Counting Associatives in a G_2 Orbifold

Eirik Eik Svanes, KCL, ICTP

Simons Center for Geometry and Physics

September 9, 2018

Based on work in collaboration with B. Acharya, A. Braun and R. Valandro

Introduction

Motivation and Overview

String model building often leaves a number of moduli to be "stabilised", i.e. the corresponding field in the effective physics must be given a mass.

Maldacena-Nunez: Perturbative effects and fluxes often fall short $\Rightarrow$ Need *non-perturbative effects*.

Branes wrapped on calibrated cycles one can produce such effects. F-theory: $M5$-instantons wrapping effective divisors in a Calabi-Yau four-fold [Donagi-Grassi-Witten '96] (DGW).

String dualitiy: There should be effects dual to DGW in string-theory and M-theory. Heterotic: World-sheet instantons [Curio-Lüst '97, Anderson etal '15]. M-theory: Euclidean $M2$-branes wrapping associative cycles [Braun-DelZotto-Halverson-Larfors-Morrison-SchaferNameki '18].

Mathematically interesting: Can lead to *interesting mathematical conjectures*.

Motivation and Overview

String model building often leaves a number of moduli to be "stabilised", i.e. the corresponding field in the effective physics must be given a mass.

Maldacena-Nunez: Perturbative effects and fluxes often fall short $\Rightarrow$ Need *non-perturbative effects*.

Branes wrapped on calibrated cycles one can produce such effects. F-theory: $M5$-instantons wrapping effective divisors in a Calabi-Yau four-fold [Donagi-Grassi-Witten '96] (DGW).

String dualitiy: There should be effects dual to DGW in string-theory and M-theory. Heterotic: World-sheet instantons [Curio-Lüst '97, Anderson etal '15]. M-theory: Euclidean $M2$-branes wrapping associative cycles [Braun-DelZotto-Halverson-Larfors-Morrison-SchaferNameki '18].

Mathematically interesting: Can lead to *interesting mathematical conjectures*.

Overview:

- Explicit construction of associative in a G_2 orbifold.
- The weak coupling limit to type IIA and Special Lagrangians.
- Mirror symmetry and divisors in type IIB. Lift to F-theory.
- Conclusions and Outlook.

Some Calibrated Geometry

Given (M, Φ), where Φ is some p-form denoting some extra structure associated to M (e.g. CY, G_2, etc where we also insist that $\mathrm{d}\Phi = 0$).

Φ is a calibration if for $x \in M$ we have $\Phi_x = \lambda \mathrm{vol}_\xi$ where $\lambda \leq 1$ $\forall$ oriented p-dim $\xi \subseteq T_x M$.

A p-dimensional sub-manifold $N \subseteq M$ is calibrated w.r.t. Φ if $\Phi|_N = \mathrm{vol}_N$. We then have

$$\mathrm{Vol}(N) = \int_N \mathrm{vol}_N = \int_N \Phi = \int_{\tilde{N}} \Phi \leq \int_{\tilde{N}} \mathrm{vol}_{\tilde{N}} = \mathrm{Vol}(\tilde{N}) \,,$$

where N and $\tilde{N}$ are in the same homology class.

In this sense, calibrated sub-manifolds are *minimal surfaces*. BPS conditions $\Rightarrow$ Can wrap branes on minimal surfaces.

Some Calibrated Geometry

Given (M, Φ), where Φ is some p-form denoting some extra structure associated to M (e.g. CY, G_2, etc where we also insist that $\mathrm{d}\Phi = 0$).

Φ is a calibration if for $x \in M$ we have $\Phi_x = \lambda \mathrm{vol}_\xi$ where $\lambda \leq 1$ $\forall$ oriented p-dim $\xi \subseteq T_x M$.

A p-dimensional sub-manifold $N \subseteq M$ is calibrated w.r.t. Φ if $\Phi|_N = \mathrm{vol}_N$. We then have

$$\mathrm{Vol}(N) = \int_N \mathrm{vol}_N = \int_N \Phi = \int_{\tilde{N}} \Phi \leq \int_{\tilde{N}} \mathrm{vol}_{\tilde{N}} = \mathrm{Vol}(\tilde{N}) \,,$$

where N and $\tilde{N}$ are in the same homology class.

In this sense, calibrated sub-manifolds are *minimal surfaces*. BPS conditions $\Rightarrow$ Can wrap branes on minimal surfaces.

Examples:

- Kähler manifold: Normalised powers of the Kähler form. Calibrated sub-manifolds are complex submanifolds, e.g. *effective divisors*.
- Calabi-Yau: The real part of a holomorphic volume form. Calibrated submanifolds are *special Lagrangian*.
- G_2: The associative/co-associative three- and four-form. Calibrated submanifolds are *associative* and *co-associative*.

G_2 Orbifolds and Associative Cycles

The G_2 Orbifold

Our main objects of interest are G_2 orbifolds of the form

$$M = (X \times T^3) \Big/ \mathbb{Z}_2 \times \mathbb{Z}_2 \, ,$$

where X is a $K3$ surface with hyper-Kähler structure $\{\omega_1, \omega_2, \omega_3\}$. The G_2 three-form Φ is

$$\Phi = dx_1 \wedge dx_2 \wedge dx_3 + \sum_i \omega_i \wedge dx_i \, .$$

The G_2 Orbifold

Our main objects of interest are G_2 orbifolds of the form

$$M = (X \times T^3) \Big/ \mathbb{Z}_2 \times \mathbb{Z}_2 \ ,$$

where X is a $K3$ surface with hyper-Kähler structure $\{\omega_1, \omega_2, \omega_3\}$. The G_2 three-form Φ is

$$\Phi = dx_1 \wedge dx_2 \wedge dx_3 + \sum_i \omega_i \wedge dx_i \ .$$

To get a non-trivial G_2 orbifold, we are looking for a commuting pair of involutions $\{\alpha, \beta\}$ which act by giving the following signs

$$\begin{array}{ccccccc} & \omega_1 & \omega_2 & \omega_3 & dx_1 & dx_2 & dx_3 \\ \alpha & + & - & - & + & - & - \\ \beta & - & - & + & - & - & + \end{array} \ .$$

We associate ω_1 with the Kähler form J_2 of X, and $\omega_2 + i\,\omega_3$ with the holomorphic two-form Ω_2. Note that α is a holomorphic involution while β is anti-holomorphic.

The Algebraic $K3$ Surface

We make a choice of involutions α and β which have an explicit algebraic realisation. Consider a family of elliptic K3 surfaces realized as the complete intersection

$$\begin{aligned} y^2 &= x^3 + xw^4 f_4(z) + w^6 g_6(z) \\ \xi^2 &= z_1 z_2 \, , \end{aligned}$$

where f_4 and g_6 are homogeneous polynomials in $\{z_1, z_2\}$ of indicated degree. We impose the toric weight system

x	y	w	z_1	z_2	ξ	Sum of degrees	W	$\xi^2 = z_1 z_2$
2	3	1	0	0	0	6	6	0
2	3	0	1	1	1	8	6	2

The sum of the degrees equals the sum of the equations, so this is indeed a $K3$.

The Algebraic Involutions

Go to a point in moduli space where f_4 and g_6 have real coefficients.

Let us choose the involutions to act algebraically as

$$\begin{aligned} \alpha : &\quad \xi \to -\xi \\ \beta : &\quad (y, x, w, z_1, z_2, \xi) \to (\bar{y}, \bar{x}, \bar{w}, \bar{z}_1, \bar{z}_2, \bar{\xi}) \, , \end{aligned}$$

where indeed α acts holomorphically, while β acts anti-holomorphically. Both involutions fit in the classification of Nikulin.

The Algebraic Involutions

Go to a point in moduli space where f_4 and g_6 have real coefficients.

Let us choose the involutions to act algebraically as

$$\begin{aligned} \alpha : &\quad \xi \rightarrow -\xi \\ \beta : &\quad (y, x, w, z_1, z_2, \xi) \rightarrow (\bar{y}, \bar{x}, \bar{w}, \bar{z}_1, \bar{z}_2, \bar{\xi}) \,, \end{aligned}$$

where indeed α acts holomorphically, while β acts anti-holomorphically. Both involutions fit in the classification of Nikulin.

α has two tori as it's fixed point locus $\Rightarrow$ acts as $(10, 8, 0)$ involution.

In terms of the second homology lattice of X,

$$H_2(X, \mathbb{Z}) = -E_8^+ \oplus -E_8^- \oplus U_1 \oplus U_2 \oplus U_3 \,,$$

we can insist that (modulo automorphisms)

$$\alpha \circlearrowleft \frac{E_8^+ \quad E_8^- \quad U_1 \quad U_2 \quad U_3}{E_8^- \quad E_8^+ \quad U_1 \quad -U_2 \quad -U_3} \,.$$

The Algebraic Involutions

In terms of α, we see that we must choose a hyper-Kähler structure so that

$$J_2 = \omega_1 \in \left(U_1 \oplus [E_8^+ + E_8^-]\right) \otimes \mathbb{R}$$
$$\Omega_2 = \omega_2 + i\,\omega_3 \in \left(U_2 \oplus U_3 \oplus [E_8^+ - E_8^-]\right) \otimes \mathbb{R}\,.$$

The Algebraic Involutions

In terms of α, we see that we must choose a hyper-Kähler structure so that

$$J_2 = \omega_1 \in \Big(U_1 \oplus [E_8^+ + E_8^-]\Big) \otimes \mathbb{R}$$
$$\Omega_2 = \omega_2 + i\,\omega_3 \in \Big(U_2 \oplus U_3 \oplus [E_8^+ - E_8^-]\Big) \otimes \mathbb{R}\,.$$

Fixed points of β are determined by our choice of homogeneous polynomials f_4 and g_6.

Choose f_4 and g_6 so that β also acts as a representative of the $(10, 8, 0)$ involution, though now anti-holomorphically:

$$\beta \circlearrowleft \frac{\begin{matrix} E_8^+ & E_8^- & U_1 & U_2 & U_3 \end{matrix}}{\begin{matrix} -E_8^- & -E_8^+ & -U_1 & -U_2 & U_3 \end{matrix}}\,.$$

Indeed β acts as $\Omega_2 \to -\bar{\Omega}_2$ if we further restrict the hyper-Kähler structure to be

$$\omega_2 \in U_2 \otimes \mathbb{R}$$
$$\omega_3 \in \Big(U_3 \oplus [E_8^+ - E_8^-]\Big) \otimes \mathbb{R}\,.$$

Sections of $K3$

Let $U_1 = \langle e_1, e^1 \rangle$. We may identify the class of the zero section of the elliptic fibration of X with $\sigma_0 = e_1 - e^1$ and the class of the fibre with $F = e^1$.

For every lattice element γ_+ in E_8^+ with $\gamma_+^2 = -2n$ and $\alpha(\gamma_+) = \gamma_-$, we may now consider the class

$$\sigma_\gamma = \sigma_0 + 2nF + \gamma_+ + \gamma_- \ .$$

Note that $\sigma_\gamma^2 = -2$ and $F \cdot \sigma_\gamma = 1$.

Sections of $K3$

Let $U_1 = \langle e_1, e^1 \rangle$. We may identify the class of the zero section of the elliptic fibration of X with $\sigma_0 = e_1 - e^1$ and the class of the fibre with $F = e^1$.

For every lattice element γ_+ in E_8^+ with $\gamma_+^2 = -2n$ and $\alpha(\gamma_+) = \gamma_-$, we may now consider the class

$$\sigma_\gamma = \sigma_0 + 2nF + \gamma_+ + \gamma_- \, .$$

Note that $\sigma_\gamma^2 = -2$ and $F \cdot \sigma_\gamma = 1$.

Each such class σ_γ corresponds to a section Σ_γ of the elliptic fibration on X:

- Modding out by α gives a rational elliptic surface S. S has a holomorphic section for each element in E_8, who's class is described by the quotient of σ_γ.
- The base of the elliptic fibration on this S is given by $[z_1 : z_2]$, so that there are holomorphic functions $y(z), x(z), w(z)$ for every element of E_8.
- The double cover giving X affects only the base of the elliptic fibration, i.e. supplies two values of ξ for each $[z_1 : z_2]$, so that all of these sections lift to sections of X.

The action of β on Σ_γ

We want to show that the Σ_γ's are preserved as sub-manifolds under β:

- β is anti-holomorphic: Sending every point on $\Sigma_\gamma \subset S$ to its complex conjugate:

$$\beta : (y(z), x(z), w(z), z_1, z_2) \rightarrow (\overline{y(z)}, \overline{x(z)}, \overline{w(z)}, \bar{z}_1, \bar{z}_2)\,.$$

The action of β on Σ_γ

We want to show that the Σ_γ's are preserved as sub-manifolds under β:

- β is anti-holomorphic: Sending every point on $\Sigma_\gamma \subset S$ to its complex conjugate:

$$\beta : (y(z), x(z), w(z), z_1, z_2) \to (\overline{y(z)}, \overline{x(z)}, \overline{w(z)}, \bar{z}_1, \bar{z}_2) \, .$$

- Preserving Σ_γ as a sub-manifold is equivalent to

$$(\overline{y(z)}, \overline{x(z)}, \overline{w(z)}, \bar{z}_1, \bar{z}_2) = (y(\bar{z}), x(\bar{z}), w(\bar{z}), \bar{z}_1, \bar{z}_2) \, ,$$

as reversing the orientation, $z \to \bar{z} \Rightarrow$ end up at a different point on Σ_γ.

- If this was not the case:

$$(\overline{y(z)}, \overline{x(z)}, \overline{w(z)}, \bar{z}_1, \bar{z}_2) = (y'(\bar{z}), x'(\bar{z}), w'(\bar{z}), \bar{z}_1, \bar{z}_2) \, ,$$

we can reverse orientation again and get a different representative holomorphic section representing σ_γ, contradiction.

Associatives in M

We are now ready to define the associatives in the G_2 orbifold M. Consider

$$C_\gamma = (\Sigma_\gamma \times S^1_{x_1}) \big/ \alpha \times \beta \, .$$

$S^1_{x_1}$ is untouched under α, while Σ_γ descends to a section of S.

Σ_γ is preserved as a sub-manifold of S under β, and we can describe C_γ as an S^2 sitting over an interval which degenerates at the ends $\Rightarrow$ β is seen to produce a three-manifold with the topology of a sphere S^3.

Associatives in M

We are now ready to define the associatives in the G_2 orbifold M. Consider

$$C_\gamma = (\Sigma_\gamma \times S^1_{x_1})\Big/\alpha \times \beta \,.$$

$S^1_{x_1}$ is untouched under α, while Σ_γ descends to a section of S.

Σ_γ is preserved as a sub-manifold of S under β, and we can describe C_γ as an S^2 sitting over an interval which degenerates at the ends $\Rightarrow$ β is seen to produce a three-manifold with the topology of a sphere S^3.

As the Σ_γ are calibrated by the Kähler form ω_1 on X, it is easy to show that

$$Vol(C_\gamma) = \int_{C_\gamma} \Phi \,.$$

Furermore, $*\Phi|_{C_\gamma} = 0$, and all of the C_γ are associatives of M.

Wrapping Euclidean $M2$ branes $\Rightarrow$ superpotential in M-theory.
String duality: Should exist de-singularisation of M that preserves the associatives.

Expect de-singularisation to produce an additional E_8 lattice worth of associatives [Curio-Lüst '97, Braun et al '18].

The Type IIA Limit

The type IIA Orbifold

Type IIA limit: Size of $S^1_{x_3}$ is sent to zero. Note: α is the type IIA orientifold.

The type IIA Calabi-Yau is a Voisin-Borcea (VB) orbifold

$$Z^\vee = (X^\vee \times T^2)\Big/\beta\,,$$

which is the Schoen (split bi-cubic) at an orbifold point. Here T^2 has coordinate $z = x_2 + i\,x_1$, and $X^\vee$ is a $K3$ surface where we have performed the hyper-Kähler rotation

$$\begin{aligned}\omega_X &\to \mathrm{Im}(\Omega^{(2,0)}_{X^\vee})\\ \mathrm{Im}(\Omega^{(2,0)}_X) &\to -\omega_{X^\vee}\\ \mathrm{Re}(\Omega^{(2,0)}_X) &\to \mathrm{Re}(\Omega^{(2,0)}_{X^\vee})\,.\end{aligned}$$

The Calabi-Yau structure forms are

$$\begin{aligned}\omega_{Z^\vee} &= \omega_{X^\vee} + \mathrm{vol}(T^2)\\ \Omega^{(3,0)}_{Z^\vee} &= \Omega^{(2,0)}_{X^\vee} \wedge dz\,.\end{aligned}$$

Special Lagrangian's

The corresponding calibrated cycles within $Z^\vee$ are now given by

$$C_\gamma^\vee = (\Sigma_\gamma \times S^1_{x_1})\Big/\beta\,.$$

It is easy to check that these cycles are *special Lagrangian*:

$$\int_{C_\gamma} \mathrm{Im}(\Omega^{3,0}_{Z^\vee}) = \int_{C_\gamma^\vee} \left(\mathrm{Re}(\Omega^{(2,0)}_{X^\vee}) \wedge dx_1 + \mathrm{Im}(\Omega^{(2,0)}_{X^\vee}) \wedge dx_2\right) = 0\,,$$

in addition to the Lagrangian condition that $\omega_{Z^\vee}|_{C_\gamma^\vee} = 0$.

Special Lagrangian's

The corresponding calibrated cycles within $Z^\vee$ are now given by

$$C_\gamma^\vee = (\Sigma_\gamma \times S^1_{x_1}) \Big/ \beta \,.$$

It is easy to check that these cycles are *special Lagrangian*:

$$\int_{C_\gamma} \mathrm{Im}(\Omega^{3,0}_{Z^\vee}) = \int_{C_\gamma^\vee} \left(\mathrm{Re}(\Omega^{(2,0)}_{X^\vee}) \wedge dx_1 + \mathrm{Im}(\Omega^{(2,0)}_{X^\vee}) \wedge dx_2 \right) = 0 \,,$$

in addition to the Lagrangian condition that $\omega_{Z^\vee}|_{C_\gamma^\vee} = 0$.

Again, it is expected that the special Lagrangians persist under an appropriate de-singularisation of $Z^\vee$.

In particular, a small resolution of $Z^\vee$ introduces a set of divisors orthogonal to $\Sigma_\gamma \in H_2(Z^\vee) \cong H^4(Z^\vee) \Rightarrow$ expect C_γ to remain Lagrangian. Furthermore, a small resolution does not change the complex structure $\Rightarrow$ expect C_γ to remain special Lagrangian.

The String Junction Picture

$X^\vee$ is again elliptically fibered, now with the base represented by $\sigma_0^\vee = e_3 - e^3$ and fiber $F^\vee = e^3$.

β is now a holomorphic involution of $X^\vee$, and we can represent β as a rotation of the base $\Sigma_0^\vee$ by 180 degrees. This fixes one elliptic curve at the "north pole" and one at the "south pole".

The String Junction Picture

$X^\vee$ is again elliptically fibered, now with the base represented by $\sigma_0^\vee = e_3 - e^3$ and fiber $F^\vee = e^3$.

β is now a holomorphic involution of $X^\vee$, and we can represent β as a rotation of the base $\Sigma_0^\vee$ by 180 degrees. This fixes one elliptic curve at the "north pole" and one at the "south pole".

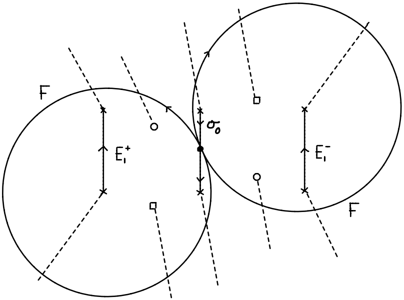
The elliptic fiber degenerates over 24 points in the base.

- String junctions between these points generate the remaining U- and E_8-lattices.
- A string junction of "charge" $[p, q]$ can start and end at a degeneration of type $[p, q]$. p and q determine the $SL(2, \mathbb{Z})$ transformation of fibre around locus.
- String junctions can be added, subtracted, deformed, etc, according to Hanany-Witten rules [Hanany-Witten '96].

We can separate $\Sigma_0^\vee$ into an "eastern" and "western" hemisphere, and go to a point in moduli space where E_8^+ lives on the westerns hemisphere, and E_8^- on the eastern.

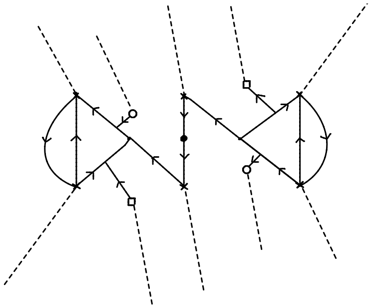
The classes σ_γ then correspond to irreducible string junctions. We illustrate this for the example $\sigma_1 = \sigma_0 + 2F + E_1^+ + E_1^-$ on the next slides. Here $E_1^\pm$ are simple roots, i.e. $\left(E_1^\pm\right)^2 = -2$. Note that $\beta(E_1^+) = -E_1^-$.

$\sigma_1 = \sigma_0 + 2F + E_1^+ + E_1^-$



- The black dot represents a fixed point of β.
- The dotted lines are branch cuts corresponding to the monodromy change.
- The circle and square represent additional unspecified degeneration loci.
- We can assume σ_0 wraps the $[p, q] = [1, 0]$ circle and F is of type $[p, q] = [0, 1]$ at the fixed point so that $F \cdot \sigma_0 = 1$.

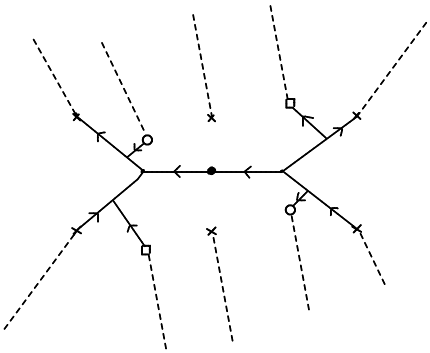
$\sigma_1 = \sigma_0 + 2F + E_1^+ + E_1^-$



- A string junction undergoes a monodromy when it encircles a degeneration locus, or equivalently crosses a branch-cut of such a locus.
- Moving a string junction crossing a branch cut across the corresponding degeneration locus produces a junction which begins (counter-clockwise) or ends (clockwise) at the locus.

$\sigma_1 = \sigma_0 + 2F + E_1^+ + E_1^-$

The final irreducible junction:



Note that this junction is inverted under β.

The Type IIB Limit

The type IIB Orbifold

Mirror symmetry to type IIB gives another VB orbifold

$$Z = (X \times T^2)\Big/\gamma \, ,$$

where γ acts as α on X, and as $z \to -z$ on T^2. Another copy of the Schoen.

The Calabi-Yau structure forms are again

$$\omega_Z = \omega_X + \mathrm{vol}(T^2)$$
$$\Omega_Z^{(3,0)} = \Omega_X^{(2,0)} \wedge dz \, .$$

The type IIB Orbifold

Mirror symmetry to type IIB gives another VB orbifold

$$Z = (X \times T^2)\Big/\gamma\,,$$

where γ acts as α on X, and as $z \to -z$ on T^2. Another copy of the Schoen.

The Calabi-Yau structure forms are again

$$\omega_Z = \omega_X + \mathrm{vol}(T^2)$$
$$\Omega_Z^{(3,0)} = \Omega_X^{(2,0)} \wedge dz\,.$$

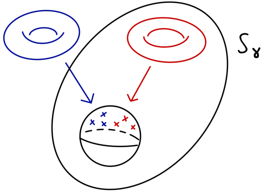
The corresponding calibrated cycles, dual to the type IIA special Lagrangians are divisors of the form

$$S_\gamma = (\Sigma_\gamma \times T^2)\Big/\gamma\,,$$

the divisors S_γ are rational elliptic surfaces at an orbifold point. Euclidean $D3$ baranes wrapping these divisors give the type IIB superpotential.

The type IIB Calabi-Yau

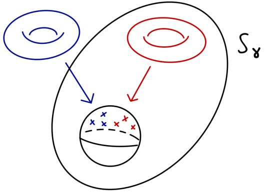
The divisors persist away from the orbifold limit. Indeed, we can describe the smooth Schoen as a double elliptic fibration:



The blue and red dots denote degeneration loci of the corresponding elliptic fibrations.

The type IIB Calabi-Yau

The divisors persist away from the orbifold limit. Indeed, we can describe the smooth Schoen as a double elliptic fibration:



The blue and red dots denote degeneration loci of the corresponding elliptic fibrations.

Fixing a holomorphic section of one elliptic fibration leaves a calibrated divisor S_γ.

Get two E_8 lattices worth of calibrated divisors. However, only one of them are invariant under the type IIB orientifold.

The orbifold limit corresponds to the point in moduli space where the degeneration loci of S_γ come together at the fixed points of γ on the base.

The F-Theory Lift

The F-Theory Four-Fold

At the The type IIB orientifold action denoted by κ acts as α on the $K3$ lattice at the VB orbifold point

$$\kappa \circlearrowleft \frac{E_8^+ \quad E_8^- \quad U_1 \quad U_2 \quad U_3}{E_8^- \quad E_8^+ \quad U_1 \quad -U_2 \quad -U_3},$$

and acts trivially on z. The type IIB orientifold is hence given by

$$B = (X \times T^2)\Big/\kappa \times \gamma = S \times \mathbb{P}^1 ,$$

where $S = X/\kappa$ is a rational elliptic surface, and $\mathbb{P}^1 = T^2/\gamma$.

The divisors S_γ reduce to surfaces of the form $\Sigma_\gamma \times \mathbb{P}^1$.

The F-Theory Four-Fold

At the The type IIB orientifold action denoted by κ acts as α on the $K3$ lattice at the VB orbifold point

$$\kappa \circlearrowleft \frac{E_8^+ \quad E_8^- \quad U_1 \quad U_2 \quad U_3}{E_8^- \quad E_8^+ \quad U_1 \quad -U_2 \quad -U_3},$$

and acts trivially on z. The type IIB orientifold is hence given by

$$B = (X \times T^2)\Big/\kappa \times \gamma = S \times \mathbb{P}^1 ,$$

where $S = X/\kappa$ is a rational elliptic surface, and $\mathbb{P}^1 = T^2/\gamma$.

The divisors S_γ reduce to surfaces of the form $\Sigma_\gamma \times \mathbb{P}^1$.

The F-theory four-fold is an elliptic fibration over B, and the corresponding calibrated divisors lift to

$$D_\gamma = E \to \Sigma_\gamma \times \mathbb{P}^1 .$$

Wrapping these divisors by Euclidean $M5$ branes produces the non-perturbative superpotential of DGW.

Conclusions and Outlook

Conclusions:

- We have given an explicit construction of an infinite number of associatives in a G_2 orbifold.
- We conjecture that the G_2 orbifold can be de-singularised, giving a smooth geometry with infinitely many associatives parametrised by an E_8 lattice.
- Euclidean $M2$ branes wrapping the associatives gives rise to a non-perturbative superpotential. This has analogs in type IIA in terms of Euclidean $D2$ branes wrapping special Lagrangians, in type IIB in terms of Euclidean $D3$ branes wrapping calibrated divisors, and in F-theory in terms of Euclidean $M5$ branes wrapping calibrated divisors.

Conclusions and Outlook

Conclusions:

- We have given an explicit construction of an infinite number of associatives in a G_2 orbifold.
- We conjecture that the G_2 orbifold can be de-singularised, giving a smooth geometry with infinitely many associatives parametrised by an E_8 lattice.
- Euclidean $M2$ branes wrapping the associatives gives rise to a non-perturbative superpotential. This has analogs in type IIA in terms of Euclidean $D2$ branes wrapping special Lagrangians, in type IIB in terms of Euclidean $D3$ branes wrapping calibrated divisors, and in F-theory in terms of Euclidean $M5$ branes wrapping calibrated divisors.

Outlook and work in progress:

- The we conjecture the existence of calibrated sub-manifolds in type IIA and M-theory. Can we make any progress towards such a de-singulairisation?
- Can we connect the (smooth) geometry and associatives to the corresponding TCS construction and associatives conjectured by [Braun etal '18]?

Thank you for your attention!