Simons Collaboration on Special Holonomy :
Geometry, Topology and Singular Special Holonomy Spaces
Freiburg University, June 6-9, 2022

Introduction, Part 2: Getting High on Gluing Orbifolds

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Motivation/Goals

- Motivation: M-theory on (non-)compact special holonomy spaces X and higher form symmetries:
  - non-compact spaces →
    superconformal quantum field theories (SCFTs) →
    higher-form symmetries global (``flavor'' branes)
  - compact spaces →
    quantum field theory (QFT) w/ gravity →
    higher-form symmetries gauged or broken

- Goals: Identify geometric origin of higher-form symmetries in non-compact special holonomy spaces X

Part II: beyond orbifolds & compact examples

Based on M. C., J. J. Heckman, M. Hübner and E. Torres, ``0-Form, 1-Form and 2-Group Symmetries via Cutting and Gluing of Orbifolds,'' 2203.10102 & work to appear

I. Introduction

[some repetition]

Defect Group for M-theory on non-compact X

- Defect Group for extended *p*-dim operators associated with M2 and M5 branes: $\mathcal{D}_p = \mathcal{D}_p^{\mathrm{M2}} \oplus \mathcal{D}_p^{\mathrm{M5}}$

- M2, M5 in X live on relative cycles:

[Morrison, Schäfer-Nameki, Willett, 2020],
[Albertini, Del Zotto, Garcia Etxebarria, Hosseini, 2020],
[Del Zotto, Heckman, Park, Rudelius, 2015]

$$\mathcal{D}_p^{\mathrm{M2}} = \frac{H_{3-p}(X,\partial X)}{H_{3-p}(X)} \cong H_{3-p-1}(\partial X)|_{\mathrm{triv}}$$

$$\mathcal{D}_p^{\mathrm{M5}} = \frac{H_{6-p}(X,\partial X)}{H_{6-p}(X)} \cong H_{6-p-1}(\partial X)|_{\mathrm{triv}}$$

- Defect pairing $\langle \cdot\,,\cdot \rangle$ in X ⟺ Linking Pairing $\ell(\cdot\,,\cdot)$ in ∂X

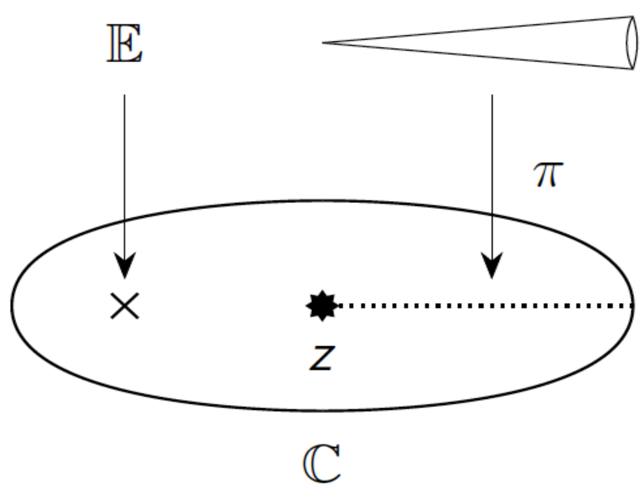
Example: non-compact K3

[M.C., Dierigl, Lin, Zhang, 2021, 2022]

- Local elliptically fibered K3: $\mathbb{E} \hookrightarrow X \rightarrow \mathbb{C}$

- Singular fiber of Kodaira type ϕ at $z \in \mathbb{C}$ w/ monodromy M

$$\mathcal{D}_1^{\mathrm{M2}} = \mathcal{D}_4^{\mathrm{M5}} = H_2(X, \partial X)/H_2(X) \cong \mathrm{Tor\,Coker}(M-1) = \langle \mathfrak{T} \rangle$$



$\mathfrak{T}$ -``Kodaira Thimble''
→ Max's talk

- X engineers 7D SYM with gauge algebra g_ϕ w/
  Defect group $\mathcal{D} = \langle \mathfrak{T} \rangle_1^{\mathrm{M2}} \oplus \langle \mathfrak{T} \rangle_4^{\mathrm{M5}}$

Example (continued):

- Non-trivial self-linking/intersection: $\ell(\partial\mathfrak{T},\partial\mathfrak{T}) = \mathfrak{T}\cdot\mathfrak{T} \neq 0$
- Elements of $\mathcal{D}_1^{M2}$, $\mathcal{D}_4^{M5}$ typically mutually non-local
- Choose electric polarization $\mathcal{D}_1^{M2}$ [for the rest of the talk]
- Gauge group is simply connected G_ϕ w/ algebra g_ϕ (ADE)
- Resulting 7D SYM theory w/ gauge group G_ϕ
- (Wilson) Line operators $\mathcal{D}_1^{M2}$ acted on by 1-form symmetry $Z_{G\phi}$ [fix group topology]

Now, turn to higher-form structures for non-compact spaces X in higher dimensions ($D \geq 6$)

→ leads to new phenomena

c.f., talks by Jonathan & Ethan

Outline for the rest of the talk:

II. Summary of key points in Jonathan's & Ethan's talks

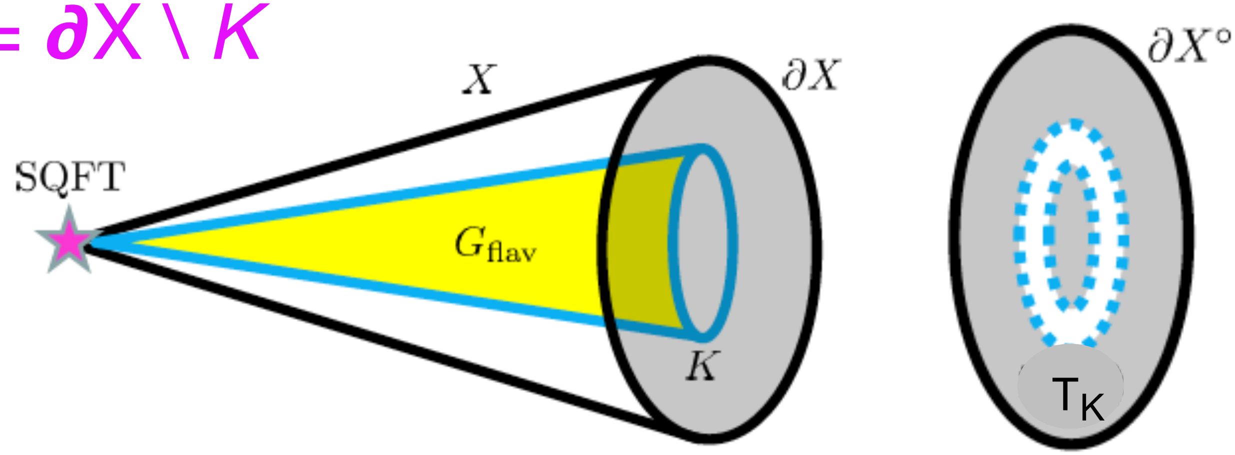
III. Application to elliptically fibered Calabi-Yau n-folds

IV. M-theory on G_2 spaces:
Circle fibered G_2 &
Type IIA on Calabi-Yau three-fold w/ D6 branes

IV. Compact examples

Summary of key points in Jonathan's & Ethan's talks

- Geometries: Singular non-compact space X w/ ADE singularities K extending to the asymptotic boundary ∂X and $\partial X^{\circ} = \partial X \setminus K$



- Key Mayer-Vietoris Sequence in homology (wrt ∂X° & T_K):

$$0 \to \underbrace{\ker(\iota_1)}_{\text{flavor center}} \to \underbrace{H_1(\partial X^{\circ} \cap T_K)}_{\text{naive flavor center}} \xrightarrow{\iota_1} \underbrace{H_1(\partial X^{\circ})}_{\text{naïve 1-form symmetry}} \oplus H_1(T_K) \to \underbrace{H_1(\partial X)}_{\text{1-form symmetry}} \to 0$$

- Motivated by orbifold homology: $H_1(\partial X^{\circ}) = H_1^{\text{orb}}(\partial X)$

Summary (continued)

- M2 branes on non-compact two-cycles → charged operators
- Symmetries, Pontryagin dual to charge operators nested as

$$0 \to \mathcal{A} \to \widetilde{\mathcal{A}} \to Z_{\widetilde{G}_F} \to Z_{G_F} \to 0$$

- Split the homology sequence (physically motivated):

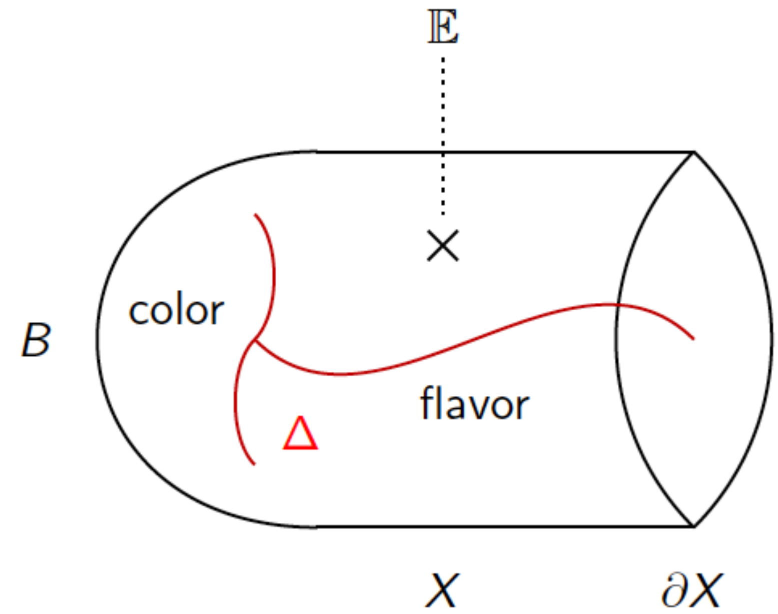
$$0 \to \ker(\iota_1) \to H_1(\partial X^\circ \cap T_K) \xrightarrow{\iota_1} \frac{H_1(\partial X^\circ \cap T_K)}{\ker(\iota_1)} \to 0\,,$$

$$0 \to \frac{H_1(\partial X^\circ \cap T_K)}{\ker(\iota_1)} \to H_1(\partial X^\circ) \oplus H_1(T_K) \to H_1(\partial X) \to 0$$

- When the bottom sequence does not split →
  2-group mixing 0-form and 1-form (flavor) symmetries

III. Elliptically fibered Calabi-Yau n-folds

- : $\mathbb{E} \hookrightarrow X_n \to B_{n-1}$



- Non-compact discriminant locus Δ

[Hübner, Morrison, Schäfer-Nameki, Wang, 2022]

- Homology groups of ∂X, ∂X° & $\partial X_F = \partial X \setminus$ {singular fibers}
- Deformation retractions of ∂B \ Δ lift to ∂X_F
- Glue singular fibers back in

$$\text{Tor}\, H_1\left(\partial X^{\circ}\right) = \text{Tor}\, H_1\left(\partial X_F\right) \oplus \text{Tor}\, H_1(\partial B)$$

→ More details, including explicit examples in Max's talk

IV. M-theory on circle fibered G_2 & Type IIA on Calabi-Yau threefold w/D6 branes

- M-theory on circle S^1 fibered G_2 holonomy space
- As size of $S^1 \rightarrow 0$:
  Type IIA string theory on Calabi-Yau three-fold w/ D6 branes
- Co-dimension 4 singularities in $G_2 \leftrightarrow$
  D6 branes in Calabi-Yau three-fold
  source gauge and flavor symmetries; detailed map c.f.,
  [M.C., Shiu, Uranga 2001]
- Metric hard, but topological issues can be addressed $\rightarrow$

Boundary Topology

Type IIA on CY3 w/D6-branes:

- Flavor Branes source RR $F = dC_1$
- Excise Flavor Branes $\partial X_6 \rightarrow \partial X_6^o$
- Expand Poincaré dual of F in 2-cycles of ∂X_6^o

M-theory on G_2:

- Construct circle fibration ∂X_7^o with Euler class F
- Gysin sequence
- Glue orbifold loci back in

More details → Max's talk

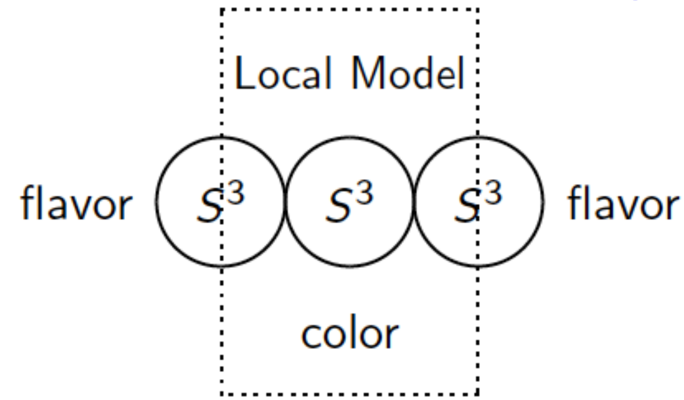
SQCD-Like Geometries

- Supersymmetric three-spheres in Calabi-Yau threefold

[Feng, He, Kennaway, Vafa, 2008],
[Del Zotto, Oh, Zhou, 2021]

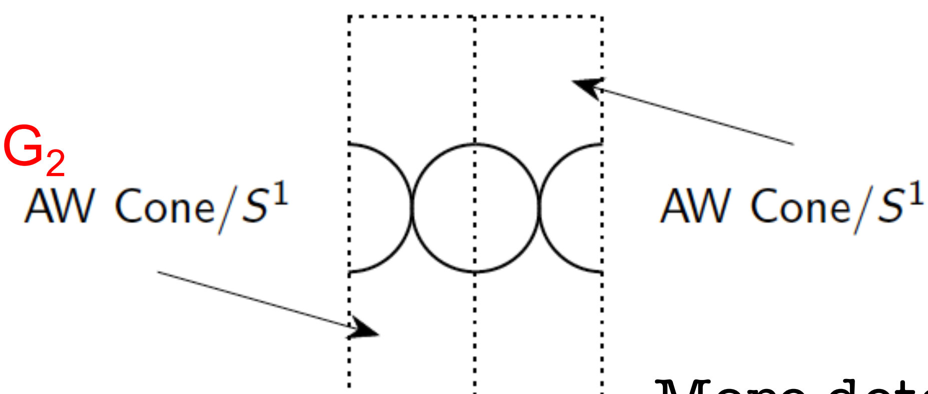
- Local geometry of color three-sphere gives 4D SQCD-like

Type IIA on local CY3
w/ D6-branes



- Topology matches two glued Acharya-Witten cones

M-theory on Local G_2



More details → Max's talk

V. Compact Models

- Compact singular space X → theory that includes quantum gravity & global symmetries gauged or broken
- What is M-theory gauge group?
- Elliptically fibered geometries:

  - Non-Abelian group algebras – ADE Kodaira classification
  Group topology → Mordell-Weil torsion

  [Aspinwall, Morrison, 1998], [Mayrhofer, Morrison,Till, Weigand, 2014], [M.C., Lin, 2017]

  - Abelian groups → Mordell-Weil ``free'' part

  [Morrison, Park 2012], [M.C., Klevers, Piragua, 2013], [Borchmann, Mayrhofer, Palti, Weigand, 2013]…

  - Total group topology → Shoida map of Mordell-Weil

  [M.C., Lin, 2017]

$$\frac{U(1)^r \times G_{\text{non-ab}}}{\prod_{i=1}^{r} \mathbb{Z}_{m_i} \times \prod_{j=1}^{t} \mathbb{Z}_{k_j}}$$

Compact Geometries (continued):

- Decompose $X = \cup_n X_n$ into local models X_n
  Converse:

  Glue $\{X_n\}$ to X ←→ Couple $\{SQFT_n\}$ to resulting one
  & includes gravity

- Relative Cycles in X_n compactify →
  (some) defects in $SQFT_n$ become dynamical - ``gauged''

Compact Geometries (continued):

- Mayer-Vietoris Sequence for covering $\{X_n\}$:

$$\partial_2 : H_2(X) \rightarrow \oplus_n H_1(\partial X_n)$$

- Decomposition of compact two-cycles into a sum of relative cycles associated with each local model

- Elliptically fibered geometries:
  torsional cycles associated w/ Mordell-Weil group
  decomposition into relative cycles of $\{X_n\}$

- Arguments extend beyond elliptically fibered models, e.g., $T^4/\mathbb{Z}_2$

→ Max's talk

Thank you!