New G_2-manifolds and Their Physical Interpretation

Physics of Codimension Six Singularities in G_2-holonomy Spaces

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5d Superconformal Field Theories and Singular spaces of SU(3) holonomy

- Work of Seiberg ('96) showed the existence of non-trivial 5d theories with supersymmetry and conformal symmetry
- D. Morrison and N. Seiberg: interpret these as M-theory on a singular (eg conical) Calabi-Yau threefold

certain singular CY 3-folds $\longleftrightarrow$ 5d SCFT's

- These theories are mysterious and have exotic matter and global symmetries
- Important to try and classify them in some way

Work in this collaboration along these lines

- S. Schafer-Nameki and Collaborators (Lin, Wang, Apruzzi, Eckhard, Lawrie, Bhardwaj, Closset)

- Other groups also
  C. Vafa and collaborators
  J. Heckman, M. Del Zotto . . .

Such "Calabi-Yau singularities" can arise naturally inside G_2-holonomy spaces and a VERY broad question is

$$\boxed{\text{What is the physics of codimension six singularities in } G_2\text{-holonomy spaces?}}$$

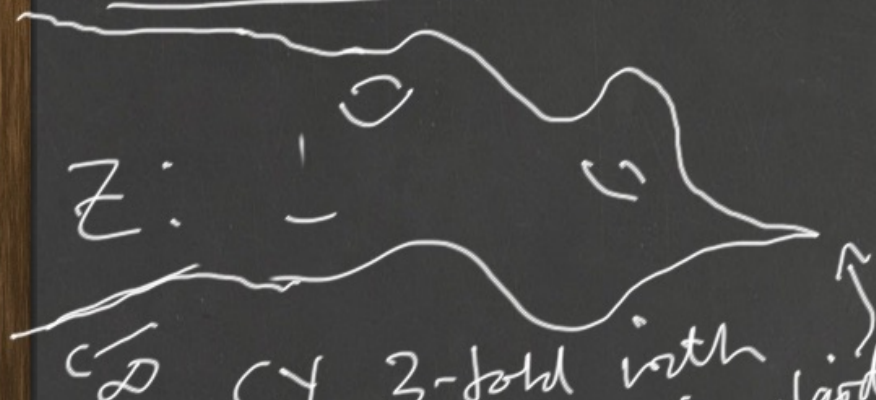
This will take some years to explore. Here I will make some general remarks, introduce a (large) class of examples and study one particular case.

Based on BSA, to appear

BSA + J. Lotay, in progress

Also BSA, J. Heckman, M. Del Zotto
to appear.

5d SCFT's from CY 3-folds

Z:  $\longleftrightarrow$ 5d SCFT on $\mathbb{R}^{4,1}$

CY 3-fold with codim six singularity

M theory on $Z \times \mathbb{R}^{4,1}$

Symmetries: $G \times Spin(4,1)$ plus $N=1$ $d=5$ supersymmetry and conformal symm

G = global symmetry of theory

G: codimension four singularities of Z, supported on non-compact curves.

(compactly supported ADE sing give <u>gauge</u> symmetries).

What if this singularity occurs inside a G_2-holonomy space X?

M-theory on $X \times \mathbb{R}^{3,1}$

General Properties :

$5d\ N=1$ SUSY $\longrightarrow$ $4d\ N=1$ SUSY

$Spin(4,1) \longrightarrow Spin(3,1)$

$G \longrightarrow G$? DEPENDS ON DETAILS

Consider our 5d theory, but on a spacetime with only 4d Lorentz symmetry:

5d SCFT on	$\mathbb{R} \times \mathbb{R}^{3,1}$	0
	$\mathbb{R}^{+} \times \mathbb{R}^{3,1}$	1
	$S^1 \times \mathbb{R}^{3,1}$	2
	$I \times \mathbb{R}^{3,1}$	3

- We will consider case 1 and 3 here
- In general, details depend on the metric on the 5-manifold and

this is determined by the G_2-hol metric in the UV description.

- For simplicity, stick to a flat metric here.

- Paper with Heckman / Del Zotto considers "warped products".

For flat spacetime we have:

0 $\mathbb{R} \times \mathbb{R}^{3,1} = \mathbb{R}^{4,1}$ with full symmetry.

1 $\mathbb{R}^+ \times \mathbb{R}^{3,1} = \frac{\mathbb{R}}{\mathbb{Z}_2} \times \mathbb{R}^{3,1}$

2 $S^1 \times \mathbb{R}^{3,1} = S^1 \times \mathbb{R}^{3,1}$ is just reduction of 5d theory on S^1 has $N=2$ susy

3 $I \times \mathbb{R}^{3,1} = \frac{S^1}{\mathbb{Z}_2} \times \mathbb{R}^{3,1}$

$\Rightarrow$ Stick to cases 1 and 3.

Q: How Does $\mathbb{Z}_2$ act on states in the theory?

By assumption we want to preserve $N=1$ 4d susy (G_2).

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A: $\mathbb{Z}_2$ = CP symmetry of 5d theory

$P: y \to -y$ $\quad$ C = charge conjugation

CP symmetry

C takes particles in rep^n R to $\bar{R}$

P reflects some number of space coords.

CP is violated in nature leading to subtle differences between particles and anti particles.

1) $\dfrac{\mathbb{R}}{\mathbb{Z}_2} \times \mathbb{R}^{3,1} \Longleftrightarrow$ M-theory on $X = \dfrac{\mathbb{R} \times Z}{\mathbb{Z}_2}$

3) $\dfrac{S^1}{\mathbb{Z}_2} \times \mathbb{R}^{3,1} \Longleftrightarrow$ M-theory on $X = \dfrac{S^1 \times Z}{\mathbb{Z}_2}$

If (Z, ω, Ω) is CY metric then X has G_2-structure $\varphi = dy \wedge \omega + \mathrm{Re}\,\Omega$

$$\mathbb{Z}_2 : (y, Z) \longrightarrow (-y, \sigma(Z))$$

σ : real structure on Z

$\sigma : (\omega, \Omega) \longrightarrow (-\omega, \overline{\Omega})$.

The fact that σ acts as "complex conjugation" naturally identifies it as a "charge conjugation operator" in 5d theory.

In particular σ reverses the orientation of Z.

· Obviously require that (Z, ω, Ω) has a real structure σ.

- Such G_2-orbifolds were considered by Joyce and Joyce-Karigiannis in compact setting.
- Here we are interested in both non-compact and compact setting

Using this geometric embedding, we have the following general results:

In the 5d theories from Z, key role is played by compact, holomorphic divisors $D \subset Z$ which collapse at the singularity. M5-branes wrapping such D's give BPS strings in 5d whose mass/length $\rightarrow 0$.

Tensionless Strings in 4d

G : typically will act as a real structure on D, which therefore preserves $[D]$.

- Hence D is coassociative in X.
- Hence, such strings are also BPS in 4d theory with $M/L \to 0$
- Hence, we expect an interesting 4d theory.

General Properties continued . . . .

SU(2) global symmetry.

In general, Z^6 = special Lagrangian submanifold $\subset Z$
L

Along L, X has a codim four A_1-singularity $\Rightarrow$ 4d theory has SU(2) global symmetry! (at $(0, L) \subset (y, Z)$)

- If eg $L \cap D \neq \emptyset$ we might expect something interesting to happen. ?

A (large) Class of Examples: $\frac{\mathbb{R}^7}{\mathbb{Z}_2 \ltimes \Gamma}$

- Here $\dfrac{\mathbb{R}^7}{\mathbb{Z}_2 \ltimes \Gamma} = \dfrac{\left(\mathbb{R} \times \frac{\mathbb{C}^3}{\Gamma}\right)}{\mathbb{Z}_2}$ FLAT $\mathcal{Q}$.

Γ = <u>Any</u> finite subgroup of $SU(3)$

$\mathbb{Z}_2 : (y, z_{1,2,3}) \longrightarrow (-y, z^*_{1,2,3})$

$\mathbb{Z}_2 \ltimes \Gamma$ is clearly a finite subgroup of G_2, for all Γ.

- $\frac{\mathbb{C}^3}{\Gamma}$ admits (at least one) crepant resolution $Z_\Gamma \cong \widehat{\frac{\mathbb{C}^3}{\Gamma}}$ (S.S. Roan) ⋮ Ito, Reid, . . .

- $\frac{\mathbb{C}^3}{\Gamma}$ has codim six conical singularity and this gives a 5d SCFT.

- Only well studied for Abelian Γ's (Some non-Abelian cases being studied in (BSA, Lambert, Najjar, Svanes)

- Resolving $\mathbb{C}^3/\Gamma$ to $\mathbb{Z}_\Gamma$ physically means we are on the Coulomb branch, where a weakly coupled IR gauge theory description exists

Question: What is the physics of M theory on $\dfrac{\mathbb{R}^7}{G'}$ $\quad G' \subset G_2$?
finite

Question: When can we desingularise $\dfrac{\mathbb{R}^7}{\mathbb{Z}_2 \times \Gamma}$ within G_2-holonomy?

Example (w J. Lotay)

$\Gamma = \mathbb{Z}_3$ = centre of $SU(3)$

$\widehat{\mathbb{C}^3/\mathbb{Z}_3} = \mathcal{O}(-3)|_{\mathbb{CP}^2}$ and admits a (highly symmetric) complete metric of $SU(3)$ holonomy, due to Calabi.

. Metric is Asymptotically Conical

$$g_{r\to\infty} \simeq dr^2 + r^2 d\Omega_5^2 \quad \mathbb{R}^+ \times S^5/\mathbb{Z}_3$$

$L \cong Z^6_r = \mathcal{O}(-3)|_{\mathbb{CP}^2}{}^6 = \mathcal{O}(-3)_{\mathbb{RP}^2}$

$\mathcal{O}(-3)_{\mathbb{RP}^2} = \dfrac{\mathbb{R} \times S^2}{\mathbb{Z}_2}$ ($\mathbb{Z}_2$ is not our involution)

We have $\hat{A}_1$-singularity along $\dfrac{\mathbb{R} \times S^2}{\mathbb{Z}_2}$.

<u>Joyce-Karigiannis</u>: if Z^6 (compact) has <u>nowhere zero</u> harmonic 1-form (or $\mathbb{Z}_2$-twisted)

$\Rightarrow$ can resolve singularities by gluing in $M^4_{EH} \times L$ (or $\dfrac{M^4 \times \hat{L}}{\mathbb{Z}_2}$) where M^4_{EH} is T^*S^2.

J-K is in compact case.

However, if 1-form α in non-compact case is L^2-normalisable and decays sufficiently fast, their results should also go through.

However, now have to ensure that (L, g_L) has appropriate behaviour at ∞ $\Rightarrow$ $g_L \to$ ALE at ∞.

Remarkably, this turns out to be true for $Z_{\mathbb{Z}_3} = \mathcal{O}(-3)|_{\mathbb{CP}^2}$ with Calabi metric!

- (L, g_L) admits a nowhere zero, $\mathbb{Z}_2$-twisted harmonic 1-form, with fast decay at ∞
- g_L is flat at ∞

$\Rightarrow$ This will give rise to . .

two new G_2-holonomy manifolds

$$X_1 = \frac{\mathbb{R} \times \mathcal{O}(-3)|_{\mathbb{CP}^2}}{\mathbb{Z}_2} \quad \text{and} \quad X_2 = \frac{S^1 \times \mathcal{O}(-3)_{\mathbb{CP}^2}}{\mathbb{Z}_2}$$

$b^2(X) \sim 0 \quad b^3(X_1) = 2 \qquad b^2(X_2) = 0 \quad b^3(X_2) = 3.$

In general one can consider, for any finite $\Gamma \subset SU(3)$,

$$\frac{(\mathbb{R} \times Z_\Gamma)}{\mathbb{Z}_2} = \frac{\left(\mathbb{R} \times \widetilde{\frac{\mathbb{C}^3}{\Gamma}}\right)}{\mathbb{Z}_2}$$

and would like to know about the G_2-holonomy metrics and possible desingularisations. But this of course requires a better understanding of Calabi-Yau metrics on $\widetilde{\frac{\mathbb{C}^3}{\Gamma}}$.

Compact case: a surprise

- Instead of $\dfrac{\left(S^1 \times \frac{\mathbb{C}^3}{\mathbb{Z}_3}\right)}{\mathbb{Z}_2}$, can

consider $\dfrac{\left(S^1 \times \frac{T^6}{\mathbb{Z}_3}\right)}{\mathbb{Z}_2}$

where $T^6 = \left(\frac{\mathbb{C}}{\Lambda}\right)^3$ $\quad \Lambda$ = Hexagonal Lattice

- Locally the singularities are all either modelled on $\frac{\mathbb{C}^3}{\mathbb{Z}_3}$ or $\frac{\left(\mathbb{R}\times\mathbb{C}^3/\mathbb{Z}_3\right)}{\mathbb{Z}_2}$ which we know how to resolve

- <u>But</u> for the latter we still require a nowhere zero harmonic 1-form

· In fact in $\left(\dfrac{S^1 \times \widetilde{\dfrac{T^6}{\mathbb{Z}_3}}}{\mathbb{Z}_2} \right)$, the singularity at the origin is along

$$L = \underbrace{T^3 \# \frac{S^1 \times S^2}{\mathbb{Z}_2}}$$

In local case this was $\mathbb{R}^3 \# \dfrac{\mathbb{R} \times S^2}{\mathbb{Z}_2} \cong \mathcal{O}(-3)_{\mathbb{RP}^2}$

So to use Joyce-Karigiannis need a nowhere zero harmonic 1-form on

$$T^3 \# \frac{S^1 \times S^2}{\mathbb{Z}_2}$$

General topological considerations show that extending our local model 1-form into the T^3 directions will <u>always</u> produce a zero.

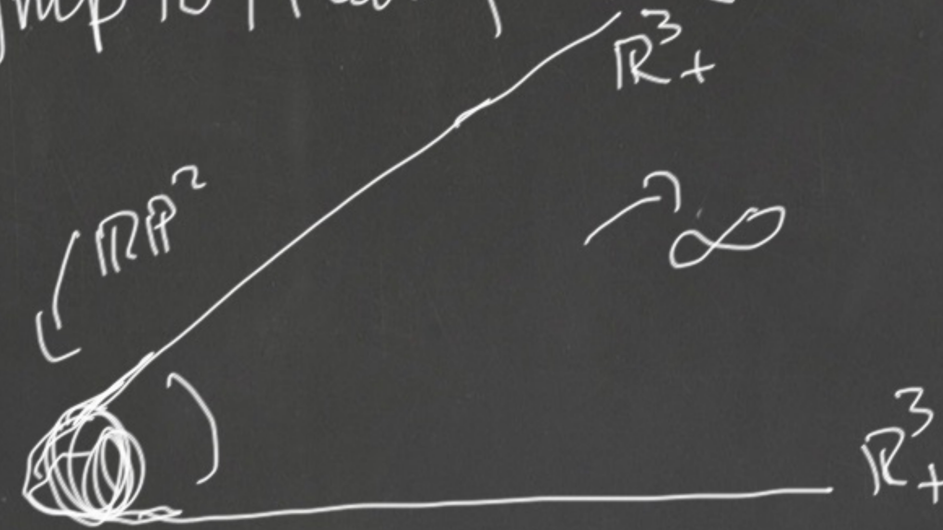
- If this zero is "generic", it will give a codimension 7 singularity with topology $\mathbb{R}^+ \times \mathbb{CP}^3$
- If one were able to have a controlled model for this one might be able to produce compact G_2-holonomy spaces with codimension 7 singularities.

A Physical Interpretation

The fixed pt set of σ in $\mathbb{C}^3/\mathbb{Z}_3$ is $\mathbb{R}^3$. But $\mathbb{R}^3 = \mathbb{R}^3_+ \cup \mathbb{R}^3_+$ with a <u>conical</u> singularity at $\{0\}$.

Resolving to $O(-3)_{\mathbb{CP}^2}$, replaces $\{0\}$ in $\mathbb{R}^3$ by $\mathbb{RP}^2$ ie a real blow up

But asymptotically we have:

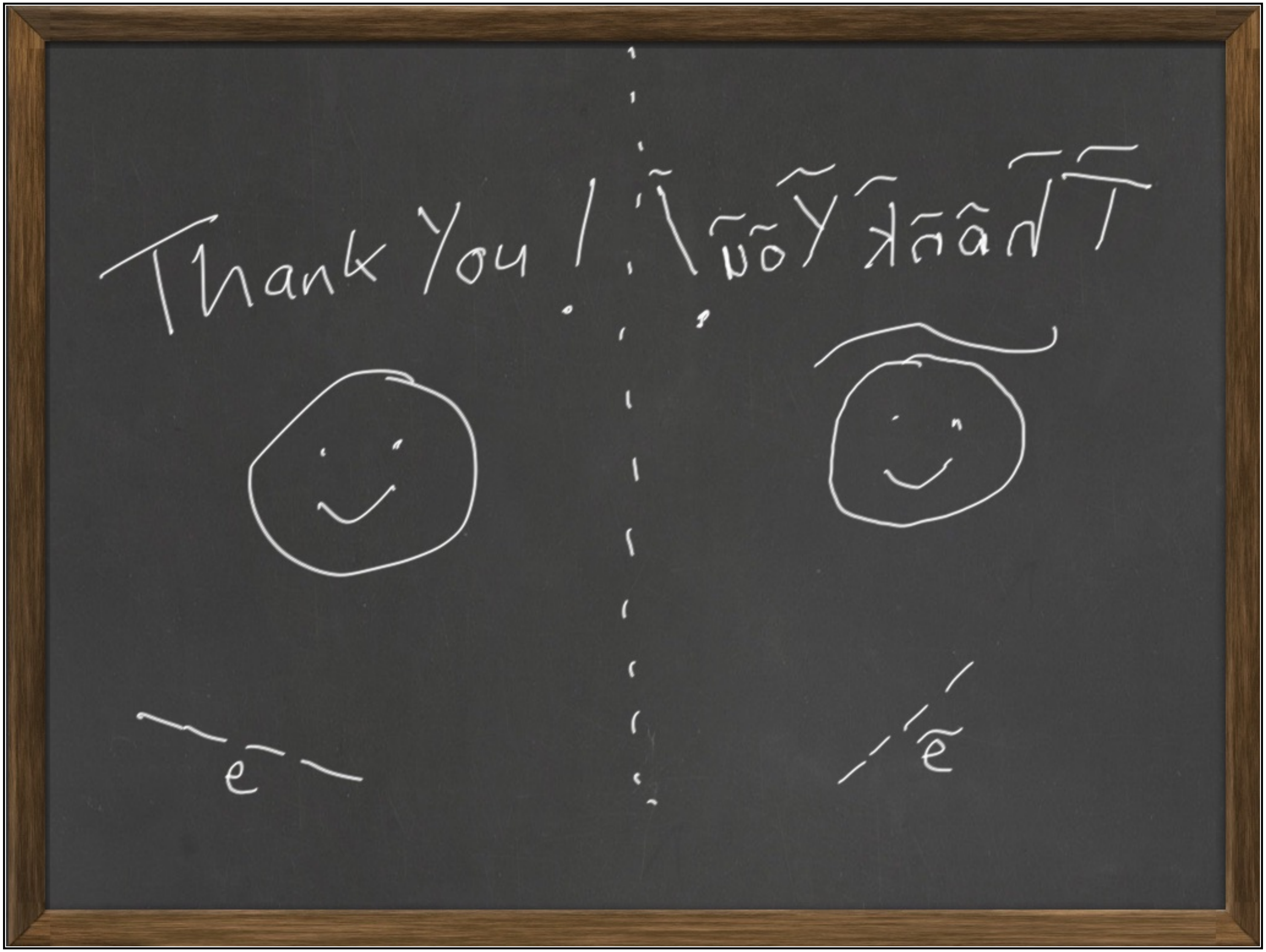


The two ends can be interpreted as D6-branes in Type IIA theory (plus O6-planes)

In the compact model, the 2 D6-brane ends have no choice but to intersect somewhere and this produces <u>chiral</u> fermions and codimension 7 singularity

- So this (and examples like it) could be promising for understanding better codimension 7 singularities in G_2-holonomy.

- In general, understanding codimension six singularities in G_2-hol spaces seems both physically and mathematically interesting.


Thank You !