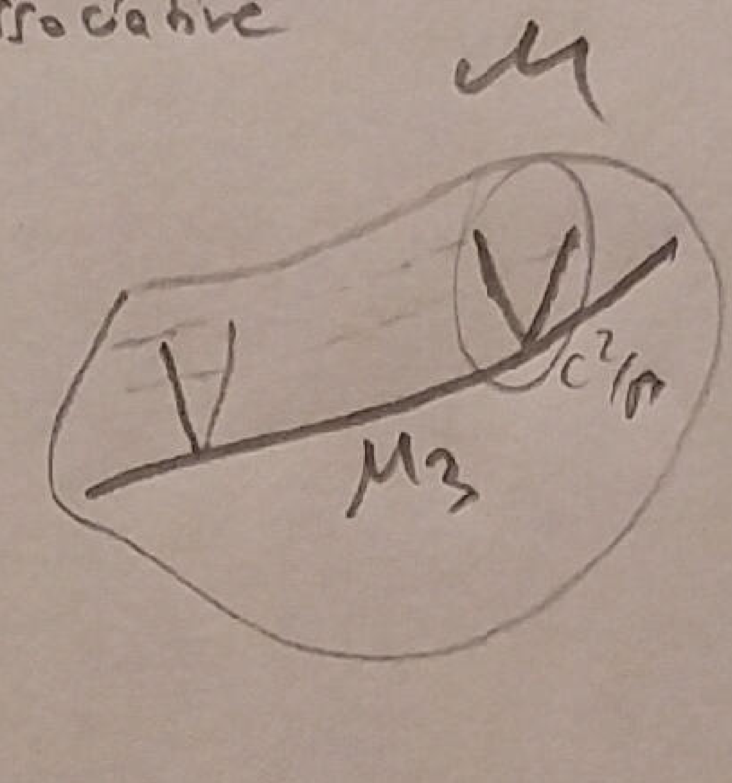
Higgs bundles & exceptional holonomy

assume associative



setup: G_2 "variety" M with singular locus M_3, transverse is $\mathbb{C}^2/\Gamma$ (partially deformed)

Γ finite subgroup of $SU(2)$

- M-theory on M gives 7D gauge theory localized on M_3
- to get started, only need SUSY 7D gauge theory

[Acharya '98]

[Pantev, Wijnholt '09]

[AB, Cizel, Hübner, Schäfer-Nameki '18]

"stack" of D6 branes

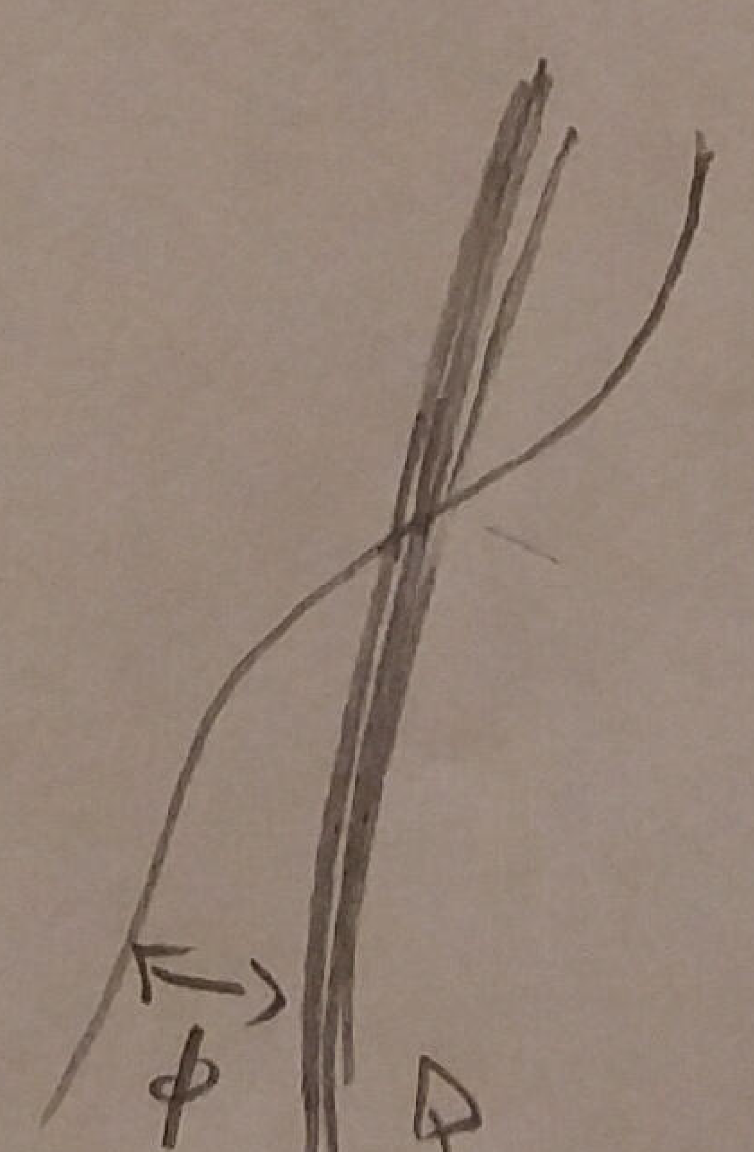
7D fields

A_μ $\mu = 1..7$

ϕ_i $i = 1..3$ $\circlearrowleft SU(2)_R$

λ fermions $\circlearrowleft SU(2)_R$

ε susy parameter $\circlearrowleft SU(2)_R$



A_μ lives here

$SO(3)_M$ breaks susy

$\to$ twist by letting

ε be section of diag$(SO(3)_M, SU(2)_R)$

$\to$ 4 real supercharges in 4D $(\mathcal{N}=1)$

$\langle \delta_\varepsilon \lambda \rangle = 0$ gauge bdl. on M_3 one form (analogue of $\to$ Lam) on M_3

$$\Rightarrow \quad F_A = i[\phi, \phi] \quad \text{on } M_3$$

$$D_A \phi = D_A^\dagger \phi = 0$$

"Hitchin eqs"

2 options

a) "flat" bundles with $F_A = 0$

b) "fluxed" bundles with $F_A \neq 0$

[Barbosa, Cvetic, Heckman, Lawrie, Torres]

a) $[\phi_i, \phi_j] = 0 \rightarrow$

can simultaneously diagonalize

ϕ_i & map data to

local geometry

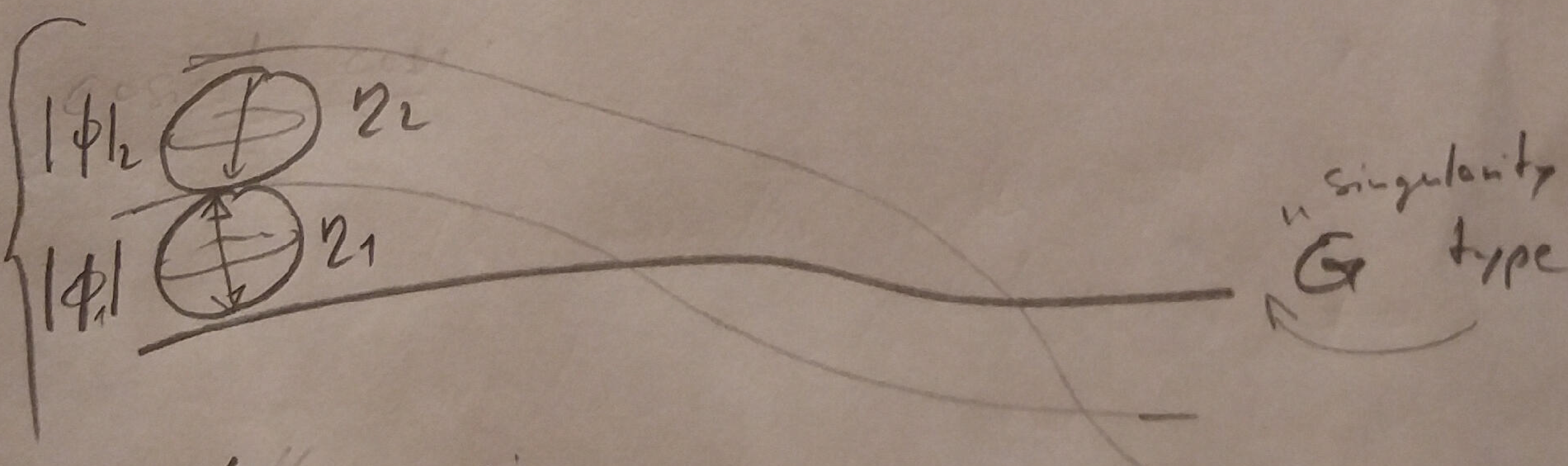
consider $\mathbb{C}^2/\tilde{\Gamma}$ $\qquad \Gamma \sim \tilde{G}$

$\phi^{\perp}$ in $\tilde{G} = G$

$(\phi_i)_\alpha = \int_{\eta_\alpha} \omega_i$ ← HK triplet of ALE space

partial desf.

η_α generators of $H^2(\widetilde{\mathbb{C}^2/\tilde{\Gamma}}, \mathbb{Z})$



setting $\phi \equiv 0$ gives singularity of $\tilde{G}$ type

<u>easiest case</u> : $\quad G^{\perp}$ in $\tilde{G} = U(1)$

let $\pi^1(M_3) = 0 \Rightarrow \omega = 0$

$d\phi = d^\dagger\phi = 0 \Rightarrow \phi = 0$

but there should be non-trivial solutions!

$\Rightarrow$ consider $\underbrace{M_3 \setminus \Sigma}$ ← tubular neighborhood of closed subset S of M_3

ϕ regular only on

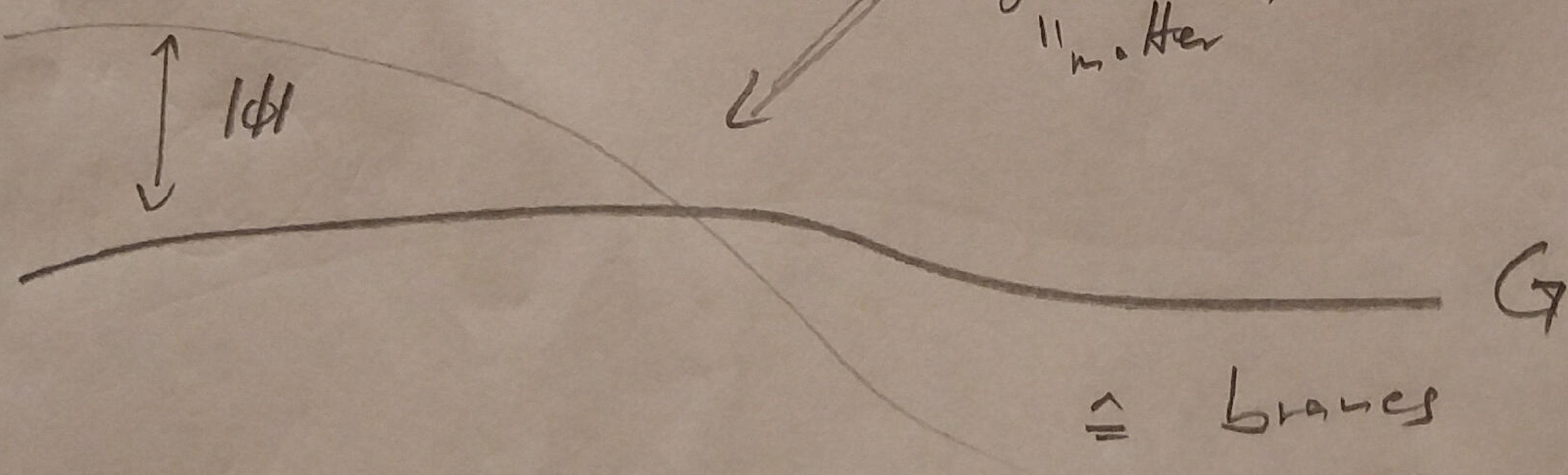
set $\phi = df$ $\quad \Delta f = \rho$

↑ localized on S

electrostatics

expect such loci to give rise to 4D "matter"

another brane



≙ branes

(e.g. D6 on SLag lifted to M-theory)

on support of ρ

$\int_\Sigma \phi \to \infty \Rightarrow$ brane moves to ∞

≙ cannot be seen close to M_3

matter $\hat{=}$ rep R of G

generic case: $\phi = 0$ over pts. of M_3
$\Longleftrightarrow$ "chiral matter" (R but not $\bar{R}$ at same time)
[Acharya, Witten '01]

$\hat{M}_3 = M_3 \setminus \Sigma$ $\qquad \Sigma = \Sigma_+ \cup \Sigma_-$

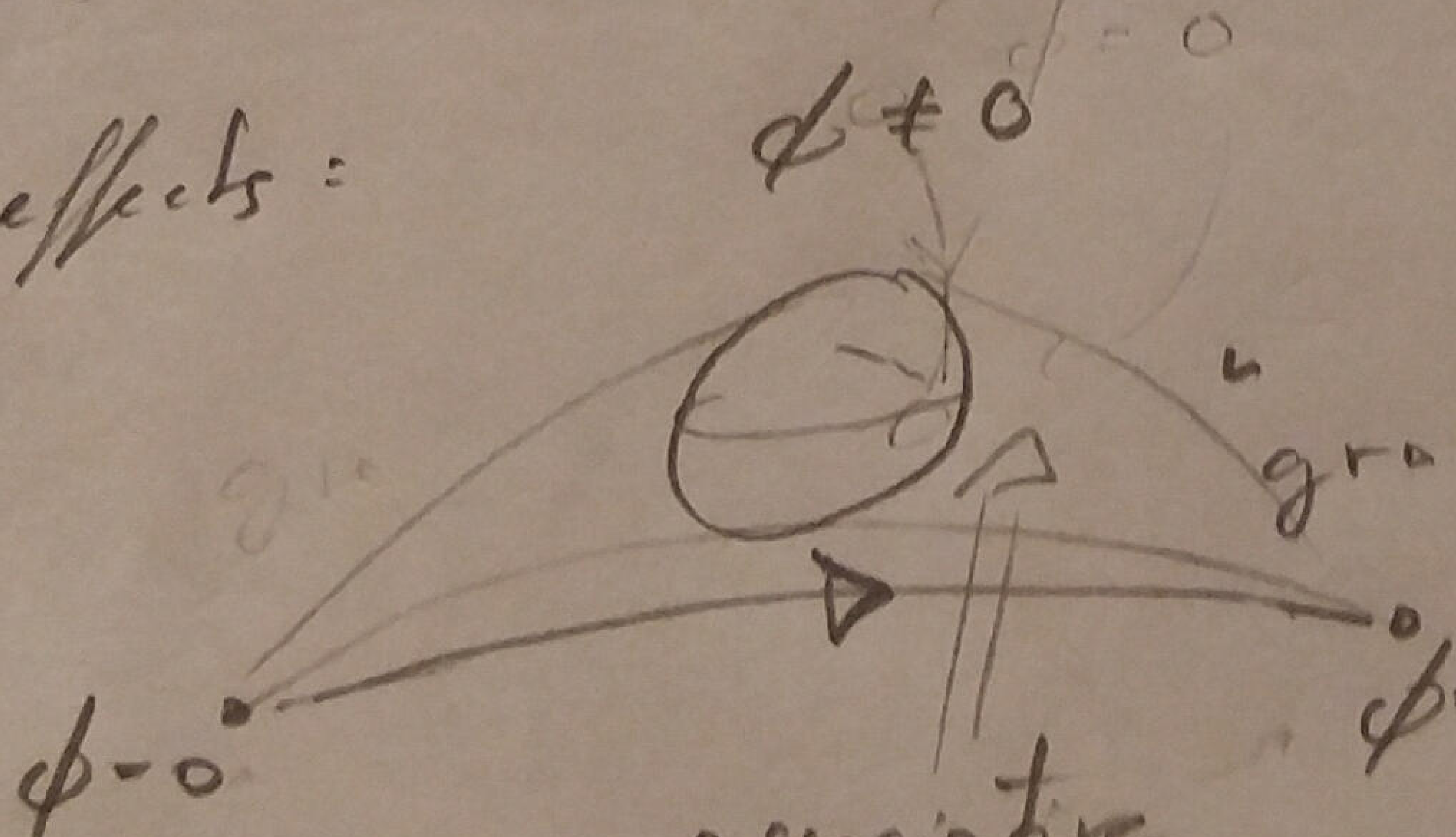
ρ pos. or neg.

(*) $R \qquad H^1(\hat{M}_3, \Sigma_-)$

$\bar{R} \qquad H^2(\hat{M}_3, \Sigma_-)$ $\qquad$ ... boundary conditions ...

relation to "enhancement loci"
$\Updownarrow$
SQM $\qquad$ "perturbative matter"

np effects:



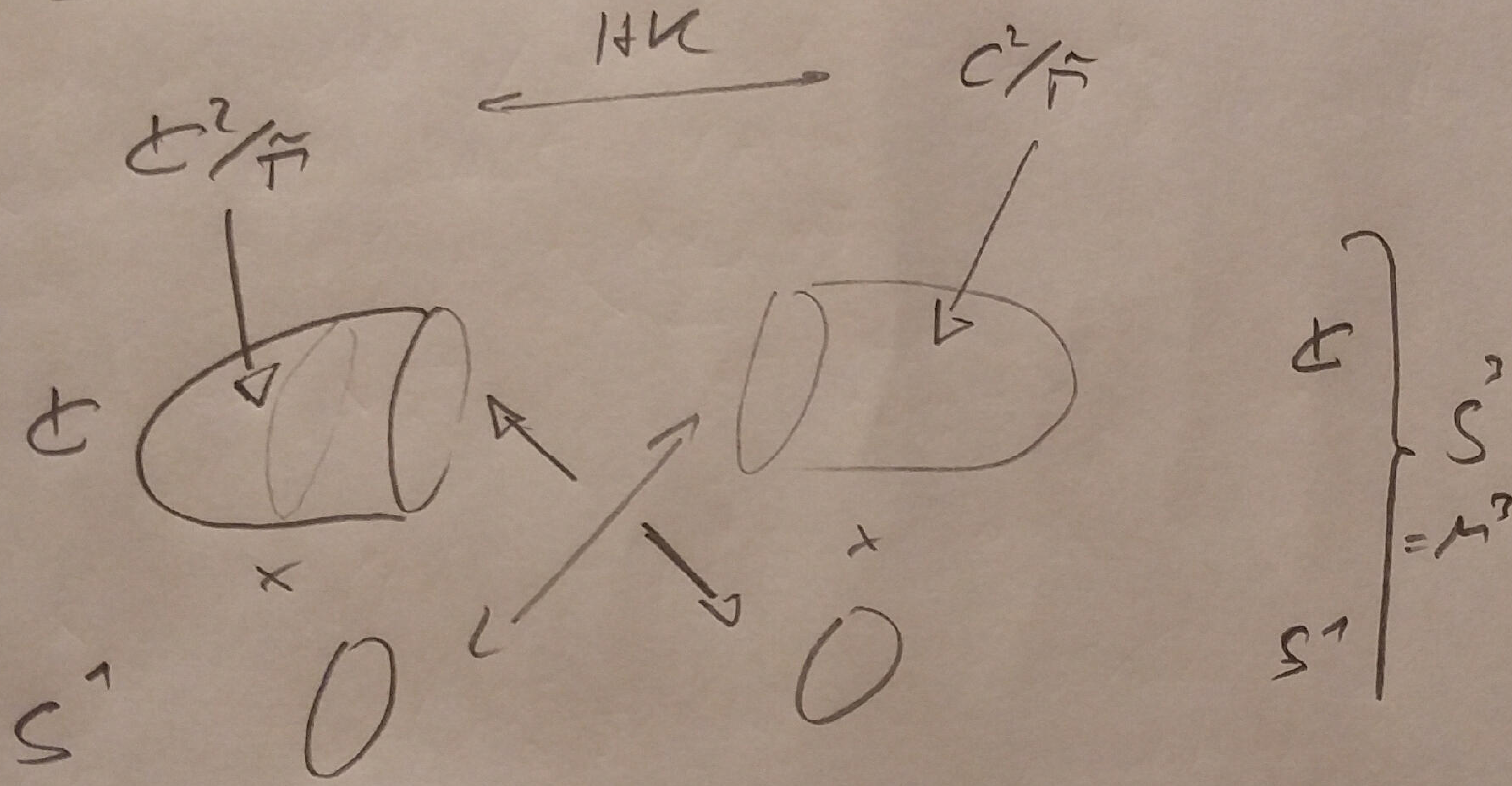
$\dot{\gamma}_i \sim \partial_i f$

S^3, mass-term from membrane instanton

$\Rightarrow$ recover (*)

generalized to "Morse – Bott" situation where $\phi = df$ vanishes along $S^1 \to$ always $R \oplus \bar{R}$

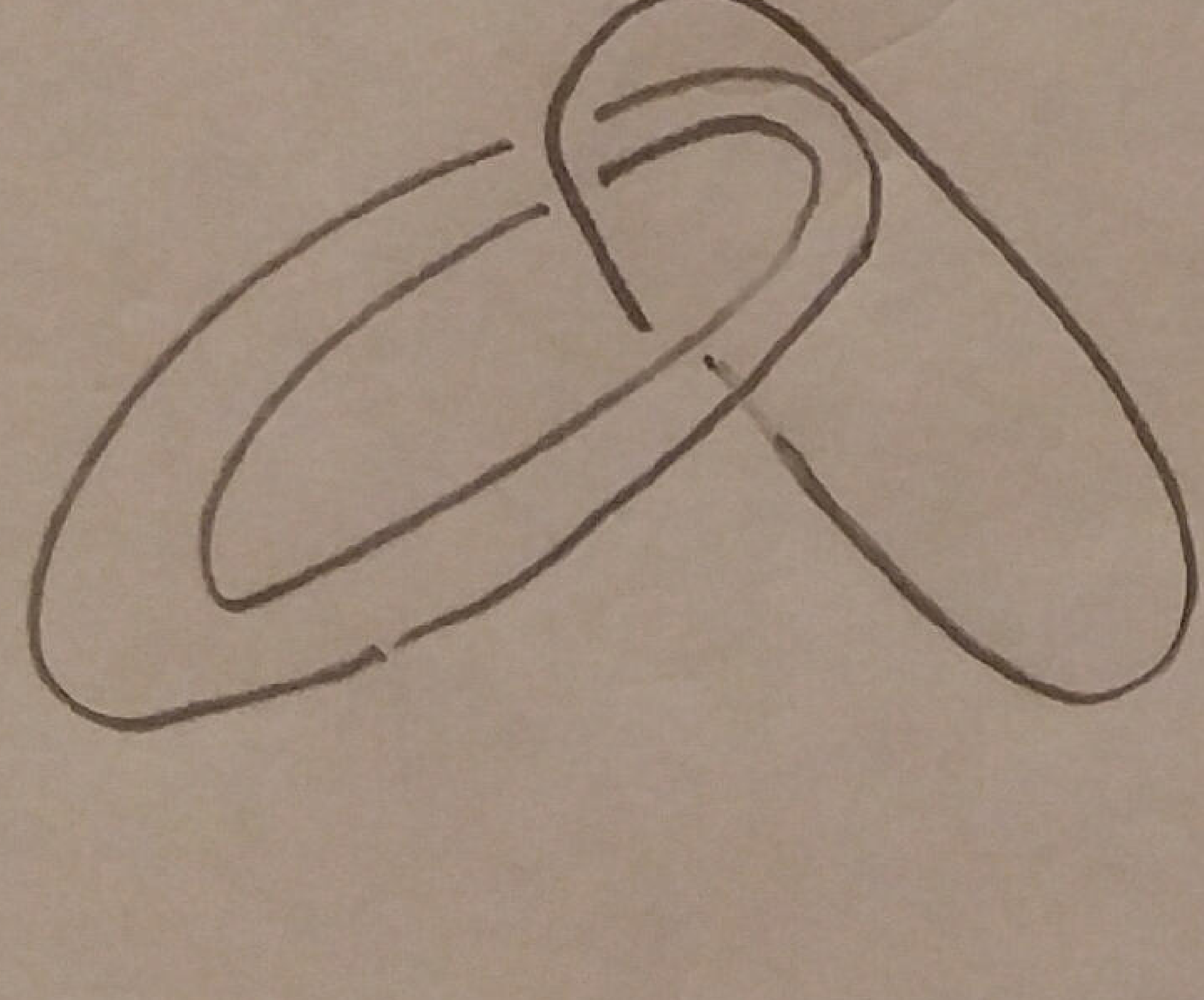
application: TCS



only ϕ_1, ϕ_2
nontrivial
$\phi_3 = \text{const.}$

ϕ_1, ϕ_3 nontrivial
$\phi_2 = \text{const}$

critical loci / charges ρ
are S^1_3, naively:



no
chirality

but gluing perturbs this situation ... (7)

however can still describe using

$$S = \bigcup_i S^1_i$$

$\Rightarrow$ no chirality after perturbation

need to "fuse" charges / destroy TCS

alternative = [Barbosa et al '19]

$F \neq 0 \Rightarrow$ T-branes

chirality without pt.-like singularities

Spin(7) [Cvetic et al. '20]

ALE $\downarrow$ M_4 Cayley

$$D_A \phi = 0$$

$$F_A + \phi \times \phi = 0$$

hierarchy:

	TCS	
$SU(3)$	$\subset G_2$	$\subset Spin(7)$
$SU(3)$	$\subset SU(4)$	$\subset Spin(7)$

"GCS"

glue Spin(7) from CY_4 & $G_2 \times S^1$ [AR, Schäfer-Nameki] issues ⊗ so far: $U(1)^n = G^\perp \subset \tilde{G}$

spectral covers are nice for non-abelian bundles

$$\det[\mathbb{1} \cdot t - \phi(x)]$$

describes cover of M_4 in F-theory

G_2 version? Some progress: [Hübner] [Haydys]

Spin(7): $CY_4 / \mathbb{Z}_2$ might be useful

$\Uparrow$

resolution gives G_2 part

⊗ moduli spaces?

⊗ compact G_2?