Getting High On Gluing Orbifolds (Introduction)

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Plan For The Talks:

- JJH's Talk: Introduction to Ethan's Talk

- Ethan's Talk: Compute 0-form, 1-form, 2-group
  for 5D SCFTs via $\mathbb{C}^3/\Gamma_{SU(3)}$

- Mirjam's Talk: Introduction to Max's Talk

- Max's Talk: "..." for non-cpct Elliptic CY_3, G_2
  and some cpct examples

Based On:

Main Focus:

hep-th/2203.10102 w/ Cvetič, Hübner, Torres

hep-th/????.????? w/ Cvetič, Hübner, Torres

Background:

hep-th/1503.04806 w/ Del Zotto, Park, Rudelius

hep-th/2201.08372 w/ Del Zotto, Moscrop, Meynet, Zhang

Physics Motivation

Main Aim:

- Use M-theory to Construct New QFTs

- Use M-theory to Study Old and New QFTs

"Geometric Engineering"

JJH Review Talks: https://scgp.stonybrook.edu/archives/25133

Go to "Videos" Menu...

Quantum Field Theory

A Universal Language Used By Theorists

Broadly Encompasses Many Physical Situations

- Subatomic Particles
- Cosmology
- Condensed Matter Systems

Many Applications To Pure Math As Well

In QFT...

Main Players:

- A Hilbert Space $\mathcal{H}$ / $\mathbb{C}$

  $\mathbb{C}$-Linear Operators $\mathcal{O} : \mathcal{H} \to \mathcal{H}$

- Local Operators $\mathcal{O}(x)$

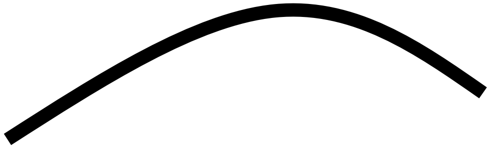
  (supported at points $x \in$ Spacetime)

- Line Operators $\mathcal{O}(L)$

  (supported at lines $L \subset$ Spacetime)

Locals & Lines

Local: 

Line: 

Higher-Form Symmetry

(For our purposes today, general enough...)

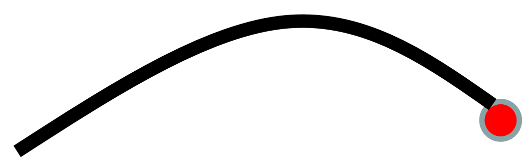
0-form Symm: $\mathcal{O}(x) \mapsto \mathcal{O}'(x)$ linear group action on local operators

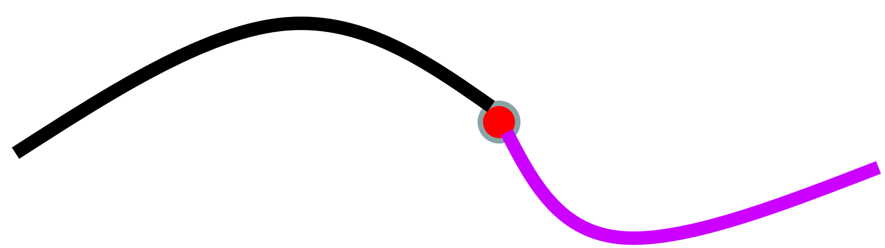
1-form Symm: $\mathcal{O}(L) \mapsto \mathcal{O}'(L)$ linear group action on line operators

$$\vdots$$

Locals & Lines

Local: 

Line Ending on Local: 

Line Changing Via Local: 

2-Group

(For our purposes today, general enough...)

(see e.g. Kapustin Thorngren '13; Benini Cordova Hsin '18)

4-tuple: $(G_{0-form}, \mathcal{A}_{1-form}, \rho, P)$

Hom $\rho : G \to \mathrm{Aut}(\mathcal{A})$

$P \in H^3(BG, (\mathcal{A})_\rho)$

Main Aim:

- Use M-theory to Construct New QFTs
- Use M-theory to Study Old and New QFTs
- Focus Today: Higher Symmetries
  "Topological Data"

Philosophy

- Metric Data is Important (But Difficult)
- Topological Data Also Important (And Easier)
- Topology: Also Protected From Quantum Effects

What is M-Theory?

Full definition of M-theory: Unknown...

But at Long Distances, 11D supergravity

Key Objects: Graviton (metric) and “Branes”

	0	1	2	3	4	5	6	7	8	9	10
M2-Brane	$\times$	$\times$	$\times$								
M5-Brane	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$					

Building QFTs

Product Space: $\mathbb{R}^{D-1,1} \times X_{11-D}$

Physically, *WE WANT* Singularities on X!

QFT Limit (no Gravity) $\Rightarrow$ X Non-Compact

Examples: X_7 Local G_2 Space

X_6 Local Calabi-Yau threefold

X_4 Local Calabi-Yau twofold

Example: $\mathbb{C}^3/\mathbb{Z}_3$

Consider M-th on $\mathbb{R}^{4,1} \times \mathbb{C}^3/\mathbb{Z}_3$

Blowup: M-th on $\mathbb{R}^{4,1} \times \mathcal{O}_{\mathbb{P}^2}(-3)$

$\mathrm{Vol}(\mathbb{P}^2) \to 0$ Limit:

Example of a "Non-Lagrangian"

5D Superconformal Field Theory

More Generally:

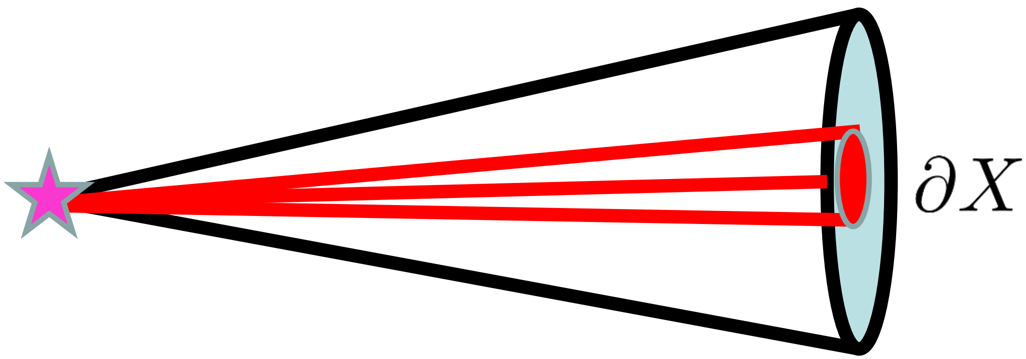
5D SCFTs via M-th on CY_3 canonical singularities

Objects in $\mathbb{C}^3/\mathbb{Z}_3$ Model

	$\mathbb{R}^{4,1}$					$\mathbb{P}^2$				$\mathcal{O}(-3)$	
	0	1	2	3	4	5	6	7	8	9	10
Particle	×					×	×				
“Defect$_1$”	×									×	×
String	×	×				×	×	×	×		
“Defect$_2$”	×	×				×	×			×	×

Definition: Defect Group

$$\mathbb{D} = \bigoplus_n \mathbb{D}^{(n)} \qquad \mathbb{D}^{(n)} = \bigoplus_{\text{p-branes}} \left(\bigoplus_{p-k+1=n} \frac{H_k(X,\partial X)}{H_k(X)} \right)$$



Defect Group of $\mathbb{C}^3/\mathbb{Z}_3$

$$\mathbb{D}^{(n)} = \bigoplus_{\text{p-branes}} \left(\bigoplus_{p-k+1=n} \frac{H_k(X,\partial X)}{H_k(X)} \right)$$

From M2: $\mathbb{D}^{(1)} = \frac{H_2(X,\partial X)}{H_2(X)} \simeq H_1(S^5/\mathbb{Z}_3) \simeq \mathbb{Z}_3^{(1)}$
"electric"

From M5: $\mathbb{D}^{(2)} = \frac{H_4(X,\partial X)}{H_4(X)} \simeq H_3(S^5/\mathbb{Z}_3) \simeq \mathbb{Z}_3^{(2)}$
"magnetic"

$\mathbb{D}$ and Higher Symmetries

Our Defects are acted on by higher-form symms.

Split $\mathbb{D}$ to elec. and mag. Polarization

$\Rightarrow \mathcal{A}^{\mathrm{elec}} \subset \mathbb{D}^{\vee}$ (dual: $\mathbb{D}^{\vee} = \mathrm{Hom}(\mathbb{D} \to U(1))$)

Aharony Witten '98; Belov Moore '05 '06; Kapustin Saulina '10; Tachikawa '14;
Del Zotto JJH Park Rudelius '15; Garcia Etxebarria Heidenreich Regalado '19;
Morrison Schafer-Nameki Willett '20; Albertini Del Zotto Garcia-Etxebarria Hosseini '20

Higher-Form Symmetry

(For our purposes today, general enough...)

0-form Symm: $\mathcal{O}(x) \mapsto \mathcal{O}'(x)$ linear group action on local operators

Group $= G_{0-\text{form}}$ can be non-abelian

1-form Symm: $\mathcal{O}(L) \mapsto \mathcal{O}'(L)$ linear group action on line operators

Group $= \mathcal{A}_{1-\text{form}}$ always abelian

Main Claim

Cvetic JJH Hubner Torres '22

related: Del Zotto Garcia-Etxebarria Schafer-Nameki '22

Compute 0-form, 1-form, 2-group via topology

$Z_{0\text{-Form}}$: $0 \to \mathcal{C} \to Z_{\widetilde{G}} \to Z_G \to 0$

1-Form: $0 \to \mathcal{A} \to \widetilde{\mathcal{A}} \to \mathcal{C} \to 0$

2-group: $0 \to \mathcal{A} \to \widetilde{A} \to Z_{\widetilde{G}} \to Z_G \to 0$

Extension problem: $H^3(BZ_G, \mathcal{A})$

$\pi_1(S^5/\Gamma)$ Approach to $\mathbb{D}^{(1)}_{\text{elec}}$

Del Zotto JJH Moscrop Nadir Maynet Zhang '22

$$\mathbb{D}^{(1)}_{\mathrm{elec}} = \mathrm{Ab}[\pi_1(S^5/\Gamma_{SU(3)})]$$

Consider $X = \mathbb{C}^3/\Gamma_{SU(3)}$, local CY_3

$$\mathbb{D}^{(1)}_{\mathrm{elec}} = \frac{H_2(X,\partial X)}{H_2(X)}$$

$$= H_1(\partial X)$$

$$= \mathrm{Ab}[\pi_1(S^5/\Gamma_{SU(3)})]$$

Works even if $\Gamma_{SU(3)}$ Is NOT fixed pt. free

Computing $\pi_1(S^5/\Gamma)$

Armstrong 1968: $\pi_1(S^5/\Gamma) = \Gamma/N_{\text{fixed}}$

$N_{\text{fixed}} =$ Normal Subgroup Gen$^{\text{ed}}$ by elts
which leave some points of S^5 fixed

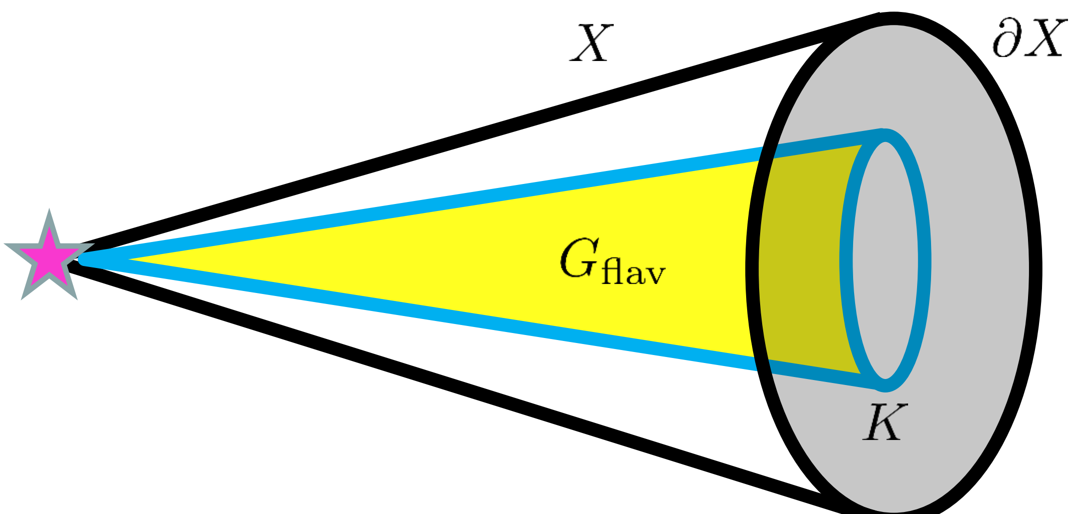
Computing $\mathbb{D}^{(1)}_{\mathrm{elec}}$

$$\mathbb{D}^{(1)}_{\mathrm{elec}} = H_1(S^5/\Gamma) = \mathrm{Ab}[\pi_1(S^5/\Gamma)] = \mathrm{Ab}[\Gamma/N_{\mathrm{fixed}}]$$

N_{fixed} Locus: Locations of "Flavor 6-Branes"

Singularities on ∂X

Often Interpretable as “Flavor Branes”



Flavor Branes (in M-theory)

Focus: $\mathbb{C}^2/\Gamma_{ADE}$ singularity $\Rightarrow$ 7D Gauge Theory

¿Gauge Group?

	$\mathbb{R}^{4,1}$					$\mathbb{R}_{\geq 0}$	K	$\mathbb{C}^2/\Gamma_{ADE}$			
	0	1	2	3	4	5	6	7	8	9	10
7D Flavor	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$				

Main Claim

Cvetic JJH Hubner Torres '22

related: Del Zotto Garcia-Etxebarria Schafer-Nameki '22

Compute 0-form, 1-form, 2-group via topology

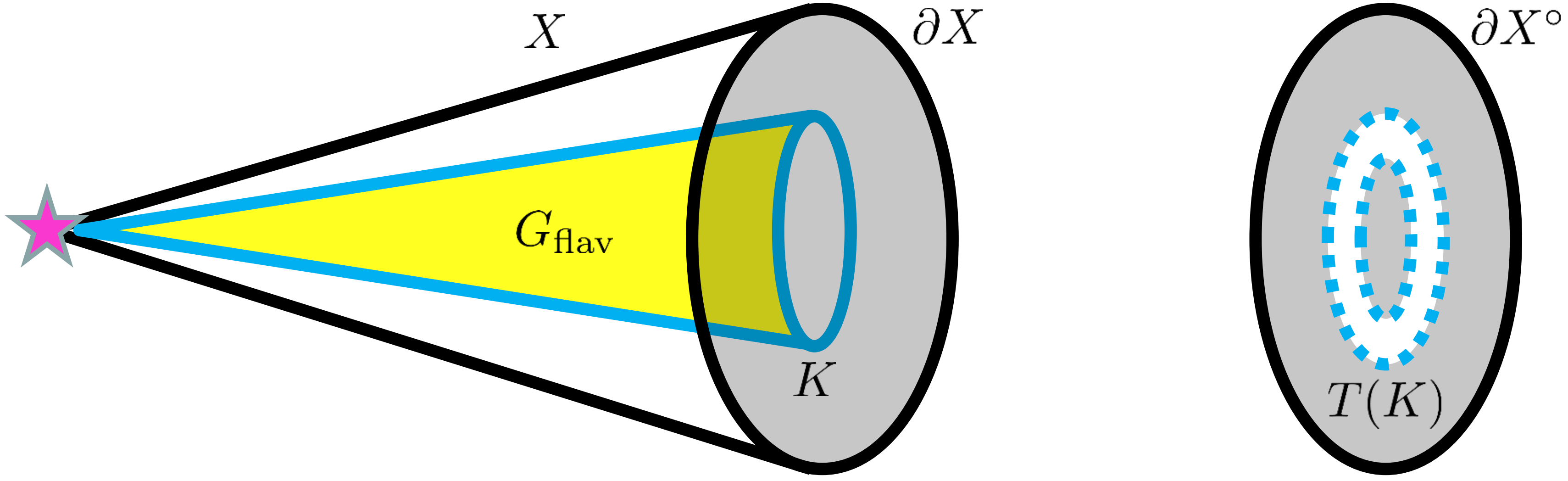
$$Z_{0\text{-Form}}:\ 0 \to \mathcal{C} \to Z_{\widetilde{G}} \to Z_G \to 0$$

$$\text{1-Form: } 0 \to \mathcal{A} \to \widetilde{\mathcal{A}} \to \mathcal{C} \to 0$$

$$\text{2-group: } 0 \to \mathcal{A} \to \widetilde{A} \to Z_{\widetilde{G}} \to Z_G \to 0$$

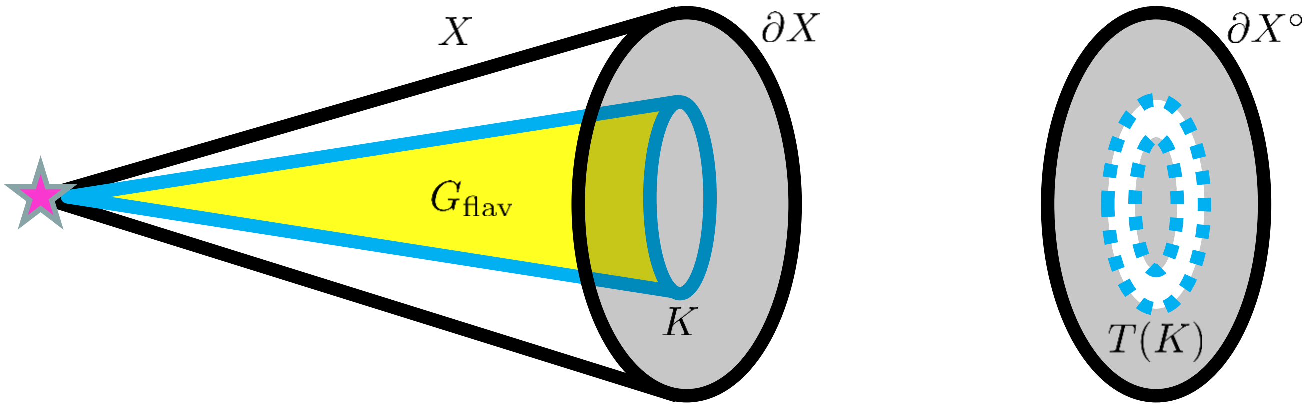
Extension problem: $H^3(BZ_G, \mathcal{A})$

“Boundary of Boundary”



Mayer '29; Vietoris '30

Mayer-Vietoris



$$\xrightarrow{\partial_{k+1}} H_k\big(\partial X^\circ \cap T(K)\big) \xrightarrow{\iota_k} H_k\big(\partial X^\circ\big) \oplus H_k\big(T(K)\big) \to H_k\big(\partial X\big)$$

$$\xrightarrow{\partial_k} H_{k-1}\big(\partial X^\circ \cap T(K)\big) \xrightarrow{\iota_{k-1}} H_{k-1}\big(\partial X^\circ\big) \oplus H_{k-1}\big(T(K)\big) \to \cdots$$

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