their existence reduces to a linear algebraic problem
  [à la localized zero modes of T-branes in Calabi-Yau fourfolds].

Cecotti, Cheng, Heckman, Vafa 0910.0477
Cecotti, Cordova, Heckman, Vafa 1010.5780

Extension and new developments

Take these 3D gauge theories fibered over another interval, thereby producing solutions to 4D gauge theories, which are expected to build local Spin(7) backgrounds given by a four-manifold of ADE singularities.

M.C., J. Heckman, T. Rochais, E. Torres and G.Zoccarato,
``Geometric Unification of Higgs Bundle Vacua,''
[arXiv:2003.13682 [hep-th]]   & work in progress

Gianluca Zoccarato's talk