Introduction, Part 2: Getting High on Gluing Orbifolds

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Motivation/Goals

• Motivation: M-theory on (non-)compact special holonomy spaces $X$ and higher form symmetries:
  - non-compact spaces $\rightarrow$ superconformal quantum field theories (SCFTs) $\rightarrow$
  higher-form symmetries global (``flavor'' branes)
  - compact spaces $\rightarrow$
  quantum field theory (QFT) w/ gravity $\rightarrow$
  higher-form symmetries gauged or broken

• Goals: Identify geometric origin of higher-form symmetries in non-compact special holonomy spaces $X$

Part II: beyond orbifolds & compact examples

Based on M. C., J. J. Heckman, M. Hübner and E. Torres,``0-Form, 1-Form and 2-Group Symmetries via Cutting and Gluing of Orbifolds,” 2203.10102 & work to appear
I. Introduction

Defect Group for M-theory on non-compact $X$

- Defect Group for extended $p$-dim operators associated with M2 and M5 branes:

$$\mathcal{D}_p = \mathcal{D}_p^{M2} \oplus \mathcal{D}_p^{M5}$$

- M2, M5 in $X$ live on relative cycles:

$$\mathcal{D}_p^{M2} = \frac{H_{3-p}(X, \partial X)}{H_{3-p}(X)} \cong H_{3-p-1}(\partial X)|_{\text{triv}}$$

$$\mathcal{D}_p^{M5} = \frac{H_{6-p}(X, \partial X)}{H_{6-p}(X)} \cong H_{6-p-1}(\partial X)|_{\text{triv}}$$

- Defect pairing $\langle \cdot, \cdot \rangle$ in $X$ ↔ Linking Pairing $\ell(\cdot, \cdot)$ in $\partial X$
Example: non-compact K3

[M.C., Dierigl, Lin, Zhang, 2021, 2022]

- Local elliptically fibered K3: \( E \hookrightarrow X \rightarrow \mathbb{C} \)

- Singular fiber of Kodaira type \( \phi \) at \( z \in \mathbb{C} \) w/ monodromy \( M \)

\[
D_1^{M^2} = D_4^{M^5} = H_2(X, \partial X)/H_2(X) \cong \text{Tor} \text{Coker}(M - 1) = \langle \Sigma \rangle
\]

- \( X \) engineers 7D SYM w/ gauge algebra \( g_\phi \) w/ Defect group \( D = \langle \Sigma \rangle_1^{M^2} \oplus \langle \Sigma \rangle_4^{M^5} \)

\( \Sigma \) - ``Kodaira Thimble''

\( \rightarrow \) Max's talk
Example (continued):

- Non-trivial self-linking/intersection: \( \ell(\partial \mathcal{I}, \partial \mathcal{I}) = \mathcal{I} \cdot \mathcal{I} \neq 0 \)

- Elements of \( \mathcal{D}_1^{M2} , \mathcal{D}_4^{M5} \) typically mutually non-local

- Choose electric polarization \( \mathcal{D}_1^{M2} \) [for the rest of the talk]

- Gauge group is simply connected \( G_\phi \) w/ algebra \( g_\phi \) (ADE)

- Resulting 7D SYM theory w/ gauge group \( G_\phi \)

- (Wilson) Line operators \( \mathcal{D}_1^{M2} \) acted on by 1-form symmetry \( Z_{G_\phi} \) [fix group topology]
Now, turn to higher-form structures for non-compact spaces $X$ in higher dimensions ($D \geq 6$) 

→ leads to new phenomena

*c.f.*, talks by Jonathan & Ethan
Outline for the rest of the talk:

II. Summary of key points in Jonathan’s & Ethan’s talks

III. Application to elliptically fibered Calabi-Yau n-folds

IV. M-theory on $G_2$ spaces:
   Circle fibered $G_2$ &
   Type IIA on Calabi-Yau three-fold w/ D6 branes

IV. Compact examples
Summary of key points in Jonathan’s & Ethan’s talks

- **Geometries**: Singular non-compact space $X$ w/ ADE singularities $K$ extending to the asymptotic boundary $\partial X$ and $\partial X^o = \partial X \setminus K$

- **Key Mayer-Vietoris Sequence in homology (wrt $\partial X^o$ & $T_K$)**:

$$0 \rightarrow \ker(\iota_1) \rightarrow H_1(\partial X^o \cap T_K) \xrightarrow{\iota_1} H_1(\partial X^o) \oplus H_1(T_K) \rightarrow H_1(\partial X) \rightarrow 0$$

  - $\ker(\iota_1)$: flavor center
  - $H_1(\partial X^o \cap T_K)$: naive flavor center
  - $H_1(\partial X^o)$: naive 1-form symmetry
  - $H_1(T_K)$: 1-form symmetry

- **Motivated by orbifold homology**: $H_1(\partial X^o) = H_1^{\text{orb}}(\partial X)$
Summary (continued)

- M2 branes on non-compact two-cycles $\rightarrow$ charged operators

- Symmetries, Pontryagin dual to charge operators nested as

  $$0 \rightarrow A \rightarrow \tilde{A} \rightarrow \mathbb{Z}_{\tilde{G}_F} \rightarrow \mathbb{Z}_{G_F} \rightarrow 0$$

- Split the homology sequence (physically motivated):

  $$0 \rightarrow \ker(\nu_1) \rightarrow H_1(\partial X^0 \cap T_K) \xrightarrow{\nu_1} \frac{H_1(\partial X^0 \cap T_K)}{\ker(\nu_1)} \rightarrow 0,$$

  $$0 \rightarrow \frac{H_1(\partial X^0 \cap T_K)}{\ker(\nu_1)} \rightarrow H_1(\partial X^0) \oplus H_1(T_K) \rightarrow H_1(\partial X) \rightarrow 0$$

- When the bottom sequence does not split $\rightarrow$ 2-group mixing 0-form and 1-form (flavor) symmetries
III. Elliptically fibered Calabi-Yau n-folds

- : $\mathbb{E} \leftrightarrow X_n \rightarrow B_{n-1}$

  - Non-compact discriminant locus $\Delta$
    
    [Hübner, Morrison, Schäfer-Nameki, Wang, 2022]

  - Homology groups of $\partial X$, $\partial X^o$, $\partial X_F = \partial X \setminus \{\text{singular fibers}\}$

  - Deformation retractions of $\partial B \setminus \Delta$ lift to $\partial X_F$

  - Glue singular fibers back in
    
    $\text{Tor } H_1(\partial X^o) = \text{Tor } H_1(\partial X_F) \oplus \text{Tor } H_1(\partial B)$

  \[ \rightarrow \text{More details, including explicit examples in Max’s talk} \]
IV. M-theory on circle fibered $G_2$ & Type IIA on Calabi-Yau threefold w/D6 branes

- M-theory on circle $S^1$ fibered $G_2$ holonomy space

- As size of $S^1 \to 0$:
  Type IIA string theory on Calabi-Yau three-fold w/ D6 branes

- Co-dimension 4 singularities in $G_2 \leftrightarrow$ D6 branes in Calabi-Yau three-fold
  source gauge and flavor symmetries; detailed map c.f.,
  [M.C., Shiu, Uranga 2001]

- Metric hard, but topological issues can be addressed
Boundary Topology

Type IIA on CY3 w/D6-branes:
• Flavor Branes source RR $F = dC_1$
• Excise Flavor Branes $\partial X_6 \rightarrow \partial X_6^\circ$
• Expand Poincaré dual of $F$ in 2-cycles of $\partial X_6^\circ$

M-theory on $G_2$:
• Construct circle fibration $\partial X_7^\circ$ with Euler class $F$
• Gysin sequence
• Glue orbifold loci back in

More details → Max’s talk
SQCD-Like Geometries

- Supersymmetric three-spheres in Calabi-Yau threefold
  [Feng, He, Kennaway, Vafa, 2008],
  [Del Zotto, Oh, Zhou, 2021]
- Local geometry of color three-sphere gives 4D SQCD-like
  Type IIA on local CY3 w/ D6-branes
- Topology matches two glued Acharya-Witten cones
  M-theory on Local G₂

More details → Max’s talk
V. Compact Models

- Compact singular space $X \rightarrow$ theory that includes quantum gravity & global symmetries gauged or broken

- What is M-theory gauge group?

- Elliptically fibered geometries:
  - Non-Abelian group algebras – ADE Kodaira classification
    Group topology $\rightarrow$ Mordell-Weil torsion
    
    \[
    \frac{U(1)^r \times G_{non-ab}}{\prod_{i=1}^{r} \mathbb{Z}_{m_i} \times \prod_{j=1}^{r} \mathbb{Z}_{k_j}}
    \]
    [Aspinwall, Morrison, 1998], [Mayrhofer, Morrison, Till, Weigand, 2014], [M.C., Lin, 2017]

  - Abelian groups $\rightarrow$ Mordell-Weil \textquotedblleft free\textquotedblright part
    [Morrison, Park 2012], [M.C., Klevers, Piragua, 2013],
    [Borchmann, Mayrhofer, Palti, Weigand, 2013]…

  - Total group topology $\rightarrow$ Shoida map of Mordell-Weil
    [M.C., Lin, 2017]
Compact Geometries (continued):

- Decompose $X = \bigcup_n X_n$ into local models $X_n$
  Converse:

  \[
  \text{Glue } \{X_n\} \text{ to } X \iff \text{Couple } \{\text{SQFT}_n\} \text{ to resulting one}
  \text{ & includes gravity}
  \]

- Relative Cycles in $X_n$ compactify $\Rightarrow$
  (some) defects in $\text{SQFT}_n$ become dynamical - ``gauged''
Compact Geometries (continued):

- **Mayer-Vietoris Sequence for covering \( \{X_n\} \):**
  \[
  \partial_2 : H_2(X) \rightarrow \bigoplus_n H_1(\partial X_n)
  \]

- **Decomposition of compact two-cycles into a sum of relative cycles associated with each local model**

- **Elliptically fibered geometries:**
  torsional cycles associated with Mordell-Weil group decomposition into relative cycles of \( \{X_n\} \)

- **Arguments extend beyond elliptically fibered models,** e.g., \( T^4 / \mathbb{Z}_2 \)
→ Max’s talk
Thank you!